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Senior School Certificate Examination

March 2017

Marking Scheme — Mathematics 65/1, 65/2, 65/3
[Outside Delhi]

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A| = 8.$ 1

2. $k = 12.$ 1

3. $-\log|\sin 2x| + c$ OR $\log|\sec x| - \log|\sin x| + c.$ 1

4. Writing the equations as $\left.\begin{array}{l} 2x - y + 2z = 5 \\ 2x - y + 2z = 8 \end{array}\right\}$ $\frac{1}{2}$

$\Rightarrow$ Distance = 1 unit $\frac{1}{2}$

SECTION B

5. Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 1

$\Rightarrow$ $|A| = -a(bc) + a(bc) = 0$ 1

OR

Since A is a skew-symmetric matrix $\therefore$ $A^T = -A$ $\frac{1}{2}$

$\therefore$ $|A^T| = |-A| = (-1)^3.|A|$ $\frac{1}{2}$

$\Rightarrow$ $|A| = -|A|$ $\frac{1}{2}$

$\Rightarrow$ $2|A| = 0$ or $|A| = 0.$ $\frac{1}{2}$

6. $f(x) = x^3 - 3x$

$\therefore$ $f'(c) = 3c^2 - 3 = 0$ $\frac{1}{2}$

$\therefore$ $c^2 = 1 \Rightarrow c = \pm 1.$ $\frac{1}{2}$

Rejecting c = 1 as it does not belong to $(-\sqrt{3}, 0)$, $\frac{1}{2}$

we get $c = -1.$ $\frac{1}{2}$

7. Let V be the volume of cube, then $\frac{dV}{dt} = 9\ \text{cm}^3/\text{s}$.

Surface area (S) of cube = $6x^2$, where x is the side.

then $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2} \cdot \frac{dV}{dt}$ 1

$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt} = 12x . \frac{1}{3x^2} \frac{dV}{dt}$ $\frac{1}{2}$

$= 4 \cdot \frac{1}{10} \cdot 9 = 3.6\ \text{cm}^2/\text{s}$ $\frac{1}{2}$

8. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$ $\frac{1}{2}$

$= 3[x^2 - 2x + 2] = 3[(x - 1)^2 + 1]$ 1

since $f'(x) > 0\ \forall\ x \in \mathbb{R}\ \therefore\ f(x)$ is increasing on $\mathbb{R}$ $\frac{1}{2}$

9. Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ $\frac{1}{2}$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$ $\frac{1}{2}$

$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3}\ \therefore$ z coord. $= -3\left(\frac{2}{3}\right) + 1 = -1.$ $\frac{1}{2} + \frac{1}{2}$

OR

P (2, 2, 1) — R (4, y, z) — Q (5, 1, –2)

Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2.$ 1

$\therefore\ z = \frac{2(-2)+1(1)}{2+1} = \frac{-3}{3} = -1.$ 1

10. Event A: Number obtained is even

B: Number obtained is red.

$P(A) = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{3}{6} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$

$P(A \cap B) = P$ (getting an even red number) $= \frac{1}{6}$ $\frac{1}{2}$

Since $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(P \cap B)$ which is $\frac{1}{6}$ $\frac{1}{2}$

$\therefore$ A and B are not independent events.

11. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize C = 300x + 400y $\frac{1}{2}$

Subject to: $\begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$ $1\frac{1}{2}$

12. $\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$ 1

$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + c$ 1

SECTION C

13. $\tan^{-1} \frac{x-3}{x-4} + \tan^{-1} \frac{x+3}{x+4} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1} \left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \frac{x-3}{x-4} \cdot \frac{x+3}{x+4}} \right) = \frac{\pi}{4}$ $1\frac{1}{2}$

$\Rightarrow \frac{2x^2 - 24}{-7} = 1 \Rightarrow x^2 = \frac{17}{2}$ $1\frac{1}{2}$

$\Rightarrow x = \pm\sqrt{\frac{17}{2}}$ 1

14. $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1+1

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding

$(a - 1)^2 \cdot (a - 1) = (a - 1)^3$. 1

OR

Let $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow 2a - c = -1, \quad 2b - d = -8$

$a = 1, \quad b = -2$ 1

$-3a + 4c = 9, \quad -3b + 4d = 22$

Solving to get $a = 1, b = -2, c = 3, d = 4$

$\therefore \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ 1

15. $x^y + y^x = a^b$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$\therefore \quad \frac{du}{dx} + \frac{dv}{dx} = 0 \qquad ...(i)$ $\frac{1}{2}$

$$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$ 1

$$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$ 1

Putting in (i) $x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$$ 1

OR

$$e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1) \cdot e^y \cdot \frac{dy}{d} = 0$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(x+1)}$$ 1

$$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$$ $1\frac{1}{2}$

16. $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta = \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}d\theta$ $\frac{1}{2}$

$$= \int \frac{dt}{(4+t^2)(1+4t^2)}, \text{ where } \sin\theta = t$$ 1

$$= \int \frac{-\frac{1}{15}}{4+t^2}dt + \int \frac{\frac{4}{15}}{1+4t^2}dt$$ 1

$$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + \frac{4}{30}\tan^{-1}(2t) + c$$ 1

$$= -\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + \frac{2}{15}\tan^{-1}(2\sin\theta) + c$$ $\frac{1}{2}$

17. $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx$ 1

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx = \pi \int_0^{\pi} \tan x (\sec x - \tan x) dx$

$I = \frac{\pi}{2} \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1) dx$ 1

$= \frac{\pi}{2} [\sec x - \tan x + x]_0^{\pi}$ 1

$= \frac{\pi(\pi - 2)}{2}$ 1

OR

$I = \int_1^4 \{|x - 1| + |x - 2| + |x - 4|\} dx$

$= \int_1^4 (x - 1) dx - \int_1^2 (x - 2) dx + \int_2^4 (x - 2) dx - \int_1^4 (x - 4) dx$ 2

$= \left.\frac{(x-1)^2}{2}\right]_1^4 - \left.\frac{(x-2)^2}{2}\right]_1^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 - \left.\frac{(x-4)^2}{2}\right]_1^4$ 1

$= \frac{9}{2} + \frac{1}{2} + 2 + \frac{9}{2} = 11\frac{1}{2}$ or $\frac{23}{2}$ 1

18. Given differential equation can be written as

$(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x \Rightarrow \frac{dy}{dx} + \frac{1}{1 + x^2} y = \frac{\tan^{-1} x}{1 + x^2}$ 1

Integrating factor = $e^{\tan^{-1} x}$. 1

$\therefore$ Solution is $y \cdot e^{\tan^{-1} x} = \int \tan^{-1} x \cdot e^{\tan^{-1} x} \frac{1}{1 + x^2} dx$ 1

$\Rightarrow y \cdot e^{\tan^{-1} x} = e^{\tan^{-1} x} \cdot (\tan^{-1} x - 1) + c$ 1

or $y = (\tan^{-1} x - 1) + c \cdot e^{-\tan^{-1} x}$

19. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ 1

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA},$ are not parallel vectors, and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ $\therefore$ A, B, C form a triangle 1

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$ $\therefore$ A, B, C form a right triangle 1

Area of $\Delta = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ 1

20. Given points, A, B, C, D are coplanar, if the

vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, i.e.

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}, \overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k}, \overrightarrow{AD} = \hat{i} + (\lambda - 9)\hat{k}$ $1\frac{1}{2}$

are coplanar

i.e., $\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda - 9 \end{vmatrix} = 0$ 1

$-2[-3\lambda + 27] + 4[-\lambda + 17] - 6(3) = 0$ 1

$\Rightarrow \lambda = 2.$ $\frac{1}{2}$

21. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore$ X :	4	6	8	10	12	1
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
xP(X) :	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{16}{6}$	$\frac{10}{6}$	$\frac{12}{6}$	
x^2P(X) :	$\frac{16}{6}$	$\frac{36}{6}$	$\frac{128}{6}$	$\frac{100}{6}$	$\frac{144}{6}$	

$$\Sigma xP(x) = \frac{48}{6} = 8 \quad \therefore \text{Mean} = 8$$ 1

$$\text{Variance} = \Sigma x^2 P(x) - [\Sigma xP(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$$ 1

22. Let E_1: Selecting a student with 100% attendance
E_2: Selecting a student who is not regular 1

A: selected student attains A grade.

$$P(E_1) = \frac{30}{100} \text{ and } P(E_2) = \frac{70}{100}$$ $\frac{1}{2}$

$$P(A/E_1) = \frac{70}{100} \text{ and } P(A/E_2) = \frac{10}{100}$$ $\frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

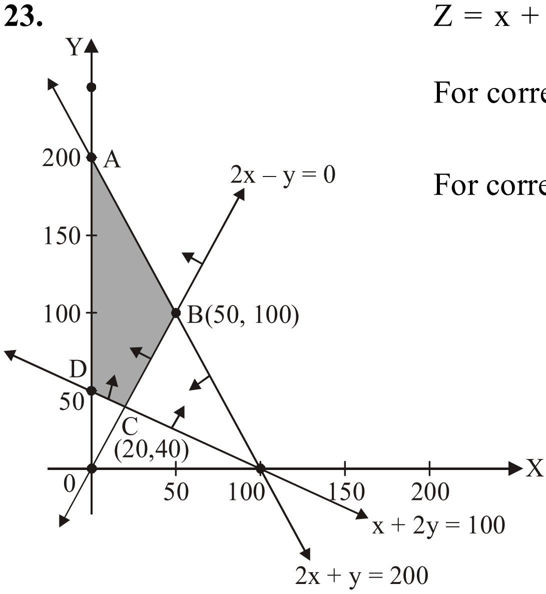
$$= \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}}$$

$$= \frac{3}{4}$$ 1

Regularity is required everywhere or any relevant value 1

23.

$Z = x + 2y$ s.t $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$



For correct graph of three lines $1\frac{1}{2}$

For correct shading $1\frac{1}{2}$

$Z(A) = 0 + 400 = 400$

$Z(B) = 50 + 200 = 250$

$Z(C) = 20 + 80 = 100$

$Z(D) = 0 + 100 = 100$

$\therefore$ Max (= 400) at $x = 0,\ y = 200$ 1

SECTION D

24. Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(i) $1\frac{1}{2}$

Given equations can be written as $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$\Rightarrow \quad AX = B$

From (i) $A^{-1} = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ 1

$\therefore \quad X = A^{-1}B = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$= \frac{1}{8}\begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ 1

$\Rightarrow x = 3, \ y = -2, \ z = -1$ $\frac{1}{2}$

25. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow \quad \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$

$\Rightarrow \quad 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow \quad 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1–1 function 2

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow \quad x = \frac{4y-3}{4-3y} \quad \therefore \ \forall\, y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is ONTO and so bijective 2

and $f^{-1}(y) = \dfrac{4y-3}{4-3y}; y \in R - \left\{\dfrac{4}{3}\right\}$ 1

$f^{-1}(0) = -\dfrac{3}{4}$ $\dfrac{1}{2}$

and $f^{-1}(x) = 2 \Rightarrow \dfrac{4x-3}{4-3x} = 2$

$\Rightarrow \quad 4x - 3 = 8 - 6x$

$\Rightarrow \quad 10x = 11 \Rightarrow x = \dfrac{11}{10}$ $\dfrac{1}{2}$

OR

$(a, b) * (c, d) = (ac, b + ad); (a, b), (c, d) \in A$

$(c, d) * (a, b) = (ca, d + bc)$

Since $b + ad \neq d + bc \Rightarrow *$ is NOT comutative $1\dfrac{1}{2}$

for associativity, we have,

$[(a,b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf)$

$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf) = (ace, b + ad + acf)$ $1\dfrac{1}{2}$

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element in A

Then $(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow \quad (ae, b + af) = (a, b) = (ae, f + be)$

$\Rightarrow \quad e = 1, f = 0 \Rightarrow (1, 0)$ is the identity element $1\dfrac{1}{2}$

(ii) Let (c, d) be the inverse element for (a, b)

$\Rightarrow \quad (a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow \quad (ac, b + ad) = (1, 0) = (ac, d + bc)$

$\Rightarrow ac = 1 \Rightarrow c = \dfrac{1}{a}$ and $b + ad = 0 \Rightarrow d = -\dfrac{b}{a}$ and $d + bc = 0 \Rightarrow d = -bc = -b\left(\dfrac{1}{a}\right)$

$\Rightarrow \left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$, $a \neq 0$ is the inverse of $(a, b) \in A$ $1\dfrac{1}{2}$

26. Let the sides of cuboid be x, x, y

$\Rightarrow$ $x^2y = k$ and $S = 2(x^2 + xy + xy) = 2(x^2 + 2xy)$ $\frac{1}{2}+1$

$\therefore$ $S = 2\left[x^2 + 2x\frac{k}{x^2}\right] = 2\left[x^2 + \frac{2k}{x}\right]$ 1

$\frac{ds}{dx} = 2\left[2x - \frac{2k}{x^2}\right]$ 1

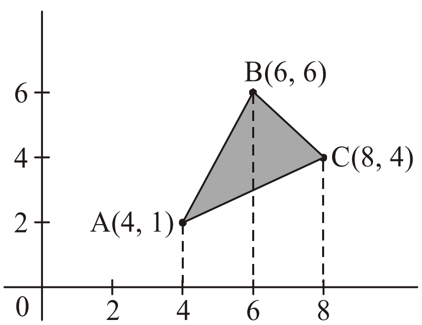
$\therefore$ $\frac{ds}{dx} = 0 \Rightarrow x^3 = k = x^2y \Rightarrow x = y$ 1

$\frac{d^2s}{dx^2} = 2\left[2 + \frac{4k}{x^3}\right] > 0$ $\therefore$ x = y will given minimum surface area 1

and x = y, means sides are equal

$\therefore$ Cube will have minimum surface area $\frac{1}{2}$

27. Figure 1



$$\left.\begin{aligned} &\text{Equation of AB}: y = \frac{5}{2}x - 9 \\ &\text{Equation of BC}: y = 12 - x \\ &\text{Equation of AC}: y = \frac{3}{4}x - 2 \end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore$ Area (A) $= \int_4^6 \left(\frac{5}{2}x - 9\right)dx + \int_6^8 (12 - x)\,dx - \int_4^8 \left(\frac{3}{4}x - 2\right)dx$ 1

$= \left[\frac{5}{4}x^2 - 9x\right]_4^6 + \left[12x - \frac{x^2}{2}\right]_6^8 - \left[\frac{3}{8}x^2 - 2x\right]_4^8$ $1\frac{1}{2}$

$= 7 + 10 - 10 = 7$ sq.units 1

OR

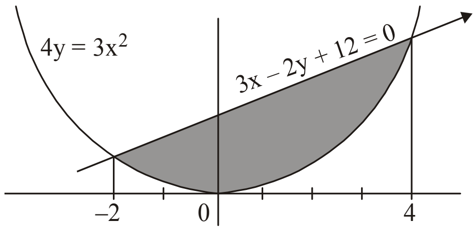


Figure 1

$$4y = 3x^2 \text{ and } 3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$$

$$\Rightarrow 3x^2 - 6x - 24 = 0 \text{ or } x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$$

$\Rightarrow$ x-coordinates of points of intersection are $x = -2$, $x = 4$ 1

$$\therefore \text{ Area (A)} = \int_{-2}^{4}\left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2\right]dx \qquad 1\frac{1}{2}$$

$$= \left[\frac{1}{2}\frac{(3x+12)^2}{6} - \frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4} \qquad 1\frac{1}{2}$$

$$= 45 - 18 = 27 \text{ sq.units} \qquad 1$$

28. $$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}} \qquad \frac{1}{2}$$

$$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \therefore\ v + x\frac{dv}{dx} = \frac{1+2v}{1-v} \qquad \frac{1}{2}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1} \Rightarrow \int\frac{v-1}{v^2+v+1}dv = -\frac{dx}{x} \qquad 1$$

$$\Rightarrow \int\frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx \Rightarrow \int\frac{2v+1}{v^2+v+1}dv - 3\int\frac{1}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int\frac{2}{x}dx \qquad 1+1$$

$$\Rightarrow \log|v^2+v+1| - 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + c \qquad 1$$

$$\Rightarrow \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = c \qquad \frac{1}{2}$$

$$x = 1,\ y = 0 \Rightarrow c = -2\sqrt{3}\cdot\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi \qquad \frac{1}{2}$$

$$\therefore \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$$

29. Equation of line through (3, – 4, – 5) and (2, –3, 1) is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \qquad \text{...(i)} \qquad 1$$

Eqn. of plane through the three given points is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$$

or $\quad 2x + y + z - 7 = 0 \qquad$ ...(ii) 2

Any point on line (i) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 1$

$\Rightarrow \quad \lambda = 2$ 1

Required point is (1, –2, 7) 1

OR

Equation of plane cutting intercepts (say, a, b, c) on the axes is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, with A(a, 0, 0), B(0, b, 0) and C(0, 0, c) 1

distance of this plane from orgin is $3p = \dfrac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \qquad \text{...(i)} \qquad 1$$

Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (x, y, z)$ 1

$\Rightarrow \quad a = 3x,\ b = 3y,\ c = 3z$, we get from (i) $\frac{1}{2}$

$$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2} \quad \text{or} \quad \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \qquad 1$$

QUESTION PAPER CODE 65/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $-\log|\sin 2x| + c$ OR $\log|\sec x| - \log|\sin x| + c$. 1

2. Writing the equations as $\left.\begin{array}{l} 2x - y + 2z = 5 \\ 2x - y + 2z = 8 \end{array}\right\}$ $\frac{1}{2}$

$\Rightarrow$ Distance = 1 unit $\frac{1}{2}$

3. $|A| = 8$. 1

4. $k = 12$. 1

SECTION B

5. Event A: Number obtained is even

B: Number obtained is red.

$P(A) = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{3}{6} = \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$

$P(A \cap B)$ = P (getting an even red number) = $\frac{1}{6}$ $\frac{1}{2}$

Since $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(P \cap B)$ which is $\frac{1}{6}$ $\frac{1}{2}$

$\therefore$ A and B are not independent events.

6. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize $C = 300x + 400y$ $\frac{1}{2}$

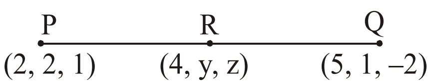
Subject to: $\left\{\begin{array}{l} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{array}\right.$ $1\frac{1}{2}$

7. Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ $\frac{1}{2}$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$ $\frac{1}{2}$

$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3}$ $\therefore$ z coord. $= -3\left(\frac{2}{3}\right)+1 = -1.$ $\frac{1}{2}+\frac{1}{2}$

OR



Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2. \qquad 1$$

$$\therefore\ z = \frac{2(-2)+1(1)}{2+1} = \frac{-3}{3} = -1. \qquad 1$$

8. $\int \frac{dx}{5-8x-x^2} = \int \frac{dx}{(\sqrt{21})^2-(x+4)^2}$ 1

$$= \frac{1}{2\sqrt{21}} \log\left|\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right| + C \qquad 1$$

9. Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 1

$\Rightarrow$ $|A| = -a(bc) + a(bc) = 0$ 1

OR

Since A is a skew-symmetric matrix $\therefore$ $A^T = -A$ $\frac{1}{2}$

$\therefore$ $|A^T| = |-A| = (-1)^3.|A|$ $\frac{1}{2}$

$\Rightarrow$ $|A| = -|A|$ $\frac{1}{2}$

$\Rightarrow$ $2|A| = 0$ or $|A| = 0.$ $\frac{1}{2}$

10. $f(x) = x^3 - 3x$

$\therefore \quad f'(c) = 3c^2 - 3 = 0$ $\frac{1}{2}$

$\therefore \quad c^2 = 1 \quad \Rightarrow \quad c = \pm 1.$ $\frac{1}{2}$

Rejecting c = 1 as it does not belong to $(-\sqrt{3}, 0)$, $\frac{1}{2}$

we get c = –1. $\frac{1}{2}$

11. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$ $\frac{1}{2}$

$= 3[x^2 - 2x + 2] = 3[(x-1)^2 + 1]$ 1

since $f'(x) > 0 \ \forall \ x \in \mathbb{R} \ \therefore$ f(x) is increasing on $\mathbb{R}$ $\frac{1}{2}$

12. Given $\frac{dx}{dt} = -5$ cm/m., $\frac{dy}{dt} = 4$ cm/m.

$A = xy \Rightarrow \frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$ 1

$= 8(4) + 6(-5) = 2$

$\therefore$ Area is increasing at the rate of 2 cm^2/minute. 1

SECTION C

13. $I = \int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx = \int_0^\pi \frac{(\pi - x)\tan x}{\sec x + \tan x} dx$ 1

$\Rightarrow 2I = \pi \int_0^\pi \frac{\tan x}{\sec x + \tan x} dx = \pi \int_0^\pi \tan x\,(\sec x - \tan x)\, dx$

$I = \frac{\pi}{2} \int_0^\pi (\sec x \tan x - \sec^2 x + 1)\, dx$ 1

$= \frac{\pi}{2} [\sec x - \tan x + x]_0^\pi$ 1

$= \frac{\pi(\pi - 2)}{2}$ 1

OR

$$I = \int_1^4 \{|x-1|+|x-2|+|x-4|\}\,dx$$

$$= \int_1^4 (x-1)\,dx - \int_1^2 (x-2)\,dx + \int_2^4 (x-2)\,dx - \int_1^4 (x-4)\,dx \qquad 2$$

$$= \left.\frac{(x-1)^2}{2}\right]_1^4 - \left.\frac{(x-2)^2}{2}\right]_1^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 - \left.\frac{(x-4)^2}{2}\right]_1^4 \qquad 1$$

$$= \frac{9}{2}+\frac{1}{2}+2+\frac{9}{2} = 11\frac{1}{2} \text{ or } \frac{23}{2} \qquad 1$$

14. $\overrightarrow{AB} = -\hat{i}-2\hat{j}-6\hat{k}, \overrightarrow{BC} = 2\hat{i}-\hat{j}+\hat{k}, \overrightarrow{CA} = -\hat{i}+3\hat{j}+5\hat{k}$ 1

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \vec{0}$ $\therefore$ A, B, C form a triangle 1

Also $\overrightarrow{BC}\cdot\overrightarrow{CA} = 0$ $\therefore$ A, B, C form a right triangle 1

Area of $\Delta = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ 1

15. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore$ X :	4	6	8	10	12	1
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
xP(X) :	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{16}{6}$	$\frac{10}{6}$	$\frac{12}{6}$	
x^2P(X) :	$\frac{16}{6}$	$\frac{36}{6}$	$\frac{128}{6}$	$\frac{100}{6}$	$\frac{144}{6}$	

$\Sigma xP(x) = \frac{48}{6} = 8 \;\therefore \text{Mean} = 8$ 1

Variance $= \Sigma x^2P(x) - [\Sigma xP(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$ 1

16. Let E_1: Selecting a student with 100% attendance
E_2: Selecting a student who is not regular
1

A: selected student attains A grade.

$P(E_1) = \frac{30}{100}$ and $P(E_2) = \frac{70}{100}$ $\frac{1}{2}$

$P(A/E_1) = \frac{70}{100}$ and $P(A/E_2) = \frac{10}{100}$ $\frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}}$$

$$= \frac{3}{4}$$ 1

Regularity is required everywhere or any relevant value 1

17. $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\frac{x-3}{x-4}\cdot\frac{x+3}{x+4}}\right) = \frac{\pi}{4}$ $1\frac{1}{2}$

$\Rightarrow \frac{2x^2-24}{-7} = 1 \Rightarrow x^2 = \frac{17}{2}$ $1\frac{1}{2}$

$\Rightarrow x = \pm\sqrt{\frac{17}{2}}$ 1

18. $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1+1

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding

$(a - 1)^2 \cdot (a - 1) = (a - 1)^3$. 1

OR

Let $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \quad 2a - c = -1, \quad 2b - d = -8$

$a = 1, \quad b = -2$ 1

$-3a + 4c = 9, \quad -3b + 4d = 22$

Solving to get $a = 1$, $b = -2$, $c = 3$, $d = 4$

$\therefore \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ 1

19. $x^y + y^x = a^b$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$\therefore \quad \dfrac{du}{dx} + \dfrac{dv}{dx} = 0 \qquad \text{...(i)}$ $\frac{1}{2}$

$$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$$ 1

$$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$ 1

Putting in (i) $x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$$ 1

OR

$$e^y \cdot (x + 1) = 1 \Rightarrow e^y \cdot 1 + (x+1) \cdot e^y \cdot \frac{dy}{d} = 0$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(x+1)}$$ 1

$$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$$ $1\frac{1}{2}$

20. $I = \int \frac{\sin\theta\, d\theta}{(4+\cos^2\theta)(2-\sin^2\theta)} = \int \frac{\sin\theta\, d\theta}{(4+\cos^2\theta)(1+\cos^2\theta)}$ $\frac{1}{2}$

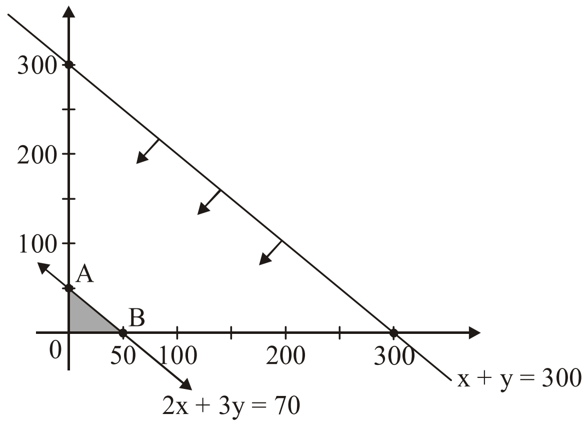
$$= -\int \frac{dt}{(4+t^2)(1+t^2)}, \text{ where } \cos\theta = t$$ 1

$$= \int \frac{1/3}{4+t^2}\, dt - \int \frac{1/3}{1+t^2}\, dt$$ 1

$$= \frac{1}{6}\tan^{-1}\frac{t}{2} - \frac{1}{3}\tan^{-1} t + c$$ 1

$$= \frac{1}{6}\tan^{-1}\left(\frac{\cos\theta}{2}\right) - \frac{1}{3}\tan^{-1}(\cos\theta) + c$$ $\frac{1}{2}$

21. Maximise: $z = 34x + 45y$ subject to $x + y \le 300$, $2x + 3y \le 70$, $x \ge 0$, $y \ge 0$



Plotting the two lines. 2

Correct shading 1

$z(A) = z\left(0, \frac{70}{3}\right) = 1050$

$z(B) = z(35, 0) = 1190$

$\Rightarrow$ max (1190) at $x = 35$, $y = 0$. 1

22. Points A, B, C and D are coplanar, then the vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{AD}$ must be coplanar.

$\overrightarrow{AB} = \hat{i} + (x-2)\hat{j} + 4\hat{k}$; $\overrightarrow{AC} = \hat{i} - 3\hat{k}$, $\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

i.e., $\begin{vmatrix} 1 & x-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$ 1

$\Rightarrow 1(9) - (x-2)(7) + 4(3) = 0 \Rightarrow x = 5.$ $1\frac{1}{2}$

23. Given differential equation can be written as

$y\frac{dx}{dy} - x = 2y^2$ or $\frac{dx}{dy} - \frac{1}{y} \cdot x = 2y$ 1

Integrating factor is $e^{-\log y} = \frac{1}{y}$ 1

$\therefore$ Solution is $x \cdot \frac{1}{y} = \int 2\,dy = 2y + c$ 2

or $x = 2y^2 + cy$.

SECTION D

24. Equation of line through (3, – 4, – 5) and (2, –3, 1) is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \qquad ...(i)$$ 1

Eqn. of plane through the three given points is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$$

or $2x + y + z - 7 = 0$ ...(ii) 2

Any point on line (i) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 1$

$\Rightarrow \lambda = 2$ 1

Required point is (1, –2, 7) 1

OR

Equation of plane cutting intercepts (say, a, b, c) on the axes is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, with A(a, 0, 0), B(0, b, 0) and C(0, 0, c) 1

distance of this plane from orgin is $3p = \dfrac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$ $1\frac{1}{2}$

$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \qquad ...(i)$ 1

Centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (x, y, z)$ 1

$\Rightarrow \quad a = 3x, b = 3y, c = 3z$, we get from (i) $\frac{1}{2}$

$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2}$ or $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$ 1

25. $\dfrac{dy}{dx} = \dfrac{x+2y}{x-y} = \dfrac{1+\dfrac{2y}{x}}{1-\dfrac{y}{x}}$ $\frac{1}{2}$

$\dfrac{y}{x} = v \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx} \quad \therefore \quad v + x\dfrac{dv}{dx} = \dfrac{1+2v}{1-v}$ $\frac{1}{2}$

$\Rightarrow \quad x\dfrac{dv}{dx} = -\dfrac{1+2v-v+v^2}{v-1} \Rightarrow \displaystyle\int \frac{v-1}{v^2+v+1}dv = -\frac{dx}{x}$ 1

$\Rightarrow \quad \displaystyle\int \frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx \Rightarrow \int \frac{2v+1}{v^2+v+1}dv - 3\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int \frac{2}{x}dx$ 1+1

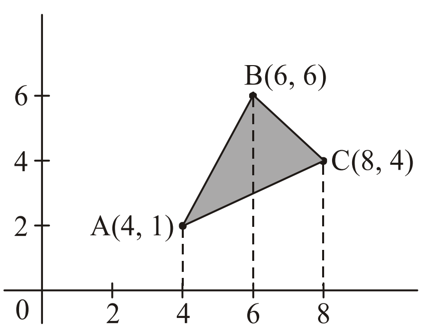
$\Rightarrow \quad \log|v^2+v+1| - 3\cdot\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + c$ 1

$\Rightarrow \quad \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\dfrac{2y+x}{\sqrt{3}x}\right) = c$ $\frac{1}{2}$

$x = 1,\ y = 0 \Rightarrow c = -2\sqrt{3}\cdot\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{3}\pi$ $\frac{1}{2}$

$\therefore \quad \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\dfrac{2y+x}{\sqrt{3}x}\right) + \dfrac{\sqrt{3}}{3}\pi = 0$

26. Figure 1



$\left.\begin{array}{l}\text{Equation of AB}: y = \dfrac{5}{2}x - 9 \\ \text{Equation of BC}: y = 12 - x \\ \text{Equation of AC}: y = \dfrac{3}{4}x - 2\end{array}\right\}$ $1\frac{1}{2}$

$\therefore \text{ Area (A)} = \displaystyle\int_4^6 \left(\frac{5}{2}x - 9\right)dx + \int_6^8 (12-x)\,dx - \int_4^8 \left(\frac{3}{4}x - 2\right)dx$ 1

$= \left[\dfrac{5}{4}x^2 - 9x\right]_4^6 + \left[12x - \dfrac{x^2}{2}\right]_6^8 - \left[\dfrac{3}{8}x^2 - 2x\right]_4^8$ $1\frac{1}{2}$

$= 7 + 10 - 10 = 7$ sq.units 1

OR

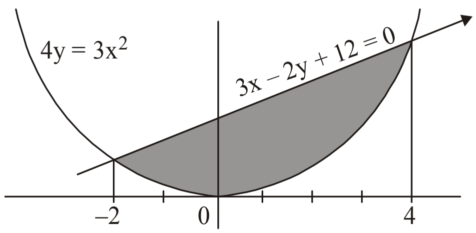


Figure 1

$4y = 3x^2$ and $3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$

$\Rightarrow 3x^2 - 6x - 24 = 0$ or $x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$

$\Rightarrow$ x-coordinates of points of intersection are $x = -2$, $x = 4$ 1

$\therefore$ Area (A) $= \int_{-2}^{4}\left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2\right]dx$ $1\frac{1}{2}$

$= \left[\frac{1}{2}\frac{(3x+12)^2}{6} - \frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4}$ $1\frac{1}{2}$

$= 45 - 18 = 27$ sq.units 1

27. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$

$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1–1 function 2

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow x = \frac{4y-3}{4-3y} \quad \therefore \forall y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is ONTO and so bijective 2

and $f^{-1}(y) = \frac{4y-3}{4-3y}; y \in R - \left\{\frac{4}{3}\right\}$ 1

$f^{-1}(0) = -\frac{3}{4}$ $\frac{1}{2}$

and $f^{-1}(x) = 2 \Rightarrow \frac{4x-3}{4-3x} = 2$

$\Rightarrow$ $4x - 3 = 8 - 6x$

$\Rightarrow$ $10x = 11 \Rightarrow x = \frac{11}{10}$ $\frac{1}{2}$

OR

$(a, b) * (c, d) = (ac, b + ad); (a, b), (c, d) \in A$

$(c, d) * (a, b) = (ca, d + bc)$

Since $b + ad \neq d + bc \Rightarrow *$ is NOT comutative $1\frac{1}{2}$

for associativity, we have,

$[(a,b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf)$

$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf) = (ace, b + ad + acf)$ $1\frac{1}{2}$

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element in A

Then $(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow$ $(ae, b + af) = (a, b) = (ae, f + be)$

$\Rightarrow$ $e = 1, f = 0 \Rightarrow (1, 0)$ is the identity element $1\frac{1}{2}$

(ii) Let (c, d) be the inverse element for (a, b)

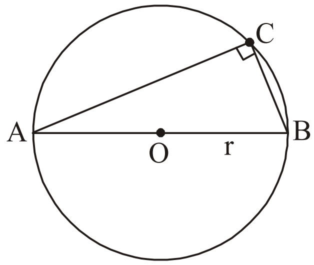
$\Rightarrow$ $(a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow$ $(ac, b + ad) = (1, 0) = (ac, d + bc)$

$\Rightarrow ac = 1 \Rightarrow c = \frac{1}{a}$ and $b + ad = 0 \Rightarrow d = -\frac{b}{a}$ and $d + bc = 0 \Rightarrow d = -bc = -b\left(\frac{1}{a}\right)$

$\Rightarrow \left(\frac{1}{a}, -\frac{b}{a}\right)$, $a \neq 0$ is the inverse of $(a, b) \in A$ $1\frac{1}{2}$

28.



Correct Figure 1

Let the length of sides of ΔABC are, $AC = x$ and $BC = y$

$\Rightarrow x^2 + y^2 = 4r^2$ and Area $A = \frac{1}{2}xy$ 1

$A = \frac{1}{2}x\sqrt{4r^2 - x^2}$ or $S = \frac{x^2}{4}(4r^2 - x^2)$

$S = \frac{1}{4}[4r^2x^2 - x^4]$ 1

$\therefore \frac{dS}{dx} = \frac{1}{4}[8r^2x - 4x^3]$

$\frac{dS}{dx} = 0 \Rightarrow 2r^2 = x^2 \Rightarrow x = \sqrt{2}\,r$ 1

and $y = \sqrt{4r^2 - 2r^2} = \sqrt{2}\,r$ $\frac{1}{2}$

and $\frac{d^2S}{dx^2} = \frac{1}{4}[8r^2 - 12x^2] = \frac{1}{4}[8r^2 - 24r^2] < 0$ 1

$\therefore$ For maximum area, $x = y$ i.e., Δ is isosceles. $\frac{1}{2}$

29. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \Rightarrow |A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$ 1

$A_{11} = 0,\ A_{12} = 2,\ A_{13} = 1$

$A_{21} = -1$, A22 $= -9,\ A_{23} = -5$ 2

$A_{31} = 2,\ A_{32} = 23,\ A_{33} = 13$

$$\Rightarrow A^{-1} = -1\begin{pmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{pmatrix}^T = -1\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$$ $\frac{1}{2}$

Given equations can be written as

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \text{ or } AX = B$$

$\Rightarrow X = A^{-1}B$ 1

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\Rightarrow x = 1,\ y = 2,\ z = 3.$ $1\frac{1}{2}$

QUESTION PAPER CODE 65/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. k = 12. 1

2. |A| = 8. 1

3. Writing the equations as $\left.\begin{array}{l}2x-y+2z=5\\2x-y+2z=8\end{array}\right\}$ $\frac{1}{2}$

$\Rightarrow$ Distance = 1 unit $\frac{1}{2}$

4. –log |sin 2x| + c OR log |sec x| – log |sin x| + c. 1

SECTION B

5. $\int\frac{dx}{5-8x-x^2} = \int\frac{dx}{(\sqrt{21})^2-(x+4)^2}$ 1

$= \frac{1}{2\sqrt{21}}\log\left|\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right|+c$ 1

6. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize C = 300x + 400y $\frac{1}{2}$

Subject to: $\begin{cases}6x+10y\geq 60\\4x+4y\geq 32\\x\geq 0, y\geq 0\end{cases}$ $1\frac{1}{2}$

7. Event A: Number obtained is even

B: Number obtained is red.

$P(A) = \frac{3}{6}=\frac{1}{2}, P(B)=\frac{3}{6}=\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$

$P(A\cap B)$ = P (getting an even red number) = $\frac{1}{6}$ $\frac{1}{2}$

Since $P(A)\cdot P(B) = \frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}\neq P(P\cap B)$ which is $\frac{1}{6}$ $\frac{1}{2}$

$\therefore$ A and B are not independent events.

8. Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ $\frac{1}{2}$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$ $\frac{1}{2}$

$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3}$ $\therefore$ z coord. $= -3\left(\frac{2}{3}\right)+1 = -1.$ $\frac{1}{2}+\frac{1}{2}$

OR

P (2, 2, 1) R (4, y, z) Q (5, 1, –2)

Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2.$$ 1

$$\therefore\ z = \frac{2(-2)+1(1)}{2+1} = \frac{-3}{3} = -1.$$ 1

9. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$ $\frac{1}{2}$

$= 3[x^2 - 2x + 2] = 3[(x-1)^2 + 1]$ 1

since $f'(x) > 0\ \forall\ x \in \mathbb{R}$ $\therefore$ f(x) is increasing on $\mathbb{R}$ $\frac{1}{2}$

10. $f(x) = x^3 - 3x$

$\therefore$ $f'(c) = 3c^2 - 3 = 0$ $\frac{1}{2}$

$\therefore$ $c^2 = 1 \Rightarrow c = \pm 1.$ $\frac{1}{2}$

Rejecting c = 1 as it does not belong to $(-\sqrt{3}, 0)$, $\frac{1}{2}$

we get c = –1. $\frac{1}{2}$

11. Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 1

$\Rightarrow$ $|A| = -a(bc) + a(bc) = 0$ 1

OR

Since A is a skew-symmetric matrix $\therefore\ A^T = -A$ $\frac{1}{2}$

$\therefore\quad |A^T| = |-A| = (-1)^3.|A|$ $\frac{1}{2}$

$\Rightarrow\quad |A| = -|A|$ $\frac{1}{2}$

$\Rightarrow\quad 2|A| = 0$ or $|A| = 0.$ $\frac{1}{2}$

12. $\frac{dV}{dt} = 8$ cm^3/s, where V is the volume of sphere i.e., $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\cdot\frac{dV}{dt}$ 1

$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r\cdot\frac{1}{4\pi r^2}\cdot 8$ $\frac{1}{2}$

$= \frac{2\times 8}{12} = \frac{4}{3}\text{cm}^2/\text{s}$ $\frac{1}{2}$

SECTION C

13. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore$ X :	4	6	8	10	12	1
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
xP(X) :	$\frac{4}{6}$	$\frac{6}{6}$	$\frac{16}{6}$	$\frac{10}{6}$	$\frac{12}{6}$	
x^2P(X) :	$\frac{16}{6}$	$\frac{36}{6}$	$\frac{128}{6}$	$\frac{100}{6}$	$\frac{144}{6}$	

$\Sigma xP(x) = \frac{48}{6} = 8 \;\therefore \text{Mean} = 8$ 1

Variance $= \Sigma x^2P(x) - [\Sigma xP(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$ 1

14. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ 1

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ $\therefore$ A, B, C form a triangle 1

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$ $\therefore$ A, B, C form a right triangle 1

Area of $\Delta = \frac{1}{2} | \overrightarrow{AB} \times \overrightarrow{BC} | = \frac{1}{2}\sqrt{210}$ 1

15. Let E_1: Selecting a student with 100% attendance
E_2: Selecting a student who is not regular 1

A: selected student attains A grade.

$P(E_1) = \frac{30}{100}$ and $P(E_2) = \frac{70}{100}$ $\frac{1}{2}$

$P(A/E_1) = \frac{70}{100}$ and $P(A/E_2) = \frac{10}{100}$ $\frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{3}{4}$$ 1

Regularity is required everywhere or any relevant value 1

16. $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \frac{x-3}{x-4} \cdot \frac{x+3}{x+4}}\right) = \frac{\pi}{4}$$ $1\frac{1}{2}$

$$\Rightarrow \quad \frac{2x^2 - 24}{-7} = 1 \;\Rightarrow\; x^2 = \frac{17}{2}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x = \pm\sqrt{\frac{17}{2}}$$ 1

17. $\Delta = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1+1

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding

$(a-1)^2 \cdot (a-1) = (a-1)^3.$ 1

OR

Let $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \begin{pmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$ 1

$\Rightarrow \quad 2a - c = -1, \quad 2b - d = -8$

$a = 1, \quad b = -2$ 1

$-3a + 4c = 9, \quad -3b + 4d = 22$

Solving to get $a = 1$, $b = -2$, $c = 3$, $d = 4$

$\therefore \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ 1

18. $x^y + y^x = a^b$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$\therefore \quad \frac{du}{dx} + \frac{dv}{dx} = 0 \qquad \text{...(i)}$ $\frac{1}{2}$

$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y \left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right]$ 1

$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x \left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$ 1

Putting in (i) $x^y \left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x \left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$ $\frac{1}{2}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$ 1

OR

$e^y \cdot (x + 1) = 1 \Rightarrow e^y \cdot 1 + (x+1) \cdot e^y \cdot \frac{dy}{d} = 0$ $1\frac{1}{2}$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{(x+1)}$ 1

$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$ $1\frac{1}{2}$

19. $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx$ 1

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx = \pi \int_0^{\pi} \tan x\,(\sec x - \tan x)\, dx$

$I = \frac{\pi}{2} \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)\, dx$ 1

$= \frac{\pi}{2} [\sec x - \tan x + x]_0^{\pi}$ 1

$= \frac{\pi(\pi - 2)}{2}$ 1

OR

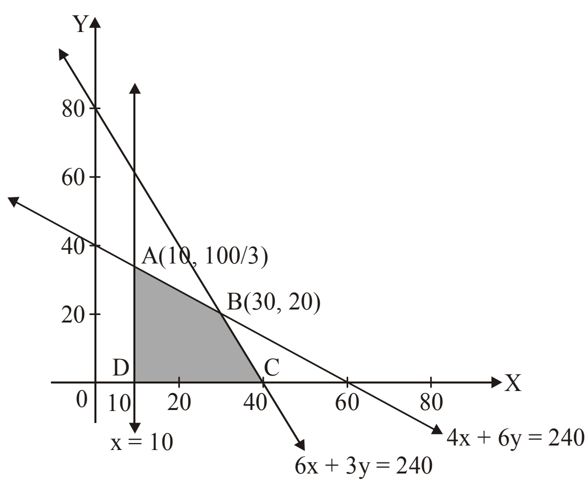
$$I = \int_1^4 \{|x-1|+|x-2|+|x-4|\}\,dx$$

$$= \int_1^4 (x-1)\,dx - \int_1^2 (x-2)\,dx + \int_2^4 (x-2)\,dx - \int_1^4 (x-4)\,dx \qquad 2$$

$$= \left.\frac{(x-1)^2}{2}\right]_1^4 - \left.\frac{(x-2)^2}{2}\right]_1^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 - \left.\frac{(x-4)^2}{2}\right]_1^4 \qquad 1$$

$$= \frac{9}{2}+\frac{1}{2}+2+\frac{9}{2} = 11\frac{1}{2} \text{ or } \frac{23}{2} \qquad 1$$

20. Maximise z = 7x + 10y, subject to $4x + 6y \le 240$; $6x + 3y \le 240$; $x \ge 10$, $x \ge 0$, $y \ge 0$



Correct graph of three lines $1\frac{1}{2}$

For correct shading $1\frac{1}{2}$

$Z(A) = Z\left(10, \frac{200}{6}\right) = 70 + 10 \times \frac{100}{3} = 403\frac{1}{3}$

Z(B) = Z(30, 20) = 210 + 200 = 410

Z(C) = Z(40, 0) = 280 + 0 = 280

Z(D) = Z(10, 0) = 70 + 0 = 70

$\Rightarrow$ Max (= 410) at x = 30, y = 20 1

21. $I = \int \frac{e^x dx}{(e^x-1)^2(e^x+2)} = \int \frac{dt}{(t+2)(t-1)^2}$ where $e^x = t$ $\frac{1}{2}$

$$= \int \frac{1/9}{(t+2)}\,dt - \int \frac{1/9}{(t-1)}\,dt + \int \frac{1/3}{(t-1)^2}\,dt \qquad 1\frac{1}{2}$$

$$= \frac{1}{9}[\log|t+2| - \log|t-1|] - \frac{1}{3(t-1)} + c \qquad 1\frac{1}{2}$$

$$= \frac{1}{9}\log\left|\frac{e^x+2}{e^x-1}\right| - \frac{1}{3(e^x-1)} + c \qquad \frac{1}{2}$$

22. $\vec{b}_1 \| \vec{a} \Rightarrow$ let $\vec{b}_1 = \lambda(2\hat{i} - \hat{j} - 2\hat{k})$ $\frac{1}{2}$

$\vec{b}_2 = \vec{b} - \vec{b}_1 = (7\hat{i} + 2\hat{j} - 3\hat{k}) - (2\lambda i - \lambda\hat{j} - 2\lambda\hat{k})$ $\frac{1}{2}$

$= (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} - (3 - 2\lambda)\hat{k}$ 1

$\vec{b}_2 \perp \vec{a} \Rightarrow 2(7 - 2\lambda) - 1(2 + \lambda) + 2(3 - 2\lambda) = 0$

$\Rightarrow \lambda = 2$ 1

$\therefore \vec{b}_1 = 4\hat{i} - 2\hat{j} - 4\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 4\hat{j} + \hat{k}$ 1

$\Rightarrow (7\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} - 2\hat{j} - 4\hat{k}) + (3\hat{i} + 4\hat{j} + \hat{k})$

23. Given differential equation is $\frac{dy}{dx} - y = \sin x$ $\frac{1}{2}$

$\Rightarrow$ Integrating factor $= e^{-x}$

$\therefore$ Solution is: $\lambda e^{-x} = \int \sin x\, e^{-x}\, dx = I_1$ 1

$I_1 = -\sin x e^{-x} + \int \cos x\, e^{-x} dx$

$= -\sin x\, e^{-x} + [-\cos x\, e^{-x} - \int + \sin x\, e^{-x} dx]$

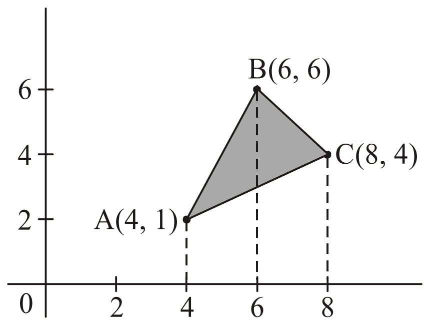
$I_1 = \frac{1}{2}[-\sin x - \cos x]e^{-x}$ $1\frac{1}{2}$

$\therefore$ Solution is $\lambda e^{-x} = \frac{1}{2}(-\sin x - \cos x)e^{-x} + c$ 1

or $y = -\frac{1}{2}(\sin x + \cos x) + ce^{x}$

SECTION D

24. Figure 1



$$\left.\begin{aligned}&\text{Equation of } AB: y=\frac{5}{2}x-9\\&\text{Equation of } BC: y=12-x\\&\text{Equation of } AC: y=\frac{3}{4}x-2\end{aligned}\right\}$$ $1\frac{1}{2}$

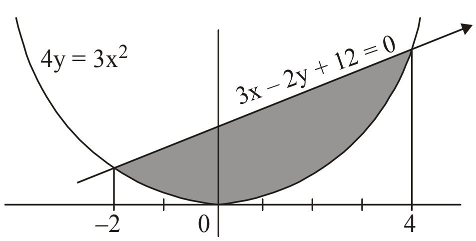
$\therefore$ Area (A) $= \int_4^6\left(\frac{5}{2}x-9\right)dx+\int_6^8(12-x)\,dx-\int_4^8\left(\frac{3}{4}x-2\right)dx$ 1

$= \left[\frac{5}{4}x^2-9x\right]_4^6+\left[12x-\frac{x^2}{2}\right]_6^8-\left[\frac{3}{8}x^2-2x\right]_4^8$ $1\frac{1}{2}$

$= 7 + 10 - 10 = 7$ sq.units 1

OR

Figure 1



$4y = 3x^2$ and $3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$

$\Rightarrow 3x^2 - 6x - 24 = 0$ or $x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$

$\Rightarrow$ x-coordinates of points of intersection are $x = -2$, $x = 4$ 1

$\therefore$ Area (A) $= \int_{-2}^{4}\left[\frac{1}{2}(3x+12)-\frac{3}{4}x^2\right]dx$ $1\frac{1}{2}$

$= \left[\frac{1}{2}\frac{(3x+12)^2}{6}-\frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4}$ $1\frac{1}{2}$

$= 45 - 18 = 27$ sq.units 1

25. $\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}}$ $\frac{1}{2}$

$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \therefore \; v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$ $\frac{1}{2}$

$\Rightarrow \quad x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1} \Rightarrow \int \frac{v-1}{v^2+v+1}dv = -\frac{dx}{x}$ 1

$\Rightarrow \quad \int \frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx \Rightarrow \int \frac{2v+1}{v^2+v+1}dv - 3\int \frac{1}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int \frac{2}{x}dx$ 1+1

$\Rightarrow \quad \log|v^2+v+1| - 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x|^2 + c$ 1

$\Rightarrow \quad \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = c$ $\frac{1}{2}$

$x = 1,\; y = 0 \Rightarrow c = -2\sqrt{3}\cdot\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi$ $\frac{1}{2}$

$\therefore \quad \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$

26. Equation of line through (3, – 4, – 5) and (2, –3, 1) is

$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ ...(i) 1

Eqn. of plane through the three given points is

$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$

or $2x + y + z - 7 = 0$ ...(ii) 2

Any point on line (i) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 1$

$\Rightarrow \quad \lambda = 2$ 1

Required point is (1, –2, 7) 1

OR

Equation of plane cutting intercepts (say, a, b, c) on the axes is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$, with A(a, 0, 0), B(0, b, 0) and C(0, 0, c) 1

distance of this plane from orgin is $3p = \frac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$ $1\frac{1}{2}$

$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} = \frac{1}{9p^2} \qquad ...(i)$ 1

Centroid of ΔABC is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right) = (x, y, z)$ 1

$\Rightarrow \quad a = 3x, b = 3y, c = 3z$, we get from (i) $\frac{1}{2}$

$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2} = \frac{1}{9p^2}$ or $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} = \frac{1}{p^2}$ 1

27. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow \quad \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$

$\Rightarrow \quad 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow \quad 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1–1 function 2

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow \quad x = \frac{4y-3}{4-3y} \quad \therefore \; \forall\, y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is ONTO and so bijective 2

and $f^{-1}(y) = \dfrac{4y-3}{4-3y}; y \in R - \left\{\dfrac{4}{3}\right\}$ 1

$f^{-1}(0) = -\dfrac{3}{4}$ $\dfrac{1}{2}$

and $f^{-1}(x) = 2 \Rightarrow \dfrac{4x-3}{4-3x} = 2$

$\Rightarrow \quad 4x - 3 = 8 - 6x$

$\Rightarrow \quad 10x = 11 \Rightarrow x = \dfrac{11}{10}$ $\dfrac{1}{2}$

OR

$(a, b) * (c, d) = (ac, b + ad); (a, b), (c, d) \in A$

$(c, d) * (a, b) = (ca, d + bc)$

Since $b + ad \neq d + bc \Rightarrow *$ is NOT comutative $1\dfrac{1}{2}$

for associativity, we have,

$[(a,b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf)$

$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf) = (ace, b + ad + acf)$ $1\dfrac{1}{2}$

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element in A

Then $(a, b) * (e, f) = (a, b) = (e, f) * (a, b)$

$\Rightarrow \quad (ae, b + af) = (a, b) = (ae, f + be)$

$\Rightarrow \quad e = 1, f = 0 \Rightarrow (1, 0)$ is the identity element $1\dfrac{1}{2}$

(ii) Let (c, d) be the inverse element for (a, b)

$\Rightarrow \quad (a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$

$\Rightarrow \quad (ac, b + ad) = (1, 0) = (ac, d + bc)$

$\Rightarrow ac = 1 \Rightarrow c = \dfrac{1}{a}$ and $b + ad = 0 \Rightarrow d = -\dfrac{b}{a}$ and $d + bc = 0 \Rightarrow d = -bc = -b\left(\dfrac{1}{a}\right)$

$\Rightarrow \left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$, $a \neq 0$ is the inverse of $(a, b) \in A$ $1\dfrac{1}{2}$

28. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \Rightarrow |A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$ 1

$A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A22 = -9, A_{23} = -5$ 2

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$\Rightarrow A^{-1} = -1\begin{pmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{pmatrix}^T = -1\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$ $\frac{1}{2}$

Given equations can be written as

$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}$ or $AX = B$

$\Rightarrow X = A^{-1}B$ 1

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\Rightarrow x = 1,\ y = 2,\ z = 3.$ $1\frac{1}{2}$

29.

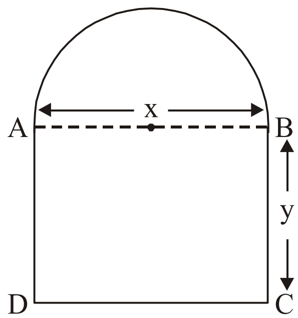


Figure 1

Let dimensions of the rectangle be x and y (as shown)

$\therefore$ Perimeter of window $p = 2y + x + \pi\frac{x}{2} = 10$ m ...(i) $\frac{1}{2}$

Area of window $A = xy + \frac{1}{2}\pi\frac{x^2}{4}$ $\frac{1}{2}$

$$A = x\left[5 - \frac{x}{2} - \pi\frac{x}{4}\right] + \frac{1}{2}\pi\frac{x^2}{4}$$

$$= 5x - \frac{x^2}{2} - \pi\frac{x^2}{8}$$ 1

$$\frac{dA}{dx} = 5 - x - \pi\frac{x}{4} = 0 \Rightarrow x = \frac{20}{4+\pi}$$ 1

$$\frac{d^2A}{dx^2} = \left(-1 - \frac{\pi}{4}\right) < 0$$ 1

$\Rightarrow$ $x = \frac{20}{4+\pi}, y = \frac{10}{4+\pi}$ will give maximum light. 1