

## অনুশীলনী - 8.1

প্রশ্ন 1.  $\triangle ABC$  ত্রিভুজৰ  $B$  কোণ সমকোণ আৰু  $AB = 24\text{cm}$ ,  $BC = 7\text{cm}$  হ'লে তলত দিয়াবিলাক উলিওৱা।

(i)  $\sin A$ ,  $\cos A$                       (ii)  $\sin C$ ,  $\cos C$

সমাধান :

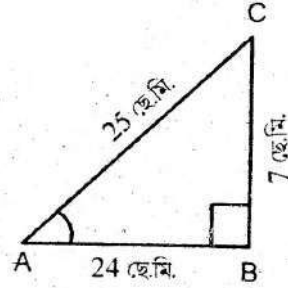
দিয়া আছে :  $AB = 24\text{cm}$ ,  $BC = 7\text{cm}$ ,  $\angle B = 90^\circ$

$$\therefore AC^2 = AB^2 + BC^2 \text{ (পিথাগোৰাচৰ সূত্রমতে)}$$

$$\Rightarrow AC^2 = (24)^2 + (7)^2$$

$$\Rightarrow AC^2 = 576 + 49 = 625$$

$$\Rightarrow AC = \sqrt{625} = 25\text{cm.}$$



$$(i) \sin A = \frac{BC}{AC} = \frac{7\text{cm}}{25\text{cm}} = \frac{7}{25} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{(উত্তৰ)}$$

$$\cos A = \frac{AB}{AC} = \frac{24\text{cm}}{25\text{cm}} = \frac{24}{25}$$

$$(ii) \sin C = \frac{AB}{AC} = \frac{24\text{cm}}{25\text{cm}} = \frac{24}{25} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{(উত্তৰ)}$$

$$\cos C = \frac{BC}{AC} = \frac{7\text{cm}}{25\text{cm}} = \frac{7}{25}$$

প্রশ্ন 2. চিত্র 8.13 ৰ পৰা  $\tan P - \cot R$  নিৰ্ণয় কৰা।

সমাধান :

$$PR = 13\text{cm}, PQ = 12\text{cm}$$

$$\therefore PR^2 = PQ^2 + QR^2$$

$$\Rightarrow (13)^2 = (12)^2 + (QR)^2$$

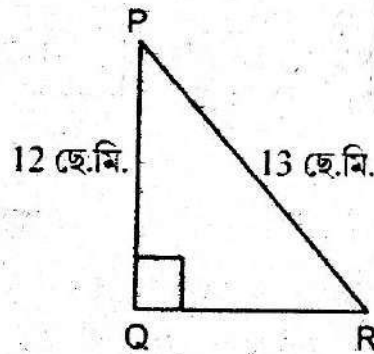
$$\Rightarrow QR^2 = 169 - 144 = 25$$

$$\Rightarrow QR = \sqrt{25} = 5\text{cm}$$

$$\therefore \tan P = \frac{RQ}{QP} = \frac{5}{12}$$

$$\therefore \cot R = \frac{RQ}{PQ} = \frac{5}{12}$$

$$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$



প্রশ্ন 3. যদি  $\sin A = \frac{3}{4}$ , তেন্তে  $\cos A$  আৰু  $\tan A$  উলিওৱা।

সমাধান :

$ABC$  এটা সমকোণী ত্ৰিভুজ।  $\angle B = 90^\circ$

$$\therefore \sin A = \frac{3}{4} = \frac{BC}{AC} = k \text{ ধৰা হ'ল। [সমানতাৰ স্ক্ৰবক]}$$

$$\therefore BC = 3k, AC = 4k$$

$$\therefore AC^2 = AB^2 + BC^2 \text{ (পিথাগোৰাচৰ সূত্ৰমতে)}$$

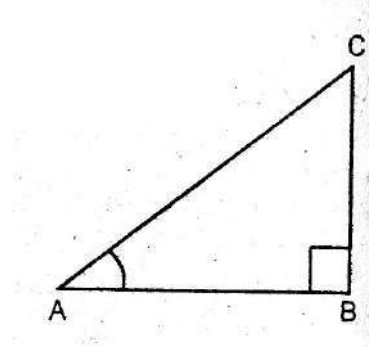
$$\Rightarrow (4k)^2 = AB^2 + (3k)^2$$

$$\Rightarrow 16k^2 - 9k^2 = AB^2$$

$$\Rightarrow AB^2 = 7k^2$$

$$\Rightarrow AB = \sqrt{7k}$$

$$\left. \begin{aligned} \therefore \cos A &= \frac{AB}{AC} = \frac{\sqrt{7k}}{4k} = \frac{\sqrt{7}}{4} \\ \tan A &= \frac{BC}{AB} = \frac{3k}{\sqrt{7k}} = \frac{3}{\sqrt{7}} \end{aligned} \right\} \text{(উত্তৰ)}$$



প্রশ্ন 4. দিয়া আছে যে,  $15 \cot A = 8$ , তেন্তে  $\sin A$  আৰু  $\sec A$  উলিওৱা।

সমাধান :

$ABC$  এটা সমকোণী ত্ৰিভুজ।  $\angle B = 90^\circ$

$$\therefore 15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15} = \frac{AB}{BC} = k \text{ ধৰা হ'ল। } k \rightarrow \text{[সমানতাৰ স্ক্ৰবক]}$$

$$\therefore AB = 8k, BC = 15k$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (8k)^2 + (15k)^2$$

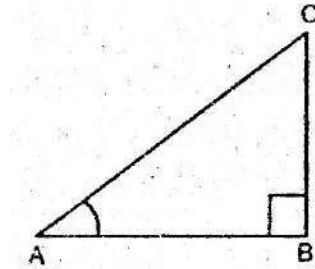
$$\Rightarrow AC^2 = 64k^2 + 225k^2$$

$$\Rightarrow AC^2 = 289k^2$$

$$\Rightarrow AC = \sqrt{289k^2}$$

$$\Rightarrow AC = 17k$$

$$\left. \begin{aligned} \therefore \cos A &= \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17} \\ \sec A &= \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8} \end{aligned} \right\} \text{(উত্তৰ)}$$



প্রশ্ন 5. দিয়া আছে যে,  $\sec \theta = \frac{13}{12}$ , আন ত্রিকোণমিতিক অনুপাতবোর গণনা কৰা ।

সমাধান :

$ABC$  এটা সমকোণী ত্ৰিভুজ  $\angle B = 90^\circ$ ,  $\angle ABC = \theta$  (সূক্ষকোণ)

$$\Rightarrow \sec \theta = \frac{13}{12} = \frac{AC}{AB} = k \text{ ধৰা হ'ল } | k \rightarrow \text{এটা [স্ফৰক]}$$

$$\therefore AC = 13k, AB = 12k$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow (13k)^2 = (12k)^2 + BC^2$$

$$\Rightarrow 169k^2 = 144k^2 + BC^2$$

$$\Rightarrow BC^2 = 169k^2 - 144k^2$$

$$\Rightarrow BC^2 = 25k^2$$

$$\Rightarrow BC = \sqrt{25k^2} = 5k$$

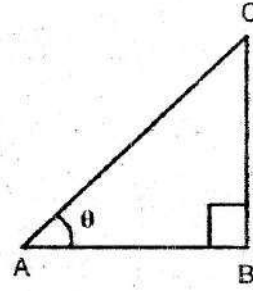
$$\therefore \sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\sin \theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

$$\cot \theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$



প্রশ্ন 7. যদি তেল্হ মান উলিওৰা ।

$$(i) \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)} \quad (ii) \cot^2 \theta$$

সমাধান :

(i)  $\angle ABC = \theta$  [সূক্ষকোণ] |  $ABC$  এটা সমকোণী ত্ৰিভুজৰ  $\angle C = 90^\circ$

$$\therefore \cot \theta = \frac{7}{8} \text{ [প্রদত্ত]}$$

$$\Rightarrow BC \cot \theta = \frac{7}{8} = \frac{BC}{AC}$$

$$\therefore \frac{BC}{AC} = \frac{7}{8} = k \text{ ধৰা হ'ল } | k \rightarrow \text{এটা [স্ফৰক]}$$

$$\therefore BC = 7k, AC = 8k$$

$$\therefore AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$\Rightarrow AB^2 = 113k^2$$

$$\Rightarrow AB = \sqrt{113k^2} = \sqrt{113}k$$

$$\therefore \sin \theta = \frac{AC}{AB} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

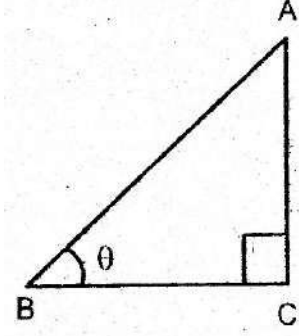
$$\cos \theta = \frac{BC}{AB} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

$$\therefore = \frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$$

$$= \frac{1-\sin^2 \theta}{1-\cos^2 \theta}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64} = \frac{49}{113} \times \frac{113}{64} = \frac{49}{64} \quad (\text{Ans.})$$



$$(ii) \therefore \cot \theta = \frac{7}{8} \quad [\text{প্রদত্ত}]$$

$$\Rightarrow \cot^2 \theta = \left(\frac{7}{8}\right)^2$$

$$\Rightarrow \cot^2 \theta = \frac{49}{64} \quad (\text{Ans.})$$

প্রশ্ন 9.  $\triangle ABC$  ব  $B$  কোণ সমকোণ। যদি  $\tan A = \frac{1}{\sqrt{3}}$  তেন্তে তলৰ মান বিলাক উলিওৱা -

$$(i) \sin A \cos C + \cos A \sin C \quad (ii) \cos A \cos C - \sin A \sin C$$

সমাধান :

$ABC$  এটা সমকোণী ত্ৰিভুজৰ  $\angle B = 90^\circ$

$$\angle A = \theta \quad [\text{সূক্ষকোণ}]।$$

$$\therefore \tan A = \frac{1}{\sqrt{3}} \quad [\text{দিয়া আছে}]$$

$$\therefore \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}} = k \text{ ধৰা হ'ল। } k \rightarrow \text{এটা } [\text{স্ফৰক}]$$

$$\therefore BC = k, AB = \sqrt{3}k$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2 = 4k^2$$

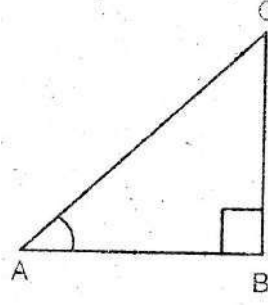
$$\Rightarrow AC = \sqrt{4k^2} = 2k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$



$$(i) \sin A \cos C + \cos A \sin C$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$$(ii) \cos A \cos C + \sin A \sin C$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0 \text{ (উত্তর)}$$

প্রশ্ন 10.  $\Delta PQR$  কোণ  $Q$  সমকোণ আৰু  $PR + QR = 25\text{cm}$  আৰু  $PQ = 5\text{cm}$ ;  $\cos P$  আৰু  $\tan P$  ৰ মান উলিওৱা।

সমাধান :

$PQR$  এটা সমকোণী ত্ৰিভুজৰ।  $\angle Q = 90^\circ$  আৰু  $\angle P \rightarrow$  সূক্ষকোণ।

$$PR + QR = 25\text{cm} \text{ আৰু } PQ = 5\text{cm}.$$

$\therefore QR$  এটা সমকোণী ত্ৰিভুজৰ পৰা পাওঁ -

$$PR^2 = QR^2 + PQ^2$$

$$\Rightarrow PR^2 = (25 - PR)^2 + (5)^2$$

$$\Rightarrow PR^2 = 625 - 50PR + PR^2 + 25$$

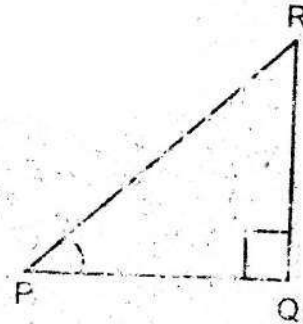
$$\Rightarrow PR^2 - PR^2 = 650 - 50PR$$

$$\Rightarrow 0 = 650 - 50PR$$

$$\Rightarrow 50PR = 650$$

$$\Rightarrow PR = \frac{650}{50} = 13\text{cm}.$$

$$\therefore QR = 25 - PR = 25 - 13 = 12\text{cm}$$



$$\left. \begin{aligned} \therefore \sin P &= \frac{QR}{PR} = \frac{12\text{cm}}{13\text{cm}} = \frac{12}{13} \\ \therefore \cos P &= \frac{PQ}{PR} = \frac{5\text{cm}}{13\text{cm}} = \frac{5}{13} \\ \therefore \tan P &= \frac{QR}{PQ} = \frac{12\text{cm}}{5\text{cm}} = \frac{12}{5} \end{aligned} \right\} \text{(উত্তৰ)}$$

অনুশীলনী - 8.2

প্ৰশ্ন 1. তলত দিয়া বিলাকৰ মান উলিওৱা -

$$(i) 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$(ii) \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} \quad (iii) \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$(iv) \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

সমাধান : (i)  $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 \times 1 + \frac{3}{4} - \frac{3}{4} = 2 \quad \text{(উত্তৰ)}$$

সমাধান : (ii)  $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}$$

$$\frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{\sqrt{2} \times \sqrt{3}(\sqrt{3}-1)}{2\sqrt{2} \times \sqrt{2}(3-1)}$$

$$= \frac{\sqrt{6}(\sqrt{3}-1)}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

সমাধান : (iii)  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$\begin{aligned}
&= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} \\
&= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} = \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} \\
&= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11} \quad (\text{উত্তর})
\end{aligned}$$

সমাধান : (iv)  $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$\begin{aligned}
&= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \left(\frac{1}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}} \\
&= \frac{\frac{5}{4} + \frac{16}{3} - 1}{1} = \frac{15+64-12}{12} = \frac{67}{12} \quad (\text{উত্তর})
\end{aligned}$$

প্রশ্ন 3. যদি  $\tan(A+B) = \sqrt{3}$  আৰু  $\tan(A-B) = \frac{1}{\sqrt{3}}$ ;  $0^\circ < A+B \leq 90^\circ$ ;  $A > B$ , তেন্তে  $A$  আৰু  $B$  উলিওৱা।

সমাধান :  $\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$$\Rightarrow A+B = 60^\circ \dots \dots \dots (i)$$

$$\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow A-B = 30^\circ \dots \dots \dots (ii)$$

$$\therefore A+B = 60^\circ$$

$$A-B = 30^\circ$$

(যোগকৰি)  $2A = 90^\circ$

$$\Rightarrow A = 45^\circ$$

এতিয়া,  $A = 45^\circ$ , (i) নং সমীকৰণত বহুৱাই পাওঁ :

$$A+B = 60^\circ$$

$$\Rightarrow 15^\circ + B = 60^\circ$$

$$\Rightarrow B = 60^\circ - 15^\circ = 45^\circ$$

$$\therefore \left. \begin{aligned} A &= 45^\circ \\ B &= 15^\circ \end{aligned} \right\} \quad (\text{উত্তর})$$

প্রশ্ন 4. তলত দিয়াবিলাক সত্য নে অসত্য কোৱা । তোমাৰ উত্তৰৰ যুক্তি দাঙি ধৰা –

$$(i) \sin(A + B) = \sin A + \sin B.$$

(ii)  $\sin\theta$  ৰ মান বাঢ়ি যায় যদি  $\theta$  ৰ মান বাঢ়ে ।

(iii)  $\cos\theta$  ৰ মান বাঢ়ি যায় যদি  $\theta$  ৰ মান বাঢ়ে ।

(iv)  $\theta$  ৰ সকলো মানৰ বাবে  $\sin\theta = \cos\theta$

(v)  $A = 0^\circ$  ৰ বাবে  $\cot A$  সংজ্ঞাবদ্ধ নহয় ।

সমাধান :

$$(i) \sin(A + B) = \sin A + \sin B \text{ অশুদ্ধ ।}$$

কাৰণ, যেতিয়া,  $A = 60^\circ, B = 30^\circ$  ধৰা হয় তেতিয়া –

$$\therefore \text{বাঁওপক্ষ} = \sin(A + B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1$$

$$\text{সোঁপক্ষ} = \sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1$$

অৰ্থাৎ,  $L.H.S. \neq R.H.S.$

সমাধান :

(ii) সত্য ।

$$\therefore \sin 0^\circ = \frac{1}{2}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.7 \text{ (প্ৰায়)}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.87 \text{ (প্ৰায়)}$$

আৰু,  $\sin 90^\circ = 1$

অৰ্থাৎ,  $\sin\theta$  বৃদ্ধি পালে,  $\theta$  -ৰ মান  $0^\circ$ -ৰ পৰা  $90^\circ$  লৈকে বৃদ্ধি পায় ।

সমাধান :

(iii) অসত্য ।

লক্ষ্য কৰিলে কেখা যায় যে, –

$$\cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.87 \text{ (প্ৰায়)}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.7 \text{ (প্ৰায়)}$$

$$\cos 60^\circ = \frac{1}{2} \quad \cos 90^\circ = 0$$

$\therefore$  দেখা যায় যে  $\theta$ -ৰ মান হ্রাস পালে,  $\theta$ -ৰ মান  $0^\circ$ -ৰ পৰা  $90^\circ$  লৈকে বৃদ্ধি পায় ।



সমাধান :

(iv) অসত্য ।

$$\therefore \sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 30^\circ \neq \cos 30^\circ$$

অকল,  $\sin 45^\circ = \cos 45^\circ$  হয় ।

সমাধান :

(v) সত্য ।

কাৰণ,  $\cot 0^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0}$  সংজ্ঞাবদ্ধ নহয় অর্থাৎ অর্থহীন ।

অনুশীলনী - 8.3

প্রশ্ন 1. মান নির্ণয় কৰা ।

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii)  $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii)  $\cos 48^\circ - \sin 42^\circ$

(iv)  $\operatorname{cosec} 31^\circ - \sec 59^\circ$

সমাধান :

(i)  $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin 18^\circ}{\cos(90^\circ - 18^\circ)} = \frac{\sin 18^\circ}{\sin 18^\circ} = 1$  (উত্তৰ)

$$[\therefore \cos(90^\circ - \theta) = \sin \theta]$$

সমাধান :

(ii)  $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan 26^\circ}{\cos(90^\circ - 26^\circ)} = \frac{\tan 26^\circ}{\tan 26^\circ} = 1$  (উত্তৰ)

$$[\therefore \cot(90^\circ - \theta) = \tan \theta]$$

সমাধান :

(iii)  $\cos 48^\circ - \sin 42^\circ$

$$= \cos(90^\circ - 42^\circ) - \sin 42^\circ \quad [\therefore \cos(90^\circ - \theta) = \sin \theta]$$

$$= \sin 42^\circ - \sin 42^\circ$$

$$= 0 \quad (\text{উত্তৰ})$$

সমাধান :

$$\begin{aligned}
 & (iv) \operatorname{cosec} 31^\circ - \sec 59^\circ \\
 & = \operatorname{cosec} 31^\circ - \sec(90^\circ - 31^\circ) \quad [\because \sec(90^\circ - \theta) = \operatorname{cosec} \theta] \\
 & = \operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ \\
 & = 0 \quad (\text{উত্তর})
 \end{aligned}$$

প্রশ্ন 3. যদি  $\tan 2A = \cot(A - 18^\circ)$ , য'ত  $2A$  সূক্ষ্মকোণ, তেন্তে  $A$  ৰ মান উলিওৱা ।

সমাধান :  $\tan 2A = \cot(A - 18^\circ)$

$$\begin{aligned}
 & = \cot(90^\circ - 2A) = \cot(A - 18^\circ) \quad [\because \cot(90^\circ - \theta) = \tan \theta] \\
 & \Rightarrow 90^\circ - 2A = A - 18 \\
 & \Rightarrow -2A - A = -18^\circ - 90^\circ \\
 & \Rightarrow -3A = -108^\circ \\
 & \Rightarrow A = \frac{-108^\circ}{-3} = 36^\circ \\
 & \therefore A = 36^\circ \quad (\text{উত্তর})
 \end{aligned}$$

প্রশ্ন 4. যদি  $\tan A = \cot B$ , প্রমাণ কৰা যে  $A + B = 90^\circ$

সমাধান :

$$\begin{aligned}
 & \therefore \tan A = \cot B \\
 & \Rightarrow \tan A = \tan(90^\circ - B) \quad [\because \tan(90^\circ - \theta) = \cot \theta] \\
 & \Rightarrow A = 90^\circ - B \\
 & \Rightarrow A + B = 90^\circ \quad (\text{উত্তর})
 \end{aligned}$$

প্রশ্ন 5. যদি  $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ , য'ত  $4A$  সূক্ষ্মকোণ, তেন্তে  $A$  ৰ মান উলিওৱা ।

সমাধান :

$$\begin{aligned}
 & \therefore \sec 4A = \operatorname{cosec}(A - 20^\circ) \\
 & \Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ) \quad [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta] \\
 & \Rightarrow 5A = 110^\circ \\
 & \Rightarrow A = \frac{110^\circ}{5} \Rightarrow A = 22^\circ \quad (\text{উত্তর})
 \end{aligned}$$

প্রশ্ন 6. যদি  $A, B$  আৰু  $C$  কোণকেইটা  $ABC$  ত্ৰিভুজৰ অস্থঃকোণ হয়, তেন্তে দেখুওৱা যে –

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

সমাধান :

আমি জামো যে,  $\triangle ABC$  ত্ৰিভুজৰ  $\angle A + \angle B + \angle C = 180^\circ$

$$\Rightarrow B + C = 180^\circ - A$$

$$\Rightarrow \frac{B+C}{2} = \frac{180^\circ - A}{2}$$

$$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right) \text{ [ উভয় পক্ষত লৈ পাওঁ । ]}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2} \text{ [ } \therefore \sin(90^\circ - \theta) = \cos\theta \text{ ]}$$

[ দেখুওৱা হ'ল ]

#### অনুশীলনী – 8.4

প্রশ্ন 1.  $\sin A$ ,  $\sec A$  আৰু  $\tan A$  এই ত্ৰিকোণমিতিক অনুপাত কেইটাক  $\cot A$  ৰ দ্বাৰা প্ৰকাশ কৰা ।

সমাধান :

$$\therefore \operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \left(\frac{1}{\sin A}\right)^2 = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

ইয়াত, ঋণাত্মক মান আমি বৰ্জন কৰিম ।

$$\therefore \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

আকৌ,  $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \pm \sqrt{\frac{1 + \cot^2 A}{\cot^2 A}} = \pm \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

$$\therefore \tan A = \frac{1}{\cot A}$$

প্রশ্ন 2.  $\sec A$  ব সহায়ত  $\angle A$  কোণৰ আন সকলোবিলাক ত্ৰিকোণমিতিক অনুপাত লিখা ।

সমাধান :

আমি জানো যে,  $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \pm \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{1}{\sec A}$$

$$1 + \tan^2 A = \sec^2 A$$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \pm \sqrt{\sec^2 A - 1}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore \tan A = \sqrt{\sec^2 A - 1}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$$

প্রশ্ন 3. মান নির্ণয় করা -

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

সমাধান :

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{\sin^2(90^\circ - 27^\circ) + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2(90^\circ - 17^\circ)}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \sin^2 17^\circ} = \frac{1}{1} = 1 \quad (\text{উত্তর})$$

$$\left[ \begin{array}{l} \sin(90^\circ - \theta) = \cos \theta \text{ আৰু} \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right]$$

সমাধান :

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= \sin 25^\circ \times \cos(90^\circ - 25^\circ) + \cos 25^\circ \times \sin(90^\circ - 25^\circ)$$

$$\therefore [\cos(90^\circ - \theta) = \sin \theta \text{ and } \sin(90^\circ - \theta) = \cos \theta]$$

$$= \sin 25^\circ \times \sin 25^\circ + \cos 25^\circ \times \cos 25^\circ$$

$$= \sin^2 25^\circ + \cos^2 25^\circ$$

$$= 1$$

প্রশ্ন 5. তলৰ অভেদ কেইটা প্রমাণ করা যদিহে ইয়াত কোণ বিলাক সূক্ষ্ম কোণ আৰু যাৰ বাবে অভেদ কেইটা সংজ্ঞাবদ্ধ হয় -

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

[ইংগিত : ইয়াত থকা পদবোৰ  $\sin \theta$  আৰু  $\cos \theta$  ত প্রকাশ করা ।

$$(iv) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$(v) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

সমাধান : (i)  $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

$$\text{বাঁওপক্ষ} = (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$$

$$\begin{aligned}
&= \left( \frac{1-\cos \theta}{\sin \theta} \right)^2 \\
&= \frac{(1-\cos \theta)^2}{\sin^2 \theta} = \frac{(1-\cos \theta)^2}{1-\cos^2 \theta} \\
&= \frac{(1-\cos \theta)(1-\cos \theta)}{(1+\cos \theta)(1-\cos \theta)} \\
&= \frac{1-\cos \theta}{1+\cos \theta} \quad \text{সৌপক্ষ (প্রমাণিত)}
\end{aligned}$$

সমাধান : (ii)  $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$$\begin{aligned}
\text{বাঁওপক্ষ} &= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} \\
&= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)} \\
&= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)} \\
&= \frac{1+1+2\sin A}{\cos A(1+\sin A)} \\
&= \frac{2+2\sin A}{\cos A(1+\sin A)} \\
&= \frac{2(1+\sin A)}{\cos A(1+\sin A)} \\
&= 2 \times \frac{1}{\cos A} = 2\sec A = \text{সৌপক্ষ (প্রমাণিত)}
\end{aligned}$$

সমাধান : (iii)  $\frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

$$\begin{aligned}
\text{বাঁওপক্ষ} &= \frac{\tan \theta}{1-\cot \theta} + \frac{\cot \theta}{1-\tan \theta} \\
&= \frac{\frac{1}{\cot \theta}}{1-\cot \theta} + \frac{\cot \theta}{1-\frac{1}{\cot \theta}} \\
&= \frac{1}{\cot \theta(1-\cot \theta)} + \frac{\cot \theta}{\frac{\cot \theta - 1}{\cot \theta}} \\
&= \frac{1}{\cot \theta(1-\cot \theta)} + \frac{\cot^2 \theta}{\cot \theta - 1} \\
&= \frac{1}{\cot \theta(1-\cot \theta)} - \frac{\cot^2 \theta}{1-\cot \theta} \\
&= \frac{1-\cot^3 \theta}{\cot \theta(1-\cot \theta)} \\
&= \frac{(1-\cot \theta)(1+\cot \theta+\cot^2 \theta)}{\cot \theta(1-\cot \theta)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1 + \cot \theta + \cot^2 \theta}{\cot \theta} \\
&= \frac{1}{\cot \theta} + 1 + \frac{\cot^2 \theta}{\cot \theta} \\
&= \frac{1}{\cot \theta} + 1 + \cot \theta \\
&= \frac{\sin \theta}{\cos \theta} + 1 + \frac{\cos \theta}{\sin \theta} \\
&= \frac{\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{(\sin^2 \theta + \cos^2 \theta) + \sin \theta \cos \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} \\
&= \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta} \\
&= \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} + 1 \\
&= \operatorname{cosec} \theta \cdot \sec \theta + 1 \\
&= 1 + \sec \theta \cdot \operatorname{cosec} \theta = \text{সৌপক্ষ (প্রমাণিত)}।
\end{aligned}$$

সমাধান : (iv)  $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\begin{aligned}
\text{বাঁওপক্ষ} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
&= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\
&= \frac{1 + \sin A}{\cos A} \\
&= \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
&= \sec A + \tan A = \text{সৌপক্ষ (প্রমাণিত)}।
\end{aligned}$$

সমাধান : (v)  $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{বাঁওপক্ষ} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$\begin{aligned}
&= \sin^2 + 2. \sin A. \operatorname{cosec} A + \operatorname{cosec}^2 A \\
&\quad + \cos^2 + 2\cos A. \sec A + \sec^2 A \\
&= \sin^2 A + \cos^2 A + 2. \sin A \times \frac{1}{\sin A} \\
&\quad + \operatorname{cosec}^2 A + 2\cos A \times \frac{1}{\sec A} + \sec^2 A \\
&= 1 + 2 + \operatorname{cosec}^2 A + 2 + \sec^2 A \\
&= 5 + 1 + \cot^2 A + 1 + \tan^2 A \\
&= 7 + \tan^2 A + \cot^2 A = \text{সৌপক্ষ (প্রমাণিত)}।
\end{aligned}$$

প্রশ্ন 6. যদি  $A = 30^\circ$ ,  $B = 60^\circ$  তেন্তে দেখুওৱা যে -

$$(i) \sin(A + B) = \sin A \cos B + \cos A \sin B$$

সমাধান :  $LHS = \sin(A + B)$

$$= \sin(30 + 60)$$

$$= \sin 90 = 1$$

$$RHS = \sin A \cos B + \cos A \sin B$$

$$= \sin 30 \cos 60 + \cos 30 \sin 60$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$$\therefore LHS = RHS.$$