QUESTION PAPER CODE 65/1/1/D

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta & 0 \\ 1 & 0 & \cos\theta \end{vmatrix} = \sin\theta\cos\theta$ $\frac{1}{2}$

$= \frac{1}{2}\sin 2\theta \therefore \text{Max value} = \frac{1}{2}$ $\frac{1}{2}$

2. $(A - I)^3 + (A + I)^3 - 7A, \quad A^2 = I \Rightarrow A^3 = A$ $\frac{1}{2}$

$= 2A - A = A$ $\frac{1}{2}$

3. $\left.\begin{array}{l} 2b = 3 \text{ and } 3a = -2 \\ b = \frac{3}{2} \text{ and } a = -\frac{2}{3} \end{array}\right\}$ $\frac{1}{2} + \frac{1}{2}$

4. Getting position vector as $2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})$ $\frac{1}{2}$

$= 3\vec{a} + 4\vec{b}$ $\frac{1}{2}$

5. $\overrightarrow{AD} = \overrightarrow{AB} + \frac{1}{2}[\overrightarrow{AC} - \overrightarrow{AB}] = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{AB})$ $\frac{1}{2}$

$|\overrightarrow{AD}| = \frac{1}{2}|3\hat{i} + 5\hat{k}| = \frac{1}{2}\sqrt{34}$ $\frac{1}{2}$

6. $\vec{r}.\frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{7} = 5$ 1

SECTION B

7. LHS $= \tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5}.\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3}.\frac{1}{8}}\right)$ 1

$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$ 1

$= \tan^{-1}\left(\frac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{325}{325}\right)$ 1

$= \tan^{-1}(1) = \frac{\pi}{4}$ 1

OR

$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\dfrac{2}{\sin x}\right)$ 2

$\Rightarrow \quad \sin x(\sin x - \cos x) = 0$ 1

$\Rightarrow \quad \sin x = \cos x$ $\frac{1}{2}$

the solution is $x = \dfrac{\pi}{4}$ $\frac{1}{2}$

8. Let the income be 3x, 4x and expenditures, 5y, 7y

$\therefore \quad \left.\begin{matrix} 3x - 5y = 15000 \\ 4x - 7y = 15000 \end{matrix}\right\}$ 1

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$\Rightarrow \quad x = 30000,\ y = 15000$ $1\frac{1}{2}$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively $\frac{1}{2}$

"Expenditure must be less than income"

(or any other relevant answer) 1

9. Here $x = a\left(\sin 2t + \dfrac{1}{2}\sin 4t\right),\ y = b(\cos 2t - \cos^2 2t)$

$\dfrac{dx}{dt} = 2a[\cos 2t + \cos 4t],\ \dfrac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t]$ 1 + 1

$\dfrac{dy}{dx} = \dfrac{b}{a}\left[\dfrac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t}\right]$ 1

$\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \dfrac{b}{a}$ $\frac{1}{2}$

and $\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{3}} = \sqrt{3}\dfrac{b}{a}$ $\frac{1}{2}$

OR

$y = x^x \Rightarrow \log y = x.\log x$ $\frac{1}{2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = (1 + \log x)$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ $\frac{1}{2}$

10. $\text{LHL} = \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{(1-\cancel{\sin x})(1+\sin x+\sin^2 x)}{3(1-\cancel{\sin x})(1+\sin x)}$ 1

$= \dfrac{1}{2}$ $\frac{1}{2}$

$\therefore \; p = \dfrac{1}{2}$ $\frac{1}{2}$

$\text{RHL} = \lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{q(1-\sin x)}{(\pi-2x)^2} = \lim\limits_{h \to 0} \dfrac{q(1-\cos h)}{(2h)^2}$, where $x - \dfrac{\pi}{2} = h$ 1

$= \lim\limits_{h \to 0} \dfrac{2q\sin^2 \frac{h}{2}}{4.4.\frac{h^2}{4}} = \dfrac{q}{8}$ $\frac{1}{2}$

$\therefore \; \dfrac{q}{8} = \dfrac{1}{2} \Rightarrow q = 4$ $\frac{1}{2}$

11. $\dfrac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3 \sin t\,(1 - \cos^2 t) = -3 \sin^3 t$ 1

$\dfrac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t\,(1 - \sin^2 t) = 3\cos^3 t$ 1

Slope of normal $= -\dfrac{dx}{dy} = \dfrac{\sin^3 t}{\cos^3 t}$ 1

Eqn. of normal is

$y - (3\sin t - \sin^3 t) = \dfrac{\sin^3 t}{\cos^3 t}[x - (3\cos t - \cos^3 t)]$ $\frac{1}{2}$

$\Rightarrow \quad y\cos^3 t - x\sin^3 t = 3\sin t \cos t\,(\cos^2 t - \sin^2 t)$

$= \dfrac{3}{4}\sin 4t$ $\frac{1}{2}$

or $\;4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$

12. $I = \displaystyle\int \frac{(3\sin\theta - 2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}\,d\theta$ $\frac{1}{2}$

$\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$\therefore \quad I = \displaystyle\int \frac{3t-2}{t^2-4t+4}\,dt = \int \frac{3t-2}{(t-2)^2}\,dt$ 1

$$= \int \frac{3(t-2)}{(t-2)^2}\,dt + 4\int \frac{1}{(t-2)^2}\,dt$$ 1

$$= 3\log|t-2| - \frac{4}{(t-2)} + C$$ 1

$$= 3\log|\sin\theta - 2| - \frac{4}{(\sin\theta - 2)} + C$$ $\frac{1}{2}$

OR

Let $I = \int_0^{\pi} \sin\left(\frac{\pi}{4} + x\right) e^{2x}\,dx$

$$= \sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\Bigg]_0^{\pi} - \int_0^{\pi}\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$$ 1

$$I = \left[\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\right\}\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} -\sin\left(\frac{\pi}{4}+x\right)\frac{e^{2x}}{2}\,dx$$ 1

$$\frac{5}{\cancel{4}} I = \left\{\frac{1}{\cancel{4}}\left[2\sin\left(\frac{\pi}{4}+x\right) - \cos\left(\frac{\pi}{4}+x\right)\right]e^{2x}\right\}_0^{\pi}$$ 1

$$I = \frac{1}{5}\left[\left\{2\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\right\}e^{2\pi} - \left\{2\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right\}\right] = \frac{-1}{5\sqrt{2}}(e^{2\pi}+1)$$ 1

13. $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\,dx$

Put $x^{3/2} = t \Rightarrow \frac{3}{2}.\,x^{1/2}\,dx = dt$ or $\sqrt{x}\,dx = \frac{2}{3}\,dt$ $1\frac{1}{2}$

$$I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$ 1

$$= \frac{2}{3}.\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C$$ 1

$$= \frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$$ $\frac{1}{2}$

14. $I = \int_{-1}^{2} |x^3 - x|\,dx$

$$= \int_{-1}^{0}(x^3 - x)\,dx + \int_0^1 -(x^3 - x)\,dx + \int_1^2 (x^3 - x)\,dx$$ $1\frac{1}{2}$

$$= \frac{x^4}{4} - \frac{x^2}{2}\Bigg]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$$ $1\frac{1}{2}$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4-2) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$$ 1

15. Given differential equation can be written as

$$\frac{(1+\log x)}{x}\,dx + \frac{2y}{1-y^2}\,dy = 0$$ 1

integrating to get, $\frac{1}{2}(1+\log x)^2 - \log|1-y^2| = C$ 2

$x = 1,\ y = 0 \Rightarrow C = \frac{1}{2}$ $\frac{1}{2}$

$\Rightarrow\ (1 + \log x)^2 - 2\log|1 - y^2| = 1$ $\frac{1}{2}$

16. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2}\,x = \frac{e^{\tan^{-1} y}}{1+y^2}$$ 1

Integrating factor is $e^{\tan^{-1} y}$ 1

$\therefore$ Solution is $x.\, e^{\tan^{-1} y} = \int e^{2\tan^{-1} y}\frac{1}{1+y^2}\,dy$ 1

$\therefore$ $x\, e^{\tan^{-1} y} = \frac{1}{2}e^{2\tan^{-1} y} + C$ 1

17. Given, that $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are coplanar

$\therefore$ $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 0$

i.e. $(\vec{a}+\vec{b}).\{(\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\} = 0$ 1

$(\vec{a}+\vec{b}).\{(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a})\} = 0$ 1

$\Rightarrow$ $\vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{a}) + \vec{a}.(\vec{c}\times\vec{a}) + \vec{b}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{a}) + \vec{b}.(\vec{c}\times\vec{a}) = 0$ $1\frac{1}{2}$

$\Rightarrow$ $2[\vec{a}, \vec{b}, \vec{c}] = 0$ or $[\vec{a}, \vec{b}, \vec{c}] = 0$ $\frac{1}{2}$

$\Rightarrow$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

18. Vector equation of the required line is

$\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \mu\,[(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})]$ 1

$\Rightarrow$ $\vec{r} = (\hat{i}+2\hat{j}-4\hat{k}) + \lambda\,[(2\hat{i}+3\hat{j}+6\hat{k})$ 2

in cartesian form, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

19. Let events are:

$$\left.\begin{array}{l} E_1 : \text{A is selected} \\ E_2 : \text{B is selected} \\ E_3 : \text{C is selected} \\ A : \text{Change is not introduced} \end{array}\right\}$$

$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$ 1

$P(A/E_1) = 0.2,\ P(A/E_2) = 0.5,\ P(A/E_3) = 0.7$ 1

$$\therefore \quad P(E_3/A) = \frac{\frac{4}{7} \times \frac{7}{10}}{\frac{1}{7} \times \frac{2}{10} + \frac{2}{7} \times \frac{5}{10} + \frac{4}{7} \times \frac{7}{10}}$$ 1

$$= \frac{28}{40} = \frac{7}{10}$$ 1

OR

$\left.\begin{array}{l} \text{Prob. of success for A} = \frac{1}{6} \\ \text{Prob. of failure for A} = \frac{5}{6} \end{array}\right\}$ 1

$\left.\begin{array}{l} \text{Prob. of success for B} = \frac{1}{12} \\ \text{Prob. of failure for B} = \frac{11}{12} \end{array}\right\}$

B can win in 2nd or 4th or 6th or....throw 1

$$\therefore \quad P(B) = \left(\frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \left(\frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{11}{12} \cdot \frac{5}{6} \cdot \frac{1}{12}\right) + \ldots.$$ 1

$$= \frac{5}{72}\left(1 + \frac{55}{72} + \left(\frac{55}{72}\right)^2 + \ldots.\right)$$

$$= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17}$$ 1

SECTION C

20. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(9x_1 + 9x_2 + 6) = 0$

$\Rightarrow \quad x_1 - x_2 = 0$ or $x_1 = x_2$ as $(9x_1 + 9x_2 + 6) \neq 0,\ x_1, x_2 \in N$

$\therefore$ f is a one-one function 2

$f: N \to S$ is ONTO as co-domain = Range 1

Hence f is invertible

$$y = 9x^2 + 6x - 5 = (3x + 1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6} - 1}{3}$$

$$\therefore \quad f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3},\ y \in S$$ 2

$$f^{-1}(43) = \frac{\sqrt{49} - 1}{3} = 2$$ $\frac{1}{2}$

$$f^{-1}(163) = \frac{\sqrt{169} - 1}{3} = 4$$ $\frac{1}{2}$

21. Using $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} y(z-x)+z^2-x^2 & x(z-y)+z^2-y^2 & xy-z^2 \\ z(x-y)+x^2-y^2 & y(x-z)+x^2-z^2 & yz-x^2 \\ x(y-z)+y^2-z^2 & z(y-x)+y^2-x^2 & zx-y^2 \end{vmatrix}$$ 2

Taking (x + y + z) common from C_1 & C_2

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} z-x & z-y & xy-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = (x+y+z)^2 \begin{vmatrix} 0 & 0 & xy+yz+zx-x^2-y^2-z^2 \\ x-y & x-z & yz-x^2 \\ y-z & y-x & zx-y^2 \end{vmatrix}$$ 1

Expanding to get

$\Delta = (x + y + z)^2 (xy + zy + zx - x^2 - y^2 - z^2)^2$ 1

Hence Δ is divisible by (x + y + z) and

the quotient is $(x + y + z) (xy + yz + zx - x^2 - y^2 - z^2)^2$ 1

OR

Writing $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_3$ $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 8 & 4 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} A$

$R_1 \to R_1 - 2R_2$
$R_3 \to R_3 - 4R_2$ $\begin{pmatrix} -3 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -4 & 0 \end{pmatrix} A$

$R_1 \to \frac{1}{3} R_1$
$R_3 \to - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & 1 & 0 \\ -1 & 4 & 0 \end{pmatrix} A$

$R_2 \to R_2 - 2R_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 0 & -1/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$ $2\frac{1}{2}$ marks for operation to get A^{-1}

$R_2 \to R_2 - R_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} A$

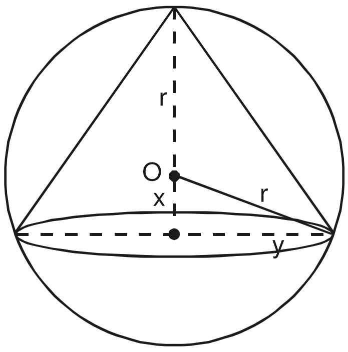
$\therefore \quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$ $\frac{1}{2}$

$AX = B \Rightarrow X = A^{-1}B$ 1

$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\therefore \quad x = 1, y = 2, z = 1$ 1

22.



Correct Figure 1

Let radius of cone be y and the altitude be r + x

$\therefore \quad x^2 + y^2 = r^2 \quad$ ...(i) $\quad \frac{1}{2}$

$$\text{Volume } V = \frac{1}{3}\pi y^2 (r + x)$$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x) \qquad 1$$

$$\frac{dV}{dx} = \frac{\pi}{3}[(r^2 - x^2)1 + (r + x)(-2x)] = \frac{\pi}{3}(r + x)(r - 3x) \qquad 1$$

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3} \qquad \frac{1}{2}$$

$$\therefore \quad \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3} \qquad \frac{1}{2}$$

$$\text{and} \quad \frac{d^2V}{dx^2} = \frac{\pi}{3}[(r + x)(-3) + (r - 3x)] = \frac{\pi}{3}[-2r - 6x] < 0 \qquad 1$$

$$\therefore \quad \text{Max. Volume} = \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right) \qquad \frac{1}{2}$$

$$= \frac{8}{27}(\text{Vol. of sphere})$$

OR

$f(x) = \sin 3x - \cos 3x,\ 0 < x < \pi$

$f'(x) = 3\cos 3x + 3\sin 3x$ $\quad 1$

$f'(x) = 0 \Rightarrow \tan 3x = -1$ $\quad \frac{1}{2}$

$\Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{4},\ n \in Z$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$ $\quad 1\frac{1}{2}$

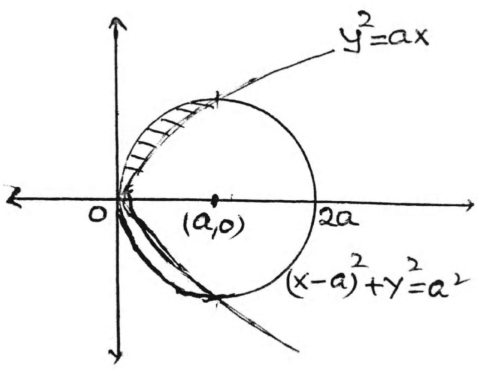
Intervals are: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$ $\quad 1$

f(x) is strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$ $\quad 1$

and strictly decreasing in $\left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$ $\quad 1$

23. $y^2 = ax,\ x^2 + y^2 = 2ax \Rightarrow x^2 - ax = 0$

$\Rightarrow \quad x = 0,\ x = a$ 1



Correct Figure 1

Shaded area $= \left[\int_0^a [\sqrt{a^2-(x-a)^2} - \sqrt{a}\sqrt{x}]dx\right.$ 1

$A = \left[\frac{x-a}{2}\sqrt{a^2-(x-a)^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a} - \sqrt{a}\,\frac{2}{3}x^{3/2}\right]_0^a$ 2

$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2}\frac{\pi}{2}\right] = \frac{\pi a^2}{4} - \frac{2a^2}{3}$ sq. units 1

24. Equation of line $AB: \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ 1

Eqn. of plane LMN: $\begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $1\frac{1}{2}$

$2(x-2) + 1\,(y-2) + 1\,(z-1) = 0$ or $2x + y + z - 7 = 0$ $\frac{1}{2}$

Any point on line AB is $(-\lambda + 3,\ \lambda - 4,\ 6\lambda - 5)$ $\frac{1}{2}$

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$ 1

$\therefore \quad$ P is (1, –2. 7) $\frac{1}{2}$

let P divides AB in K : 1

$\Rightarrow \quad 1 = \frac{2K+3}{K+1} \Rightarrow K = -2$ i.e. P divides, AB externally in 2 : 1 1

25. X = No. of red

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{1}{3}\right)^4 = \frac{1}{81}$	${}^4C_1\left(\frac{1}{3}\right)^3\frac{2}{3} = \frac{8}{81}$	${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 = \frac{24}{81}$	${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = \frac{32}{81}$	${}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{81}$
XP(X):	0	$\frac{8}{81}$	$\frac{48}{81}$	$\frac{96}{81}$	$\frac{64}{81}$
$X^2P(X)$:	0	$\frac{8}{81}$	$\frac{96}{81}$	$\frac{288}{81}$	$\frac{256}{81}$

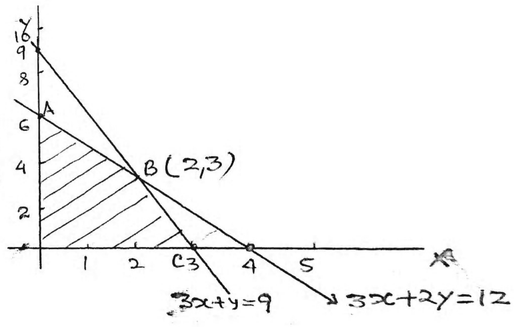
1

$2\frac{1}{2}$

Mean $= \Sigma XP(X) = \frac{216}{81} = \frac{8}{3}$ 1

Variance $= \Sigma X^2P(X) - [\Sigma XP(X)]^2 = \frac{648}{81} - \frac{64}{9} = \frac{8}{9}$ $1\frac{1}{2}$

26. Let production of A, B (per day) be x, y respectively



Maximise P = 7x + 4y 1

Subject to $\left.\begin{array}{l} 3x + 2y \le 12 \\ 3x + y \le 9 \\ x \ge 0, y \ge 0 \end{array}\right\}$ 2

Correct Graph 2

P(A) = 24

P(B) = 26

P(C) = 21

$\therefore$ 2 units of product A and 3 units of product B for maximum profit 1