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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/D, 65/2/D, 65/3/D

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question(s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/D
EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.
$$p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$$
 ½ + ½ m

2.
$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$$
 ½ + ½ m

3.
$$\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \Rightarrow \theta = \frac{\pi}{6}$$
 ½ + ½ m

4.
$$a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$$
 ½ + ½ m

5.
$$\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$$
 ½ + ½ m

6.
$$I.F = e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$$
 ½ + ½ m

SECTION - B

7. Getting
$$A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$
 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$
 1 m

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix} \quad 1 \text{ m}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} \quad 1 \text{ m}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad 2 \text{ m}$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

$$8. \quad f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \rightarrow R_2 - xR_1 \quad \text{and} \quad R_3 \rightarrow R_3 - x^2R_1$$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix} \quad (\text{For bringing 2 zeroes in any row/column}) \quad 1+1 \text{ m}$$

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2 \quad 1 \text{ m}$$

$$\begin{aligned} \therefore f(2x) - f(x) &= a[2x+a]^2 - a(x+a)^2 \\ &= a x (3x + 2a) \end{aligned} \quad 1 \text{ m}$$

$$\begin{aligned}
9. \quad \int \frac{dx}{\sin x + \sin 2x} &= \int \frac{dx}{\sin x (1 + 2 \cos x)} = \int \frac{\sin x \cdot dx}{(1 - \cos x) (1 + \cos x) (1 + 2 \cos x)} && 1 \text{ m} \\
&= - \int \frac{dt}{(1-t) (1+t) (1+2t)} \quad \text{where } \cos x = t && \frac{1}{2} \text{ m} \\
&= \int \left(\frac{-1/6}{1-t} + \frac{1/2}{1+t} - \frac{4/3}{1+2t} \right) dt && 1\frac{1}{2} \text{ m} \\
&= + \frac{1}{6} \log |1-t| + \frac{1}{2} \log |1+t| - \frac{2}{3} \log |1+2t| + c && \frac{1}{2} \text{ m} \\
&= \frac{1}{6} \log |1 - \cos x| + \frac{1}{2} \log |1 + \cos x| - \frac{2}{3} \log |1 + 2 \cos x| + c && \frac{1}{2} \text{ m}
\end{aligned}$$

OR

$$\begin{aligned}
\int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}} dx &= \int \frac{2 - 3x - (1-x^2)}{\sqrt{1-x^2}} dx && \frac{1}{2} \text{ m} \\
&= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx && 1 \text{ m} \\
&= 2 \sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} \sin^{-1}x + c && (\frac{1}{2}+1+1) \text{ m} \\
\text{or } &= \frac{3}{2} \sin^{-1}x + \frac{1}{2} (6-x)\sqrt{1-x^2} + c
\end{aligned}$$

$$\begin{aligned}
10. \quad I &= \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2 \cos ax \sin bx dx \\
&= I_1 - I_2 && \frac{1}{2} \text{ m}
\end{aligned}$$

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \quad (\text{being an even fun.}) \quad 1 \text{ m}$$

$$I_2 = 0 \quad (\text{being an odd fun.}) \quad 1 \text{ m}$$

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx \quad \frac{1}{2} \text{ m}$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi} \quad \frac{1}{2} \text{ m}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi \quad \frac{1}{2} \text{ m}$$

11. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3} \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

Let A : Getting one Red and one black ball

$$\therefore P(A|E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15}, P(A|E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15} \quad 1+1 \text{ m}$$

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45} \quad 1 \text{ m}$$

OR

	x	:	0	1	2	3	4	
--	---	---	---	---	---	---	---	--

	P(x)	:	${}^4C_0 \left(\frac{1}{2}\right)^4$	${}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)$	${}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$	${}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3$	${}^4C_4 \left(\frac{1}{2}\right)^4$	
--	------	---	--------------------------------------	---	---	---	--------------------------------------	--

	:	=	$\frac{1}{16}$	$= \frac{4}{16}$	$= \frac{6}{16}$	$= \frac{4}{16}$	$= \frac{1}{16}$	
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	xP(x)	:	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	
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	x ² P(x)	:	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$	
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$$\text{Mean} = \sum x P(x) = \frac{32}{16} = 2 \quad \frac{1}{2} \text{ m}$$

$$\text{Variance} = \sum x^2 P(x) - \left(\sum x P(x)\right)^2 = \frac{80}{16} - (2)^2 = 1 \quad \frac{1}{2} \text{ m}$$

12. $\vec{r} \times \vec{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j} \quad 1\frac{1}{2} \text{ m}$

$$\vec{r} \times \vec{j} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j} = x\hat{k} - z\hat{i} \quad 1\frac{1}{2} \text{ m}$$

$$\left(\vec{r} \times \vec{i}\right) \cdot \left(\vec{r} \times \vec{j}\right) = (z\hat{j} - y\hat{k}) \cdot (-z\hat{i} + x\hat{k}) = -xy \quad \frac{1}{2} \text{ m}$$

$$\left(\vec{r} \times \vec{i}\right) \cdot \left(\vec{r} \times \vec{j}\right) + xy = -xy + xy = 0 \quad \frac{1}{2} \text{ m}$$

13. Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3\lambda+2, 4\lambda-1, 12\lambda+2)$ 1 m

If this is the point of intersection with plane $x - y + z = 5$

then $3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Rightarrow \lambda = 0$ 1 m

\therefore Point of intersection is $(2, -1, 2)$ 1 m

Required distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$ 1 m

14. Writing $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$ ½ m

$1+x^2+2x+1=1+x^2 \Rightarrow x = -\frac{1}{2}$ ½ m

OR

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$ 1 m

$\therefore 2(\tan^{-1}x)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0$ 1½ m

$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\pi/4, -\pi/4$ 1 m

$\Rightarrow x = -1$ ½ m

15. Putting $x^2 = \cos \theta$, we get ½ m

$$y = \tan^{-1} \left(\frac{\sqrt{1 + \cos \theta} + \sqrt{1 - \cos \theta}}{\sqrt{1 + \cos \theta} - \sqrt{1 - \cos \theta}} \right) \quad \frac{1}{2} \text{ m}$$

$$= \tan^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left(\frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right) \quad 1 + \frac{1}{2} \text{ m}$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2 \quad \frac{1}{2} \text{ m}$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}} \quad 1 \text{ m}$$

16. $\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$ ½ m

$$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta \quad \frac{1}{2} \text{ m}$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{-a \sin \theta + b \cos \theta} = -\frac{x}{y} \quad 1\frac{1}{2} \text{ m}$$

$$\text{or } y \frac{dy}{dx} + x = 0$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0 \quad 1 \text{ m}$$

$$\text{Using (i) we get } y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0 \quad \frac{1}{2} \text{ m}$$

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

17. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.} \quad 1 \text{ m}$$

$$\text{Area (A)} = \frac{\sqrt{3}x^2}{4} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2/\text{s} \quad 1 \text{ m}$$

18. Writing $x + 3 = -\frac{1}{2}(-4 - 2x) + 1$ 1 m

$$\therefore \int (x+3)\sqrt{3-4x-x^2} dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx + \int \sqrt{7-(x+2)^2} dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c \quad 1+1 \text{ m}$$

19. HF. M P

$$\begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix} \begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix} \quad 1\frac{1}{2} \text{ m}$$

Funds collected by school A : Rs. 7000,

School B : Rs. 6125, School C : Rs. 7875 1 m

Total collected : Rs. 21000 ½ m

For writing one value 1 m

SECTION - C

20. $\forall a, b \in \mathbb{N}, (a, b) R (a, b)$ as $ab(b+a) = ba(a+b)$
 $\therefore R$ is reflexive (i) 2 m

Let $(a, b) R (c, d)$ for $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ (ii)

Also $(c, d) R (a, b) \therefore cb(d+a) = da(c+b)$ (using ii)

$\therefore R$ is symmetric (iii) 2 m

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, for $a, b, c, d, e, f, \in \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ and $cf(d+e) = de(c+f)$ 1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

i.e $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

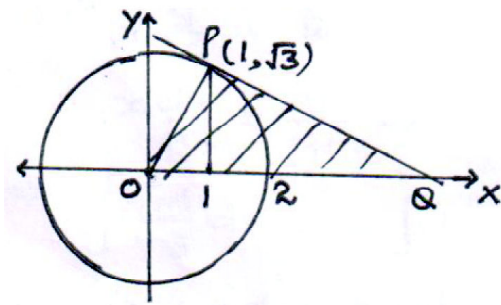
adding we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow af(b+e) = be(a+f)$

Hence $(a, b) R (e, f) \therefore R$ is transitive (iv) ½ m

Form (i), (iii) and (iv) R is an equivalence relation ½ m

21. Correct Fig. 1 m



Eqn. of normal (OP) : $y = \sqrt{3}x$ ½ + ½ m

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x-1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4-x) \quad 1 \text{ m}$$

Coordinates of Q (4, 0) ½ m

$$\therefore \text{Req. area} = \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}} (4-x) \, dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= \sqrt{3} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4 \quad 1 \text{ m}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units} \quad \frac{1}{2} \text{ m}$$

OR

$$\int_1^3 (e^{2-3x} + x^2 + 1) \, dx \quad \text{here } h = \frac{2}{n} \quad \frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [(e^{-1} + 2) + (e^{-1-3h} + 2 + 2h + h^2) + (e^{-1-6h} + 2 + 4h + 4h^2) + \dots$$

$$+ (e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^2 h^2)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [e^{-1}(1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-1)h}) + 2n + 2h(1+2+\dots+(n-1)) + h^2(1^2 + 2^2 + \dots + (n-1)^2)] \quad 1\frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h \left(e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2 \frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6} \right) \quad 1 \text{ m}$$

$$= e^{-1} \cdot \frac{(e^{-6} - 1)}{-3} + 4 + 4 + \frac{8}{3} = -e^{-1} \frac{(e^{-6} - 1)}{3} + \frac{32}{3} \quad 1 \text{ m}$$

22. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2} \quad 1 \text{ m}$$

\therefore Integrating factor is $e^{\tan^{-1}y}$ 1 m

$$\therefore \text{ Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^t dt \text{ where } \tan^{-1}y = t \quad 1 \text{ m}$$

$$= t e^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \quad 1\frac{1}{2} \text{ m}$$

$$\text{or } x = \tan^{-1}y - 1 + c e^{-\tan^{-1}y}$$

OR

Given differential equation is $\frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$

$$\text{Putting } \frac{y}{x} = v \text{ to get } v + x \frac{dv}{dx} = \frac{v}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore x \frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow \int \frac{v^2+1}{v^3} dv = - \int \frac{dx}{x} \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \log |v| - \frac{1}{2v^2} = - \log |x| + c \quad 1 \text{ m}$$

$$\therefore \log y - \frac{x^2}{2y^2} = c \quad 1 \text{ m}$$

$$x = 0, y = 1 \Rightarrow c = 0 \therefore \log y - \frac{x^2}{2y^2} = 0 \quad \frac{1}{2} \text{ m}$$

23. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$ 1 m

$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}$, hence $k = \frac{9}{2}$ 2½ m

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$
 1 m

$\Rightarrow -5(x-1) + 2(y+1) + 1(z-1) = 0$ 1 m

i.e. $5x - 2y - z - 6 = 0$ ½ m

24. $P(\bar{A} \cap B) = \frac{2}{15} \Rightarrow P(\bar{A}) \cdot P(B) = \frac{2}{15}$ 1 m

$P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\bar{B}) = \frac{1}{6}$ 1 m

$\therefore (1 - P(A))P(B) = \frac{2}{15}$ or $P(B) - P(A) \cdot P(B) = \frac{2}{15}$ (i) 1 m

$P(A)(1 - P(B)) = \frac{1}{6}$ or $P(A) - P(A) \cdot P(B) = \frac{1}{6}$ (ii) 1 m

From (i) and (ii) $P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$ ½ m

Let $P(A) = x, P(B) = y \therefore x = \left(\frac{1}{30} + y\right)$

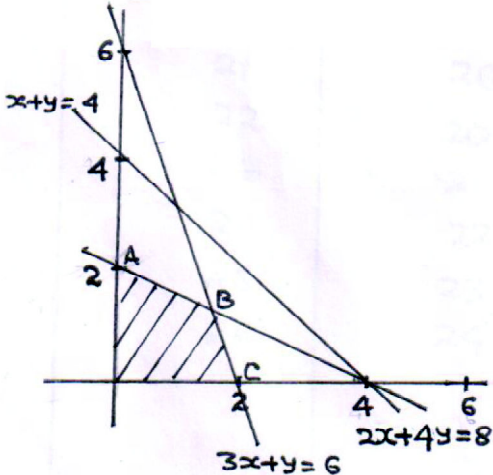
(i) $\Rightarrow y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0$ ½ m

Solving to get $y = \frac{1}{6}$ or $y = \frac{4}{5}$

$\therefore x = \frac{1}{5}$ or $x = \frac{5}{6}$ ½ m

Hence $P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$ OR $P(A) = \frac{5}{6}, P(B) = \frac{4}{5}$ ½ m

25. $f(x) = \sin x - \cos x, 0 < x < 2\pi$
- $f'(x) = 0 \Rightarrow \cos x + \sin x = 0$ or $\tan x = -1,$ 1 m
- $\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4}$ 1 m
- $f''(x) = \cos x - \sin x$ 1 m
- $f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$ i.e - ve so, $x = \frac{3\pi}{4}$ is Local Maxima 1 m
- and $f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ i.e + ve so, $x = \frac{7\pi}{4}$ is Local Minima 1 m
- Local Maximum value = $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ ½ m
- Local Minimum value = $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$ ½ m

26. 

Correct graphs of three lines 1x3 = 3 m

Correctly shading feasible region 1 m

Vertices are A(0, 2), B(1.6, 1.2), C(2, .0) 1 m

$Z = 2x + 5y$ is maximum at A(0, 2) and maximum value = 10 1 m

QUESTION PAPER CODE 65/2/D
EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$ ½ + ½ m

2. I.F = $e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$ ½ + ½ m

3. $p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$ ½ + ½ m

4. $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$ ½ + ½ m

5. $\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \Rightarrow \theta = \frac{\pi}{6}$ ½ + ½ m

6. $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$ ½ + ½ m

SECTION - B

7. $\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta$ ½ m

$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta$ ½ m

$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{a \sin \theta + b \cos \theta} = -\frac{x}{y}$ 1½ m

$$\text{or } y \frac{dy}{dx} + x = 0$$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0 \quad 1 \text{ m}$$

$$\text{Using (i) we get } y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0 \quad \frac{1}{2} \text{ m}$$

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

8. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.} \quad 1 \text{ m}$$

$$\text{Area (A)} = \frac{\sqrt{3}x^2}{4} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2/\text{s} \quad 1 \text{ m}$$

9. Writing $x + 3 = -\frac{1}{2}(-4 - 2x) + 1 \quad 1 \text{ m}$

$$\therefore \int (x+3)\sqrt{3-4x-x^2} dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx + \int \sqrt{7-(x+2)^2} dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c \quad 1 + 1 \text{ m}$$

10. HF. M P

$$\begin{matrix} A \\ B \\ C \end{matrix} \begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix} \quad 1\frac{1}{2} \text{ m}$$

Funds collected by school A : Rs. 7000,

School B : Rs. 6125, School C : Rs. 7875 1 m

Total collected : Rs. 21000 ½ m

For writing one value 1 m

11. Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad 1 \text{ m}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix} \quad 1 \text{ m}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} \quad 1 \text{ m}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad 2 \text{ m}$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

$$12. \quad f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \rightarrow R_2 - xR_1 \quad \text{and} \quad R_3 \rightarrow R_3 - x^2R_1$$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix} \quad (\text{For bringing 2 zeroes in any row/column}) \quad 1+1 \text{ m}$$

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2 \quad 1 \text{ m}$$

$$\begin{aligned} \therefore f(2x) - f(x) &= a[2x+a]^2 - a(x+a)^2 \\ &= ax(3x+2a) \end{aligned} \quad 1 \text{ m}$$

$$13. \quad \int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x \cdot dx}{(1-\cos x)(1+\cos x)(1+2\cos x)} \quad 1 \text{ m}$$

$$= -\int \frac{dt}{(1-t)(1+t)(1+2t)} \quad \text{where } \cos x = t \quad \frac{1}{2} \text{ m}$$

$$= \int \left(\frac{-1/6}{1-t} + \frac{1/2}{1+t} - \frac{4/3}{1+2t} \right) dt \quad 1\frac{1}{2} \text{ m}$$

$$= +\frac{1}{6} \log|1-t| + \frac{1}{2} \log|1+t| - \frac{2}{3} \log|1+2t| + c \quad \frac{1}{2} \text{ m}$$

$$= \frac{1}{6} \log |1 - \cos x| + \frac{1}{2} \log |1 + \cos x| - \frac{2}{3} \log |1 + 2 \cos x| + c \quad \frac{1}{2} \text{ m}$$

OR

$$\int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}} dx = \int \frac{2 - 3x - (1-x^2)}{\sqrt{1-x^2}} dx \quad \frac{1}{2} \text{ m}$$

$$= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx \quad 1 \text{ m}$$

$$= 2 \sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} \sin^{-1}x + c \quad (\frac{1}{2}+1+1) \text{ m}$$

$$\text{or } = \frac{3}{2} \sin^{-1}x + \frac{1}{2} (6-x)\sqrt{1-x^2} + c$$

$$14. \quad I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2 \cos ax \sin bx dx$$

$$= I_1 - I_2 \quad \frac{1}{2} \text{ m}$$

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \quad (\text{being an even fun.}) \quad 1 \text{ m}$$

$$I_2 = 0 \quad (\text{being an odd fun.}) \quad 1 \text{ m}$$

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx \quad \frac{1}{2} \text{ m}$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi} \quad \frac{1}{2} \text{ m}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi \quad \frac{1}{2} \text{ m}$$

15. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$$\therefore P(E_1) = \frac{1}{3}, \quad P(E_2) = \frac{2}{3} \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

Let A : Getting one Red and one black ball

$$\therefore P(A|E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15}, \quad P(A|E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15} \quad 1+1 \text{ m}$$

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45} \quad 1 \text{ m}$$

OR

	0	1	2	3	4	
x :						½ m
P(x) :	${}^4C_0 \left(\frac{1}{2}\right)^4$	${}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)$	${}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$	${}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3$	${}^4C_4 \left(\frac{1}{2}\right)^4$	1½ m
:	$= \frac{1}{16}$	$= \frac{4}{16}$	$= \frac{6}{16}$	$= \frac{4}{16}$	$= \frac{1}{16}$	½ m

xP(x) :	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	
---------	---	----------------	-----------------	-----------------	----------------	--

x ² P(x) :	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$	½ m
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$$\text{Mean} = \sum x P(x) = \frac{32}{16} = 2 \quad ½ \text{ m}$$

$$\text{Variance} = \sum x^2 P(x) - \left(\sum x P(x)\right)^2 = \frac{80}{16} - (2)^2 = 1 \quad ½ \text{ m}$$

16. $\vec{r} \times \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{i} = -y\hat{k} + z\hat{j} \quad 1½ \text{ m}$

$$\vec{r} \times \hat{j} = (x\hat{i} + y\hat{j} + z\hat{k}) \times \hat{j} = x\hat{k} - z\hat{i} \quad 1½ \text{ m}$$

$$\left(\vec{r} \times \hat{i}\right) \cdot \left(\vec{r} \times \hat{j}\right) = (z\hat{j} - y\hat{k}) \cdot (-z\hat{i} + x\hat{k}) = -xy \quad ½ \text{ m}$$

$$\left(\vec{r} \times \hat{i}\right) \cdot \left(\vec{r} \times \hat{j}\right) + xy = -xy + xy = 0 \quad ½ \text{ m}$$

17. Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3\lambda+2, 4\lambda-1, 12\lambda+2) \quad 1 \text{ m}$

If this is the point of intersection with plane $x - y + z = 5$

then $3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Rightarrow \lambda = 0$ 1 m

\therefore Point of intersection is $(2, -1, 2)$ 1 m

Required distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$ 1 m

18. Writing $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$ ½ m

$1+x^2+2x+1=1+x^2 \Rightarrow x = -\frac{1}{2}$ ½ m

OR

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$ 1 m

$\therefore 2(\tan^{-1}x)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0$ 1½ m

$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\pi/4, -\pi/4$ 1 m

$\Rightarrow x = -1$ ½ m

19. Putting $x^2 = \cos \theta$, we get ½ m

$y = \tan^{-1} \left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}} \right)$ ½ m

$$= \tan^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left(\frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right) \quad 1 + \frac{1}{2} \text{ m}$$

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2 \quad \frac{1}{2} \text{ m}$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}} \quad 1 \text{ m}$$

SECTION - C

20. $f(x) = \sin x - \cos x, 0 < x < 2\pi$
 $f'(x) = 0 \Rightarrow \cos x + \sin x = 0$ or $\tan x = -1,$ 1 m

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4} \quad 1 \text{ m}$$

$$f''(x) = \cos x - \sin x \quad 1 \text{ m}$$

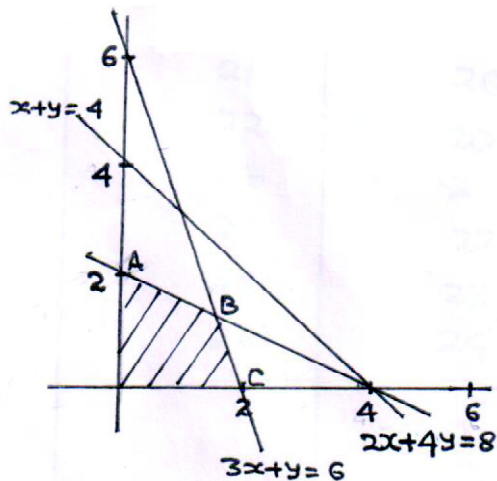
$$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \text{ i.e. -ve so, } x = \frac{3\pi}{4} \text{ is Local Maxima} \quad 1 \text{ m}$$

$$\text{and } f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ i.e. +ve so, } x = \frac{7\pi}{4} \text{ is Local Minima} \quad 1 \text{ m}$$

$$\text{Local Maximum value} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \quad \frac{1}{2} \text{ m}$$

$$\text{Local Minimum value} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} \quad \frac{1}{2} \text{ m}$$

21. Correct graphs of three lines 1x3 = 3 m
Correctly shading
feasible region 1 m



Vertices are

A (0, 2), B (1.6, 1.2), C (2, .0) 1 m

$Z = 2x + 5y$ is maximum

at A (0, 2) and maximum value = 10 1 m

22. $\forall a, b \in \mathbb{N}, (a, b) R (a, b)$ as $ab(b+a) = ba(a+b)$

$\therefore R$ is reflexive (i) 2 m

Let $(a, b) R (c, d)$ for $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ (ii)

Also $(c, d) R (a, b) \therefore cb(d+a) = da(c+b)$ (using ii)

$\therefore R$ is symmetric (iii) 2 m

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, for $a, b, c, d, e, f \in \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ and $cf(d+e) = de(c+f)$ 1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

i.e $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

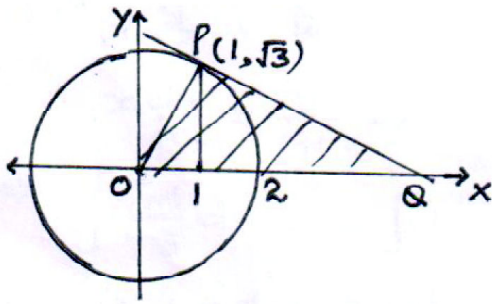
adding we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow af(b+e) = be(a+f)$

Hence $(a, b) R (e, f) \therefore R$ is transitive (iv) $\frac{1}{2}$ m

Form (i), (iii) and (iv) R is an equivalence relation $\frac{1}{2}$ m

23.



Correct Fig.

1 m

Eqn. of normal (OP) : $y = \sqrt{3}x$

$\frac{1}{2} + \frac{1}{2}$ m

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x - 1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4 - x)$$

1 m

Coordinates of Q (4, 0)

$\frac{1}{2}$ m

$$\therefore \text{Req. area} = \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}}(4 - x) \, dx$$

$\frac{1}{2} + \frac{1}{2}$ m

$$= \sqrt{3} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4$$

1 m

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units}$$

$\frac{1}{2}$ m

OR

$$\int_1^3 (e^{-3x} + x^2 + 1) \, dx \quad \text{here } h = \frac{2}{n}$$

$\frac{1}{2}$ m

$$= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)]$$

1 m

$$= \lim_{h \rightarrow 0} h [(e^{-1} + 2) + (e^{-1-3h} + 2 + 2h + h^2) + (e^{-1-6h} + 2 + 4h + 4h^2) + \dots$$

$$+ (e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^2 h^2)]$$

1 m

$$= \lim_{h \rightarrow 0} h \left[e^{-1} (1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-1)h}) + 2n + 2h(1 + 2 + \dots + (n-1)) + h^2 (1^2 + 2^2 + \dots + (n-1)^2) \right] \quad 1\frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h \left(e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3h} - 1} \cdot h + 2nh + 2 \frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6} \right) \quad 1 \text{ m}$$

$$= e^{-1} \cdot \frac{(e^{-6} - 1)}{-3} + 4 + 4 + \frac{8}{3} = -e^{-1} \frac{(e^{-6} - 1)}{3} + \frac{32}{3} \quad 1 \text{ m}$$

24. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2} \quad 1 \text{ m}$$

\therefore Integrating factor is $e^{\tan^{-1}y}$ 1 m

$$\therefore \text{Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^t dt \text{ where } \tan^{-1}y = t \quad 1 \text{ m}$$

$$= t e^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \quad 1\frac{1}{2} \text{ m}$$

$$\text{or } x = \tan^{-1}y - 1 + c e^{-\tan^{-1}y}$$

OR

$$\text{Given differential equation is } \frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

$$\text{Putting } \frac{y}{x} = v \text{ to get } v + x \frac{dv}{dx} = \frac{v}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore x \frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow \int \frac{v^2+1}{v^3} dv = - \int \frac{dx}{x} \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \log |v| - \frac{1}{2v^2} = - \log |x| + c \quad 1 \text{ m}$$

$$\therefore \log y - \frac{x^2}{2y^2} = c \quad 1 \text{ m}$$

$$x=0, y=1 \Rightarrow c=0 \therefore \log y - \frac{x^2}{2y^2} = 0 \quad \frac{1}{2} \text{ m}$$

25. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$ 1 m

$$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}, \text{ hence } k = \frac{9}{2} \quad 2\frac{1}{2} \text{ m}$$

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$\Rightarrow -5(x-1) + 2(y+1) + 1(z-1) = 0 \quad 1 \text{ m}$$

$$\text{i.e. } 5x - 2y - z - 6 = 0 \quad \frac{1}{2} \text{ m}$$

26. $P(\bar{A} \cap B) = \frac{2}{15} \Rightarrow P(\bar{A}) \cdot P(B) = \frac{2}{15}$ 1 m

$$P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\bar{B}) = \frac{1}{6} \quad 1 \text{ m}$$

$$\therefore (1-P(A))P(B) = \frac{2}{15} \text{ or } P(B) - P(A) \cdot P(B) = \frac{2}{15} \dots\dots\dots \text{(i)} \quad 1 \text{ m}$$

$$P(A)(1-P(B)) = \frac{1}{6} \text{ or } P(A) - P(A) \cdot P(B) = \frac{1}{6} \dots\dots\dots \text{(ii)} \quad 1 \text{ m}$$

$$\text{From (i) and (ii) } P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30} \quad \frac{1}{2} \text{ m}$$

$$\text{Let } P(A) = x, P(B) = y \therefore x = \left(\frac{1}{30} + y\right)$$

$$(i) \Rightarrow y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Solving to get } y = \frac{1}{6} \text{ or } y = \frac{4}{5}$$

$$\therefore x = \frac{1}{5} \text{ or } x = \frac{5}{6} \quad \frac{1}{2} \text{ m}$$

$$\text{Hence } P(A) = \frac{1}{5}, P(B) = \frac{1}{6} \text{ OR } P(A) = \frac{5}{6}, P(B) = \frac{4}{5} \quad \frac{1}{2} \text{ m}$$

QUESTION PAPER CODE 65/3/D
EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\cos^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{3} + \cos^2 \theta = 1 \Rightarrow \theta = \frac{\pi}{6}$ ½ + ½ m

2. $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$ ½ + ½ m

3. $\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r \frac{dv}{dr} = 0$ ½ + ½ m

4. I.F = $e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$ ½ + ½ m

5. $p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$ ½ + ½ m

6. $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$ ½ + ½ m

SECTION - B

7. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$\therefore P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$ ½ + ½ m

Let A : Getting one Red and one balck ball

$\therefore P(A|E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15}, P(A|E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15}$ 1+1 m

$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$

$$= \frac{1}{3} \cdot \frac{8}{15} + \frac{2}{3} \cdot \frac{7}{15} = \frac{22}{45} \quad 1 \text{ m}$$

OR

	x	:	0	1	2	3	4	
	P(x)	:	${}^4C_0 \left(\frac{1}{2}\right)^4$	${}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)$	${}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$	${}^4C_3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^3$	${}^4C_4 \left(\frac{1}{2}\right)^4$	1½ m
		:	$= \frac{1}{16}$	$= \frac{4}{16}$	$= \frac{6}{16}$	$= \frac{4}{16}$	$= \frac{1}{16}$	½ m
	x P(x)	:	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	
	x ² P(x)	:	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$	½ m

$$\text{Mean} = \sum x P(x) = \frac{32}{16} = 2 \quad ½ \text{ m}$$

$$\text{Variance} = \sum x^2 P(x) - \left(\sum x P(x)\right)^2 = \frac{80}{16} - (2)^2 = 1 \quad ½ \text{ m}$$

$$8. \quad \vec{r} \times \vec{i} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right) \times \hat{i} = -y\hat{k} + z\hat{j} \quad 1½ \text{ m}$$

$$\vec{r} \times \vec{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right) \times \hat{j} = x\hat{k} - z\hat{i} \quad 1½ \text{ m}$$

$$\left(\vec{r} \times \hat{i}\right) \cdot \left(\vec{r} \times \vec{j}\right) = \left(\hat{o}\hat{i} + z\hat{j} - y\hat{k}\right) \cdot \left(-z\hat{i} + \hat{o}\hat{j} + x\hat{k}\right) = -xy \quad ½ \text{ m}$$

$$\left(\vec{r} \times \hat{i}\right) \cdot \left(\vec{r} \times \vec{j}\right) + xy = -xy + xy = 0 \quad ½ \text{ m}$$

$$9. \quad \text{Any point on the line } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} \text{ is } (3\lambda+2, 4\lambda-1, 12\lambda+2) \quad 1 \text{ m}$$

If this is the point of intersection with plane $x - y + z = 5$

$$\text{then } 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 - 5 = 0 \Rightarrow \lambda = 0 \quad 1 \text{ m}$$

∴ Point of intersection is $(2, -1, 2)$ 1 m

$$\text{Required distance} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13 \quad 1 \text{ m}$$

10. Writing $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$ ½ m

$1+x^2+2x+1=1+x^2 \Rightarrow x = -\frac{1}{2}$ ½ m

OR

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow (\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$ 1 m

$\therefore 2(\tan^{-1}x)^2 - \pi \tan^{-1}x - \frac{3\pi^2}{8} = 0$ 1½ m

$\tan^{-1}x = \frac{\pi \pm \sqrt{\pi^2 + 3\pi^2}}{4} = 3\pi/4, -\pi/4$ 1 m

$\Rightarrow x = -1$ ½ m

11. Putting $x^2 = \cos\theta$, we get ½ m

$y = \tan^{-1} \left(\frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}} \right)$ ½ m

$= \tan^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left(\frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right)$ 1 + ½ m

$$y = \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1}x^2 \quad \frac{1}{2} \text{ m}$$

$$\frac{dy}{dx} = -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}} \quad 1 \text{ m}$$

12. $\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta \quad \frac{1}{2} \text{ m}$

$$\frac{dy}{d\theta} = a \cos \theta + b \sin \theta \quad \frac{1}{2} \text{ m}$$

$$\therefore \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{a \sin \theta + b \cos \theta} = -\frac{x}{y} \quad 1\frac{1}{2} \text{ m}$$

or $y \frac{dy}{dx} + x = 0$

$$\therefore y \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0 \quad 1 \text{ m}$$

Using (i) we get $y \frac{d^2y}{dx^2} - \frac{x}{y} \frac{dy}{dx} + 1 = 0 \quad \frac{1}{2} \text{ m}$

$$\therefore y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

13. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.} \quad 1 \text{ m}$$

$$\text{Area (A)} = \frac{\sqrt{3}x^2}{4} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt} \quad 1 \text{ m}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2/\text{s} \quad 1 \text{ m}$$

14. Writing $x + 3 = -\frac{1}{2}(-4 - 2x) + 1$ 1 m

$$\therefore \int (x+3)\sqrt{3-4x-x^2} dx = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx + \int \sqrt{7-(x+2)^2} dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= -\frac{1}{3}(3-4x-x^2)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c \quad 1+1 \text{ m}$$

15. HF. M P

$$\begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix} \begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix} \quad 1\frac{1}{2} \text{ m}$$

Funds collected by school A : Rs. 7000,

School B : Rs. 6125, School C : Rs. 7875 1 m

Total collected : Rs. 21000 \frac{1}{2} \text{ m}

For writing one value 1 m

16. Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ 1\frac{1}{2} \text{ m}

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad 1 \text{ m}$$

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix} \quad 1 \text{ m}$$

$$\therefore X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} \quad 1 \text{ m}$$

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad 2 \text{ m}$$

$$\therefore (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

$$17. \quad f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$R_2 \rightarrow R_2 - xR_1 \quad \text{and} \quad R_3 \rightarrow R_3 - x^2R_1$$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix} \quad (\text{For bringing 2 zeroes in any row/column}) \quad 1+1 \text{ m}$$

$$\therefore f(x) = a(a^2 + 2ax + x^2) = a(x+a)^2 \quad 1 \text{ m}$$

$$\begin{aligned} \therefore f(2x) - f(x) &= a[2x+a]^2 - a(x+a)^2 \\ &= ax(3x+2a) \quad 1 \text{ m} \end{aligned}$$

$$\begin{aligned}
18. \quad \int \frac{dx}{\sin x + \sin 2x} &= \int \frac{dx}{\sin x (1 + 2 \cos x)} = \int \frac{\sin x \cdot dx}{(1 - \cos x) (1 + \cos x) (1 + 2 \cos x)} && 1 \text{ m} \\
&= - \int \frac{dt}{(1-t) (1+t) (1+2t)} \quad \text{where } \cos x = t && \frac{1}{2} \text{ m} \\
&= \int \left(\frac{-1/6}{1-t} + \frac{1/2}{1+t} - \frac{4/3}{1+2t} \right) dt && 1\frac{1}{2} \text{ m} \\
&= + \frac{1}{6} \log |1-t| + \frac{1}{2} \log |1+t| - \frac{2}{3} \log |1+2t| + c && \frac{1}{2} \text{ m} \\
&= \frac{1}{6} \log |1 - \cos x| + \frac{1}{2} \log |1 + \cos x| - \frac{2}{3} \log |1 + 2 \cos x| + c && \frac{1}{2} \text{ m}
\end{aligned}$$

OR

$$\begin{aligned}
\int \frac{x^2 - 3x + 1}{\sqrt{1-x^2}} dx &= \int \frac{2 - 3x - (1-x^2)}{\sqrt{1-x^2}} dx && \frac{1}{2} \text{ m} \\
&= 2 \int \frac{1}{\sqrt{1-x^2}} dx - 3 \int \frac{x}{\sqrt{1-x^2}} dx - \int \sqrt{1-x^2} dx && 1 \text{ m} \\
&= 2 \sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} \sin^{-1}x + c && (\frac{1}{2}+1+1) \text{ m} \\
\text{or } &= \frac{3}{2} \sin^{-1}x + \frac{1}{2} (6-x)\sqrt{1-x^2} + c
\end{aligned}$$

$$\begin{aligned}
19. \quad I &= \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - \int_{-\pi}^{\pi} 2 \cos ax \sin bx dx \\
&= I_1 - I_2 && \frac{1}{2} \text{ m}
\end{aligned}$$

$$I_1 = 2 \int_0^{\pi} (\cos^2 ax + \sin^2 bx) dx \quad (\text{being an even fun.}) \quad 1 \text{ m}$$

$$I_2 = 0 \quad (\text{being an odd fun.}) \quad 1 \text{ m}$$

$$\therefore I = I_1 = \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx) dx \quad \frac{1}{2} \text{ m}$$

$$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi} \quad \frac{1}{2} \text{ m}$$

$$= \left[2\pi + \frac{1}{2a} \cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right] \text{ or } 2\pi \quad \frac{1}{2} \text{ m}$$

SECTION - C

20. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$ 1 m

$\therefore \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}$, hence $k = \frac{9}{2}$ 2½ m

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$
 1 m

$\Rightarrow -5(x-1) + 2(y+1) + 1(z-1) = 0$ 1 m

i.e. $5x - 2y - z - 6 = 0$ ½ m

21. $P(\bar{A} \cap B) = \frac{2}{15} \Rightarrow P(\bar{A}) \cdot P(B) = \frac{2}{15}$ 1 m

$P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\bar{B}) = \frac{1}{6}$ 1 m

$\therefore (1-P(A))P(B) = \frac{2}{15}$ or $P(B) - P(A) \cdot P(B) = \frac{2}{15}$ (i) 1 m

$P(A)(1-P(B)) = \frac{1}{6}$ or $P(A) - P(A) \cdot P(B) = \frac{1}{6}$ (ii) 1 m

From (i) and (ii) $P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$ ½ m

Let $P(A) = x, P(B) = y \therefore x = \left(\frac{1}{30} + y\right)$

(i) $\Rightarrow y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \therefore 30y^2 - 29y + 4 = 0$ ½ m

Solving to get $y = \frac{1}{6}$ or $y = \frac{4}{5}$

$$\therefore x = \frac{1}{5} \text{ or } x = \frac{5}{6} \quad \frac{1}{2} \text{ m}$$

Hence $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{6}$ OR $P(A) = \frac{5}{6}$, $P(B) = \frac{4}{5}$ $\frac{1}{2} \text{ m}$

22. $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

$$f'(x) = 0 \Rightarrow \cos x + \sin x = 0 \text{ or } \tan x = -1, \quad 1 \text{ m}$$

$$\therefore x = \frac{3\pi}{4}, \frac{7\pi}{4} \quad 1 \text{ m}$$

$$f''(x) = \cos x - \sin x \quad 1 \text{ m}$$

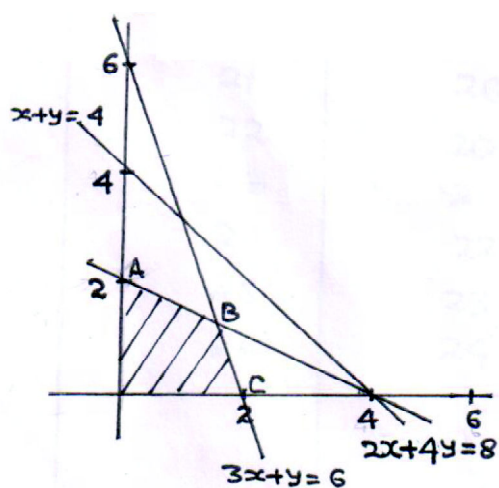
$$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \text{ i.e. -ve so, } x = \frac{3\pi}{4} \text{ is Local Maxima} \quad 1 \text{ m}$$

$$\text{and } f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ i.e. +ve so, } x = \frac{7\pi}{4} \text{ is Local Minima} \quad 1 \text{ m}$$

$$\text{Local Maximum value} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \quad \frac{1}{2} \text{ m}$$

$$\text{Local Minimum value} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} \quad \frac{1}{2} \text{ m}$$

23.



Correct graphs of three lines $1 \times 3 = 3 \text{ m}$

Correctly shading feasible region 1 m

Vertices are

$A(0, 2)$, $B(1.6, 1.2)$, $C(2, 0)$ 1 m

$Z = 2x + 5y$ is maximum

at $A(0, 2)$ and maximum value = 10 1 m

24. $\forall a, b \in \mathbb{N}, (a, b) R (a, b)$ as $ab(b+a) = ba(a+b)$
 $\therefore R$ is reflexive (i) 2 m

Let $(a, b) R (c, d)$ for $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ (ii)

Also $(c, d) R (a, b) \therefore cb(d+a) = da(c+b)$ (using ii)

$\therefore R$ is symmetric (iii) 2 m

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$, for $a, b, c, d, e, f \in \mathbb{N}$

$\therefore ad(b+c) = bc(a+d)$ and $cf(d+e) = de(c+f)$ 1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

$$\text{i.e. } \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \text{ and } \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

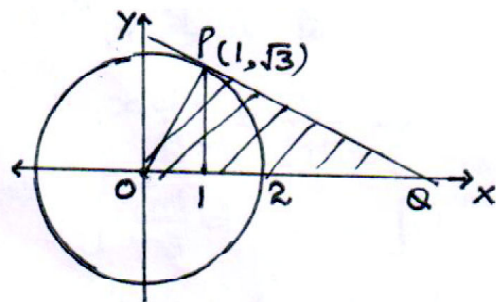
$$\text{adding we get } \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow af(b+e) = be(a+f)$$

Hence $(a, b) R (e, f) \therefore R$ is transitive (iv) $\frac{1}{2}$ m

Form (i), (iii) and (iv) R is an equivalence relation $\frac{1}{2}$ m

25. Correct Fig. 1 m



Eqn. of normal (OP) : $y = \sqrt{3}x$ $\frac{1}{2} + \frac{1}{2}$ m

Eqn. of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x-1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4-x) \quad 1 \text{ m}$$

Coordinates of Q (4, 0) $\frac{1}{2}$ m

$$\therefore \text{Req. area} = \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}} (4-x) \, dx \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$= \sqrt{3} \left[\frac{x^2}{2} \right]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4 \quad 1 \text{ m}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. units} \quad \frac{1}{2} \text{ m}$$

OR

$$\int_1^3 (e^{2-3x} + x^2 + 1) \, dx \quad \text{here } h = \frac{2}{n} \quad \frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [(e^{-1} + 2) + (e^{-1-3h} + 2 + 2h + h^2) + (e^{-1-6h} + 2 + 4h + 4h^2) + \dots]$$

$$+ (e^{-1-3(n-1)h} + 2 + 2(n-1)h + (n-1)^2 h^2)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [e^{-1}(1 + e^{-3h} + e^{-6h} + \dots + e^{-3(n-1)h}) + 2n + 2h(1 + 2 + \dots + (n-1)) + h^2(1^2 + 2^2 + \dots + (n-1)^2)] \quad 1\frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h \left(e^{-1} \cdot \frac{e^{-3nh} - 1}{e^{-3n} - 1} \cdot h + 2nh + 2 \frac{nh(nh-h)}{2} + \frac{nh(nh-h)(2nh-h)}{6} \right) \quad 1 \text{ m}$$

$$= e^{-1} \cdot \frac{(e^{-6} - 1)}{-3} + 4 + 4 + \frac{8}{3} = -e^{-1} \frac{(e^{-6} - 1)}{3} + \frac{32}{3} \quad 1 \text{ m}$$

26. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2} \quad 1 \text{ m}$$

\therefore Integrating factor is $e^{\tan^{-1}y}$ 1 m

$$\therefore \text{ Solution is : } x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow x \cdot e^{\tan^{-1}y} = \int t e^t dt \text{ where } \tan^{-1}y = t \quad 1 \text{ m}$$

$$= t e^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \quad 1\frac{1}{2} \text{ m}$$

$$\text{or } x = \tan^{-1}y - 1 + c e^{-\tan^{-1}y}$$

OR

Given differential equation is $\frac{dy}{dx} = \frac{\frac{y}{x}}{1 + \left(\frac{y}{x}\right)^2}$

$$\text{Putting } \frac{y}{x} = v \text{ to get } v + x \frac{dv}{dx} = \frac{v}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore x \frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2} \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow \int \frac{v^2+1}{v^3} dv = - \int \frac{dx}{x} \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \log |v| - \frac{1}{2v^2} = - \log |x| + c \quad 1 \text{ m}$$

$$\therefore \log y - \frac{x^2}{2y^2} = c \quad 1 \text{ m}$$

$$x = 0, y = 1 \Rightarrow c = 0 \therefore \log y - \frac{x^2}{2y^2} = 0 \quad \frac{1}{2} \text{ m}$$