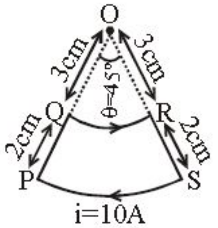
#1328959



A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of $10A$. The magnetic field at the point O will be close to :

A $1.0 \times 10^{-5} T$

B $1.5 \times 10^{-5} T$

C $1.0 \times 10^{-7} T$

D $2.0 \times 10^{-7} T$

Solution

$\vec{B} = \frac{\mu_0 i}{4\pi}\theta\left[\frac{1}{r_1} - \frac{1}{r_2}\right]\hat{k}$

$r_1 = 3cm = 3 \times 10^{-2} m$

$r_2 = 5cm = 5 \times 10^{-2} m$

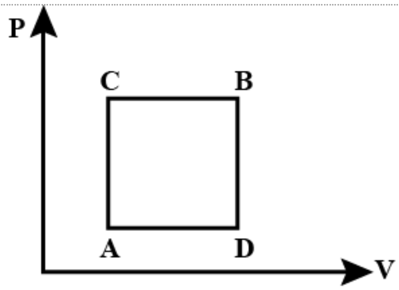
$\theta = \frac{\pi}{4}, i = 10A$

$\Rightarrow \vec{B} = \frac{4\pi \times 10^{-7}}{16} \times 10\left[\frac{1}{3 \times 10^{-2}} - \frac{1}{5 \times 10^{-2}}\right]\hat{k}$

$\Rightarrow |\vec{B}| = \frac{\pi}{3} \times 10^{-5} T$

$\approx 1 \times 10^{-5} T$

#1329052



A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used $60J$ of heat flows into the system and $30J$ of work is done by the system. If path ADB is used work done by the system is $10J$. The heat Flow into the system in path ADB is :

A $80J$

B $20J$

C $100J$

D $40J$

Solution

$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

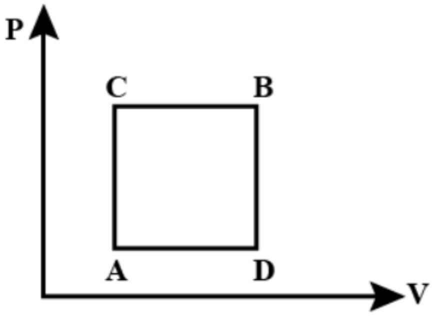
$\Rightarrow 60J = 30J + \Delta U_{ACB}$

$\Rightarrow \Delta U_{ACB} = 30J$

$\Rightarrow \Delta U_{ADB} = \Delta U_{ACB} = 30J$

$\Delta Q_{ACD} = \Delta U_{ACB} + \Delta W_{ADB}$

$= 10J + 30J = 40J$



#1329184

A plane electromagnetic wave of frequency $50MHz$ travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}V/m$. The corresponding magnetic field $\vec{B}$, at that point will be:

A $18.9 \times 10^{-8}\hat{k}T$

B $6.3 \times 10^{-8}\hat{k}T$

$\boxed{\text{C}}$ $2.1 \times 10^{-8}\hat{k}T$

D $18.9 \times 10^{8}\hat{k}T$

Solution

$|B| = \frac{|E|}{[C} = \frac{6.3}{3 \times 10^{8}} = 2.1 \times 10^{-8}T$

and $\hat{E} \times \hat{B} = \hat{C}$

$\hat{j} \times B = \hat{i}$

$\hat{B} = \hat{k}$

$\vec{B} = |B|\hat{B} = 2.1 \times 10^{8}\hat{k}T$

#1329369

Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

A $4:1$

$\boxed{\text{B}}$ $25:9$

C $6:9$

D $5:3$

Solution

$$\frac{I_{max}}{I_{min}} = 16$$

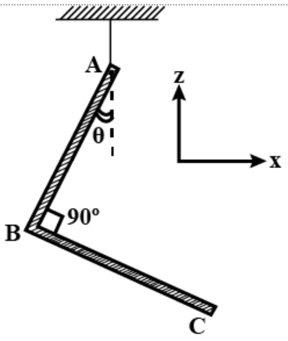
$$\Rightarrow \frac{A_{max}}{A_{min}} = 4$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$$

Using componendo & diviendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_3} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

#1329402



An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as showm=n in figure. If $AB = BC$, and the angle made by $AB = B$, and the angle made by AB with downward vertical is θ, then:

A $\tan\theta = \frac{2}{\sqrt{3}}$

B $\tan\theta = \frac{1}{3}$

C $\tan\theta = \frac{1}{2}$

D $\tan\theta = \frac{1}{2\sqrt{3}}$

Solution

Let the mass of one of is m.

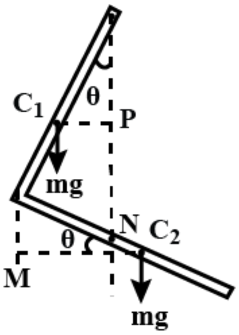
Balancing torque about the point

$$mg(C_1P) = mg(C_2N)$$

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \tan\theta = \frac{1}{3}$$



#1329454

A mixture of 2 moles of helium gas (atomic mass $= 4u$), and 1 mole of argon gas (atomic mass $= 40u$) is kept at $300K$ in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(helium)}{V_{rms}(arg\,on)}\right]$, is close to:

A 2.24

B 0.45

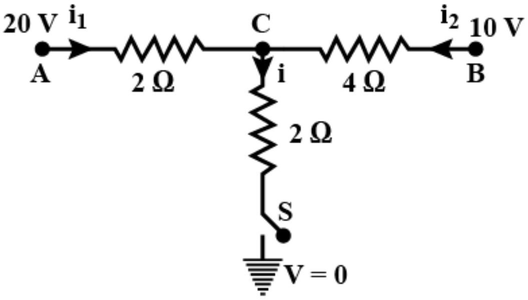
C 0.32

[✓ D] 3.16

Solution

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

#1329657



When the switch S, in the circuit shown, is closed, then the value of current i will be :

A $3A$

[✓ B] $5A$

C $4A$

D $2A$

Solution

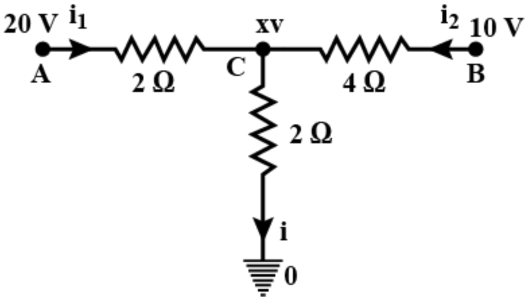
Let voltage at $C = xv$

$KCL : i_1 + i_2 = i$

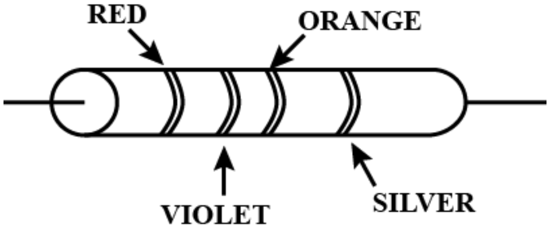
$$\frac{20 - x}{2} + \frac{10 - x}{4} = \frac{x - 0}{2}$$

$\Rightarrow x = 10$

and $i = 5Amp$.



#1329677



A resistance is shown in the figure. Its value and tolerance are given respectively by:

A $27\ K\Omega,\ 20\%$

B $270\ K\Omega,\ 5\%$

C $270\ K\Omega,\ 10\%$

[✓ D] $27\ K\Omega,\ 10\%$

Solution

Color code:

Red violet orange silver

$R = 27 \times 10^3 \Omega \pm 10\%$

$= 27\ K\Omega \pm 10\%$

#1329700

A bar magnet is demagnetized by inserting it inside a solenoid of length $0.2m$, $100 turas$, and carrying a current of $5.2A$. The coercivity of the bar magnet is:

A $1200\ A/m$

[✓ B] $2600\ A/m$

C $5200\ A/jm$

D $285\ A/m$

Solution

Coercivity $= H = \frac{B}{\mu_0}$

$ni = \frac{N}{\ell} i = \frac{100}{0.2} \times 5.2$

$= 2600 A/m$

#1329725

A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having a coefficient of linear expansion ^oC. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by $\Delta T K$. Young's modulus, Y, for this metal is :

A $\frac{F}{2A\alpha\Delta T}$

B $\frac{F}{A\alpha(\Delta T - 273)}$

[✓ C] $\frac{F}{A\alpha\Delta T}$

D $\frac{2F}{A\alpha\Delta T}$

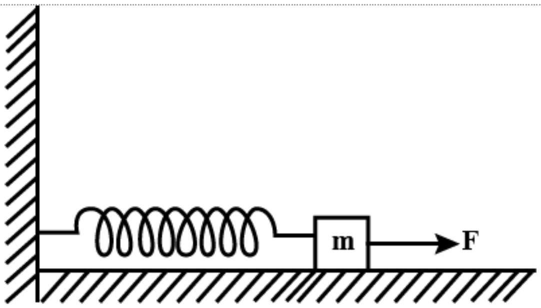
Solution

Young's modulus $y = \frac{Stress}{Strain}$

$= \frac{F/A}{(\Delta\ell/\ell)}$

$= \frac{F}{A(\alpha\Delta T)}$

#1329810



A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :

A $\frac{\pi F}{\sqrt{mk}}$

B $\frac{2F}{\sqrt{mk}}$

$\boxed{\text{C}}$ $\frac{F}{\sqrt{mk}}$

D $\frac{F}{\pi\sqrt{mk}}$

Solution

Maximum speed is at mean position(equilibrium). $F = kx$

$$x = \frac{F}{k}$$

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow V_{max} = \frac{F}{\sqrt{mk}}$$

#1329825

Three charges $+Q, q, +Q$ are placed respectively, at distance, 0, $d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, Ls zero,then value of q is :

A $\frac{+Q}{2}$

B $\frac{-Q}{2}$

$\boxed{\text{C}}$ $\frac{-Q}{4}$

D $\frac{+Q}{4}$

Solution

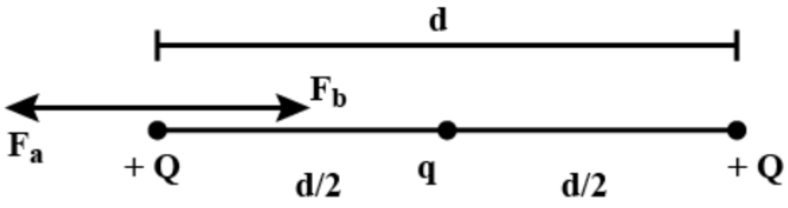
For equilibrium,

$\vec{F}_a + \vec{F}_B = 0$

$\vec{F}_a = -\vec{F}_B$

$\frac{kQQ}{d^2} = -\frac{kQq}{(d/2)^2}$

$\Rightarrow q = -\frac{Q}{4}$



#1329847

A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3} m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0s$ and $t = 10ms$ is close to:

A $0.14mC$

B $0.21mC$

C $6mC$

D $7mC$

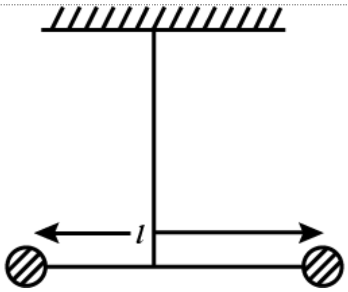
Solution

$Q = \frac{\Delta\phi}{R} = \frac{1}{10}A(B_f - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3}\left(0.4\sin\frac{\pi}{2} - 0\right)$

$= \frac{1}{10}(3.5 \times 10^{-3})(0.4 - 0)$

$= 1.4 \times 10^{-4} = 0.14mC$

#1329935



Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement 0. If the rod is rota ted by θ_0 and released, the tension in it when it passes through its mean position will be:

A $\frac{3k\theta_0^2}{l}$

B $\frac{k\theta_0^2}{2l}$

C $\frac{2k\theta_0^2}{l}$

D $\frac{k\theta_0^2}{l}$

Solution

$\omega = \sqrt{\frac{k}{I}}$

$\omega = \sqrt{\frac{3k}{m\ell^2}}$ (Ref. image 1)

$\Omega = \omega\theta_0 =$ average velocity

$T = m\Omega^2 r_1$

$T = m\Omega^2 \frac{\ell}{3}$

$= m\omega^2\theta_0^2\frac{\ell}{3}$

$= m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3}$

$= \frac{k\theta_0^2}{\ell}$

$I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2$

$= \frac{m\ell^2}{3}$ (Ref. image 2)

$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$

Image 1

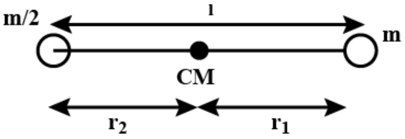


Image 2

#1329967

A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:

A 2.5%

B 0.5%

[✓ **C**] 1.0%

D 2.0%

Solution

$R = \frac{\rho\ell}{A}$ and volume $(V) = a\ell$.

$R = \frac{\rho\ell^2}{V}$

$\Rightarrow \frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 1\%$

#1330036

A parallel plate capacitor is made of two square plates of side 'a', separated by a distance $d (d << a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure.

The capacitance of this capacitor is :

A $\frac{1}{2}\frac{k\in_0 a^2}{d}$

B $\frac{k \in_0 a^2}{d} ln K$

[✓ **C**] $\frac{k \in_0 a^2}{d(K-1)} ln K$

D $\frac{k \in_0 a^2}{2d(K+1)}$

Solution

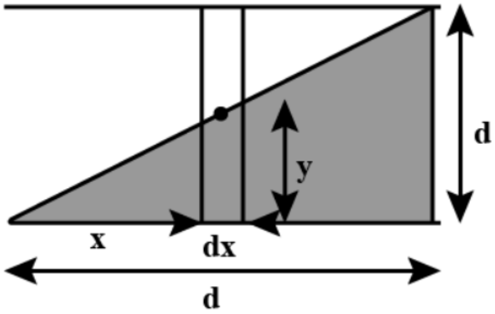
Lets consider a strip of thickness dx at a distance of x from the left end a shown in the figure.

$\frac{y}{x} = \frac{d}{a} \Rightarrow (\frac{d}{a})x$

$C_1 = \frac{\epsilon_o a\ dx}{d-y}$ and $C_2 = \frac{k\epsilon_o\ a\ dx}{y}$

$C_{eq} = \frac{C_1 . C_2}{C_1 + C_2} = \frac{k\epsilon_o adx}{kd + (1-k)y}$

On integrating it from 0 to a, we will get $\frac{k \in_0 a^2}{d(K-1)} ln K$



#1330112

The mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19} m^{-3}$ and their mobility is $1.6 m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to:

A $2\Omega m$

[✓ **B**] $0.4\Omega m$

C $4\Omega m$

D $0.2\Omega m$

Solution

$j = \sigma E = nev_d$

$\sigma = ne\frac{v_d}{E}$

$= ne\mu$

$\frac{1}{\sigma} = \rho = \frac{1}{n_e e \mu_e}$

$= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6}$

$= 0.4\Omega m$

#1330126

If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit L, about the center of the Sun, its areal velocity is:

A $\frac{4L}{m}$

B $\frac{L}{m}$

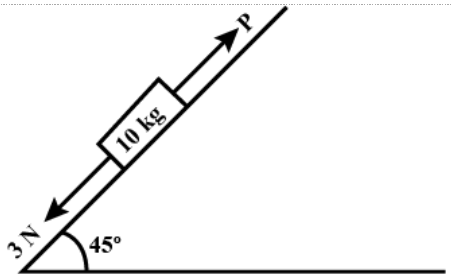
$\boxed{\text{C}}$ $\frac{L}{2m}$

D $\frac{2L}{m}$

Solution

$\frac{dA}{dt} = \frac{L}{2m}$

#1330155



A block of mass $10kg$ is kept on a rough inclined plane as shown in the figure. A force of $3N$ is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block does not move downward? (take $g = 10ms^{-2}$)

$\boxed{\text{A}}$ $32N$

B $25N$

C $44N$

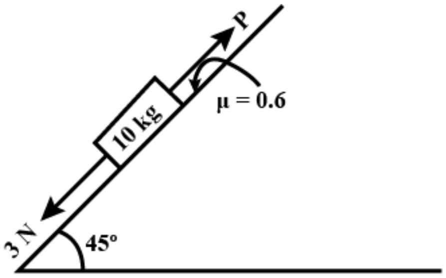
D $18N$

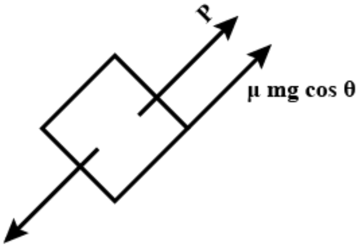
Solution

$mg\sin 45^o = \frac{100}{\sqrt{2}} = 50\sqrt{2}$

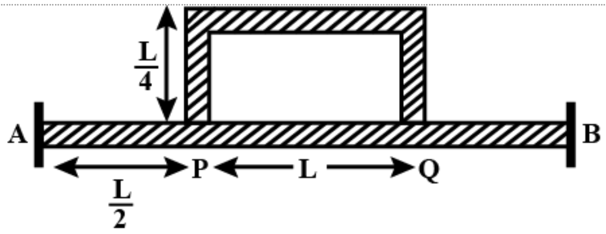
$\mu mg\cos\theta = 0.6 \times mg \times \frac{1}{\sqrt{2}} = 0.6 \times 50\sqrt{2}$

$P = 31.28 \simeq 32N$





#1330217



The temperature difference of 120^oC is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, the temperature difference between P and Q will be close to:

A $60^{o}C$

B $75^{o}C$

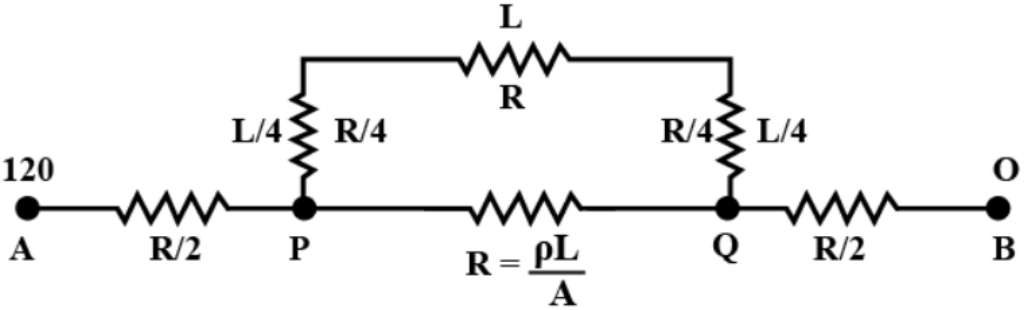
C $35^{o}C$

$\boxed{\text{D}}$ $45^{o}C$

Solution

$$\frac{\Delta T}{R_{eq.}} = I = \frac{(120)5}{8R} = \frac{120 \times 5}{8R}$$

$$\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5}R = \frac{30}{8} = 45^{o}C$$



#1330301

A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60ms^{-1}$. When the car has an acceleration a, the wave-speed increases to $60.5ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to :

$\boxed{\text{A}}$ $\frac{g}{5}$

B $\frac{g}{20}$

C $\frac{g}{10}$

D $\frac{g}{30}$

Solution

$$60 = \sqrt{\frac{Mg}{\mu}}$$

$$60.5 = \sqrt{\frac{M(g^2 + a^2)^{1/2}}{\mu}} \Rightarrow \frac{60.5}{60} = \sqrt{\sqrt{\frac{g^2 + a62}{g^2}}}$$

$$\left(1 + \frac{05}{60}\right)^4 = \frac{g^2 + a^2}{g^2} = 1 + \frac{2}{60}$$

$$\Rightarrow g^2 + a^2 = g^2 + g^2 \times \frac{2}{60}$$

$$a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47}$$

$$\simeq \frac{g}{5}$$

#1330326

A sample of radioactive material A, that has an activity of $10mCi(1\ Ci = 3.7 \times 10^{10} decays/s)$, has twice the number of nuclei as another sample of a different radioactive maternal B which has an activity of $20mCi$. The correct choices for half-life of A and B would then be respectively :

$\boxed{\text{A}}$ 20 days and 5 days

B 10 days and 20 days

C 5 days and 10 days

D 10 days and 40 days

Solution

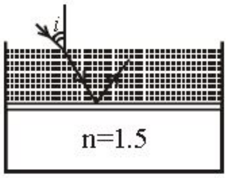
Activity $A = \lambda N$

For A $10 = (2N_0)\lambda_A$

For B $20 = N_0\lambda_B$

$\therefore \lambda_B = 4\lambda_A \Rightarrow (T_{1/2})_A = 4(T_{1/2})_B$

#1330423



Consider a tank made of glass(refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is :

$\boxed{\text{A}}$ $\frac{3}{\sqrt{5}}$

B $\frac{5}{\sqrt{3}}$

C $\sqrt{\frac{5}{3}}$

D $\frac{4}{3}$

Solution

$C < i_b$

here i_b is "brewester angle"

and c is critical angle

$\sin_c < \sin i_b$ since $\tan i_b = \mu_{0_{cel}} = \frac{1.5}{\mu}$

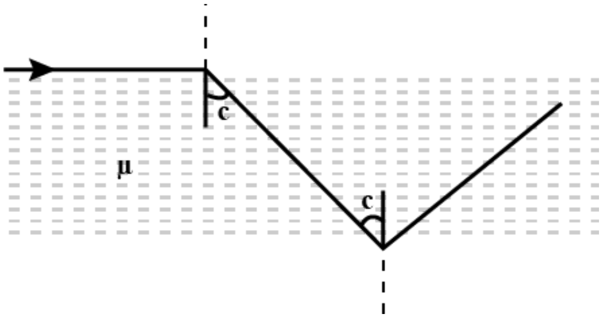
$\frac{1}{\mu} < \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$ $\therefore \sin i_b = \frac{1.5}{\sqrt{\mu^2 + (1.5)^2}}$

$\sqrt{\mu^2 \times (1.05)^2} < 1.5 \times \mu$

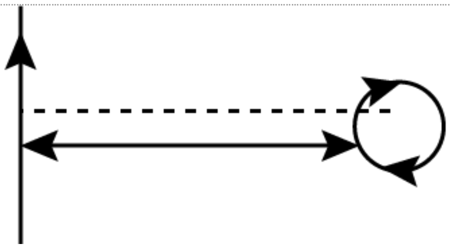
$\mu^2 + (1.5)^{<}(\mu \times 1.5)^2$

$\mu < \frac{3}{\sqrt{5}}$

slab $\mu = 1.5$



#1330480



An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d(>> a)$. If the loop applies a force F on the wire then :

A $F \propto \left(\frac{a^2}{d^3}\right)$

B $F \propto \left(\frac{a}{d}\right)$

C $F \propto \left(\frac{a}{d}\right)^2$

D $F = 0$

Solution

Eqvilent dipole of given loop

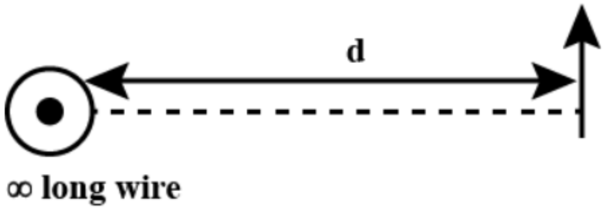
$F = m \cdot \frac{dB}{dr}$

Now, $\frac{dB}{dx} = \frac{d}{dx}\left(\frac{\mu_0 I}{2\pi x}\right)$

$\propto \frac{1}{x^2}$

$\Rightarrow$ So $F \propto \frac{M}{x^2}[\therefore M = NIA]$

$F \propto \frac{a^2}{d^2}$



#1330561

Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350 nm$ and then, by light of wavelength $\lambda_2 = 54Dnm$. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to:

(Energjr of photon $= \frac{1240}{\lambda(in\, nm)} eV$)

A 1.8

B 1.4

C 2.5

D 5.6

Solution

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v)^2$$

$$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi$$

$$\Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$

$$= 1.8eV$$

#1330603

A particle is moving with a velocity $\bar{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

A $xy = constant$

[✓ **B**] $y^2 = x^2 + constant$

C $y = x^2 + constant$

D $y^2 = x + constant$

Solution

$$\frac{dx}{dt} = ky, \ \frac{dy}{dt} = kx$$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$

$\Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

#1330672

A convex lens is put $10cm$ from a light source and it makes a sharp image on a screen, kept $10cm$ from the lens. Now a glass block (refractive index 1.5) of $1.5cm$ thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

[✓ **A**] $0.55cm$ away from the lens

B $1.1cm$ away from the lens

C $0.55cm$ towards the lens

D 0

Solution

$2f = 10cm$

$f = 5cm$

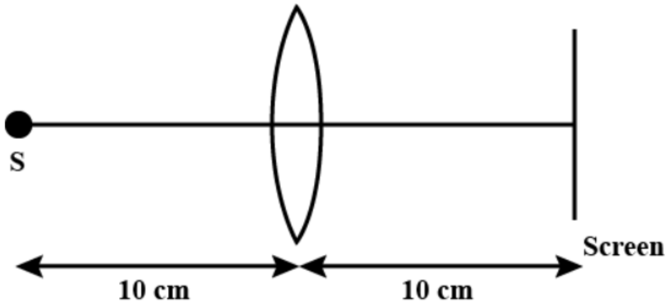
Now due to glass plate, shift = $t(1 - \frac{1}{\mu}) = 1.5(1 - \frac{2}{3}) = 0.5cm$

New $u = 10 - 0.5 = 9.5cm$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$v = \frac{47.5}{4.5}$

shift = $v - 10 = \frac{5}{9}cm$



#1330713

For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :

A $\frac{R}{\sqrt{5}}$

B R

$\boxed{\text{C}}$ $\frac{R}{\sqrt{2}}$

D $R\sqrt{2}$

Solution

Electric field on axis of ring

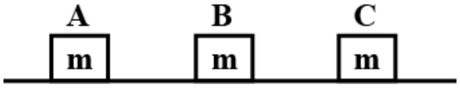
$E = \frac{kQh}{(h^2 + R^2)^{3/2}}$

for maximum electric field

$\frac{dE}{dh} = 0$

$\Rightarrow h = \frac{R}{\sqrt{2}}$

#1330794



Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given a brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}th$ of the initial kinetic energy is lost in the whole process. What is the value of $\frac{M}{m}$?

$\boxed{A}$ 4

B 5

C 3

D 2

Solution

$k_i = \frac{1}{2}mv_0^2$

From linear momentum conservation

$mv_0 = (2m + M)v_f$

$\Rightarrow v_f = \frac{mv_0}{2m + M}$

$\frac{k_i}{k_f} = 6$

$$\Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m + M)\left(\frac{mv_0}{2m + M}\right)^2} = 6$$

$\Rightarrow \frac{2m + M}{m} = 6$

$\Rightarrow \frac{M}{m} = 4$

#1330832

Drift speed of electrons, when $1.5A$ of current flows in a copper wire of cross section $5mm2$, is v. If the electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s is close to (Take charge of electron to be $= 1.6 \times 10^{-19}C$)

A 0.2

B 3

C 2

$\boxed{D}$ 0.02

Solution

$I = neAv_d$

$\Rightarrow v_d = \frac{I}{neA} = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$

$= 0.02mm/s$

#1329135

Which one of the following statements regarding Henry's law is not correct?

A The value of K_H increases with the function of the nature of the gas

[✓ B] Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids

C The partial of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

D Different gases have different K_H (Henry's law constant) values at the same temperature.

Solution

Liquid solution

$P_{gas} = K_H \times X_{gas}$

Thus, K_H is directly proportional to the pressure.

More is K_H less is solubility, lesser solubility is at higher temperature.

So more is the temperature, more is the K_H.

#1329207

The correct decreasing order for acid strength is:

[✓ A] $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

B $FCH_2COOH > NCCH_2COOH > NO_2CHCOOH > ClCH_2COOH$

C $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

D $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

Solution

EWG increases the acidic strength.

Here the strength of electron withdrawing group is in the following order:

$NO_2 > CN > F > Cl$

Thus, acidic strength is given as:

$NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

#1329285

Two complex $[Cr(H_2O_6)Cl_3]$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is:

A Δ_0 value of (A) is less than that of (B)

[✓ B] Δ_0 value of (A) and (B) are calculated from the energies of violet and yellow light, respectively

C both absorb energies corresponding to their complementary colors

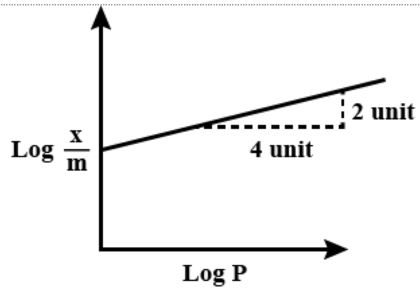
D both are paramagnetic with three unpaired electrons

Solution

Δ_0 order will be compared by spectrochemical series, not by energies of violet & yellow light so Δ_0 order is

$[Cr(H_2O)_6]Cl_3 < [Cr(NH_3)_6]Cl_3$

#1329354



Adsorption of gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas absorbed on mass m of the adsorbent at pressure p. $\frac{x}{m}$ is proportional to:

A $P^{\frac{1}{4}}$

B P^{2}

C P

[✓ D] $P^{\frac{1}{2}}$

Solution

$\frac{x}{m} = K \times P^{1/n}$

$log\frac{x}{m} = logK + \frac{1}{n}log\,P$

$m = \frac{1}{n} = \frac{2}{4} = \frac{1}{2} \Rightarrow n = 2$

So, $\frac{x}{m} = K \times P^{1/2}$

$\therefore$ $\frac{x}{m}$ is directly proportional to the $P^{\frac{1}{2}}$

#1329387

Correct statements among a to d regarding silicones are:

(a) They are polymers with hydrophobic character

(b) They are biocompatible.

(c) In general, they have high thermal stability and low dielectric strength.

(d) Usually, they are resistant to oxidation and used as greases.

A (a), (b) and (c) only

B (a), and (b) only

[✓ C] (a), (b), (c) and (d)

D (a), (b) and (d) only

Solution

- Silicones are the polymer of silicon-containing $[-(R)_3Si-O-]$ linkage.
- It is a polymer with hydrophobic character thus used in making water-resistant seals.
- They are biocompatible.
- They have high thermal stability and low dielectric strength.
- They are also resistant to oxidation. Because of there wax like taxture, they are also used in greases.

#1329707

For emission line of atomic hydrogen from $n_i = 8$ to n_f the plot of wave number $(\bar{\nu})$ against $\left(\frac{1}{n^2}\right)$ will be: (The Rydberg constnt, R_H is in wave number unit).

A linear with slope $-R_H$

B linear with intercept $-R_H$

C non linear

D linear with slope R_H

Solution

$$\frac{1}{\lambda} = \bar{v} = R_{Hz^2}\left(\frac{1}{\eta_1^2} - \frac{1}{\eta_2^2}\right)$$

$$\bar{v} = R_H \times \left(\frac{1}{\eta_1^2} - \frac{1}{8^2}\right)$$

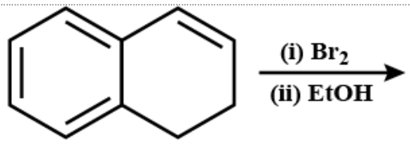
$$\bar{v} R_H \times \frac{1}{\eta^2} - \frac{R_H}{8_2}$$

$$\bar{v} R_H \times \frac{1}{\eta^2} - \frac{R_H}{64}$$

$m = R_H$

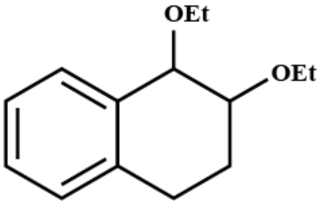
Linear with slope R_H

#1329731

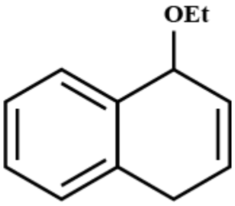


The major product the following reaction is:

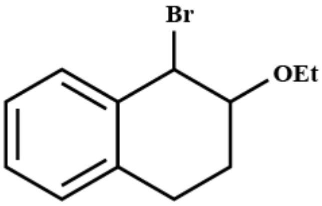
A



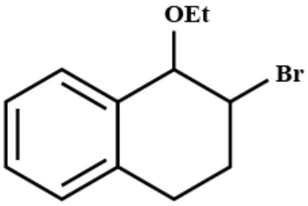
B



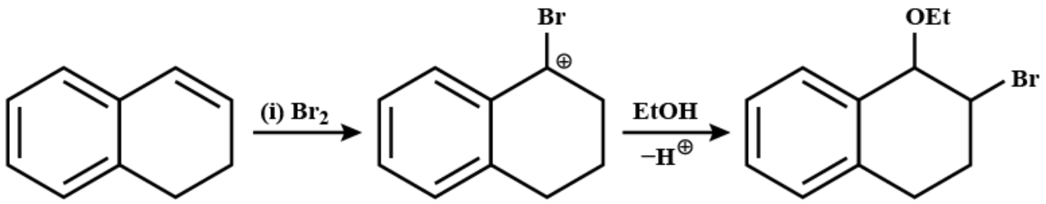
C



D



Solution



#1329792

The alkaline earth metal nitrate that does not crystallise with water molecules, is:

A $Sr(NO_3)_2$

B $Mg(NO_3)_2$

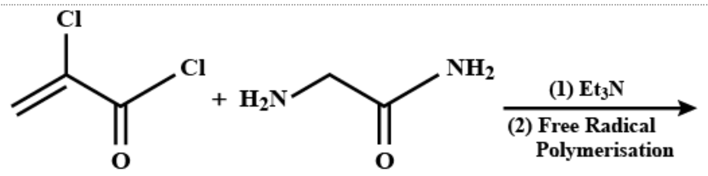
C $CA(NO_3)_2$

D $Ba(NO_3)_2$

Solution

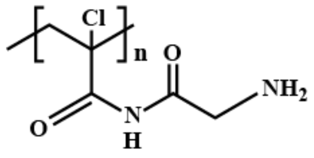
Smaller in size of center atoms more water molecules will crystallize hence $Ba(NO_3)_2$ is answer due to its largest size of $' + ve'$ ion.

#1329830

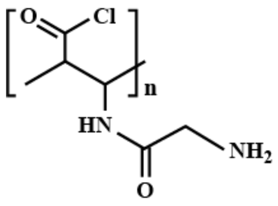


Major product of the following reaction is:

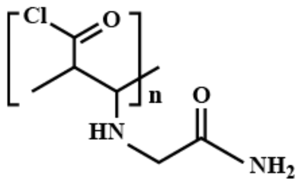
A



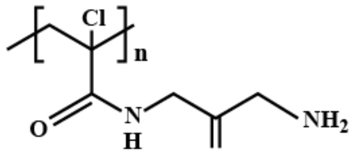
B



C



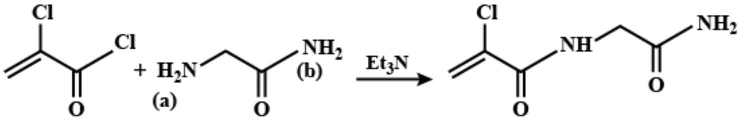
D



Solution

NH_2 (a) will wact as nucleophile as (b) is having

delocalised lonepair.



NH_2 (a) will wact as nucleophile as (b) is having delocalised lonepair.

#1329853

The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is :

$\boxed{\text{A}}$ 5.92

B 3.87

C 6.93

D 4.90

Solution

$\mu = \sqrt{n(n+2)}$ B.M

$n =$ Number of unpaired electrons

$n =$ Maximum number of unpaired electron $= 5$

$EX: Mn^{2+}$ complex.

#1329974

20 mL of $0.1 M H_2SO_4$ solution is added to 30 mL of $0.2 M\, NH_4OH$ solution. The pH of the resultant mixture is: $[pk_b \text{ of } NH_4OH = 4.7]$.

A 9.4

B 5.0

$\boxed{\text{C}}$ 9.0

D 5.2

Solution

20 ml 0.1M $H_2SO_4 \Rightarrow \eta_{H^+} = 4$

30 ml 0.2M $NH_4OH \Rightarrow \eta_{NH_4OH=6}$

$NH_4OH + H^+ \rightleftharpoons NH^{\oplus} + H_2O$

$\Rightarrow$ 6 4 0 0

$\Rightarrow$ 2 0 4 4

Solution is basic buffer

$$pOH = pK_b + log\frac{NH_4^+}{NH_4OH}$$

$= 4.7 + log 2$

$= 4.7 + 0.3 = 5$

$pH = 14 - 5 = 9$

#1330039

0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a a container of volume $10m3$ at 1000 K. given R is the gas constant in J_K^1 mol^{1}m, x is:

A $\frac{2R}{4+12}$

B $\frac{2R}{4-12}$

[✓ C] $\frac{4-R}{2R}$

D $\frac{4+R}{2R}$

Solution

$n_T = (0.5 + x)$

$PV = n \times R \times T$

$200 \times 10 = (0.5 + x) \times R \times 1000$

$2 = (0.5 + x)R$

$\frac{2}{R} = \frac{1}{2} + x$

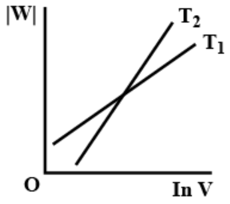
$\frac{4}{R} - 1 = 2x$

$\frac{4-R}{2R} = x$

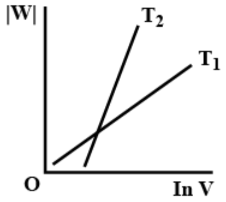
#1330148

Consider the reversible isothermal exapnsion of an ideal gas in a closed systeam at two different temperatures T_1 and $T_2 (T_1 < T_2)$. The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

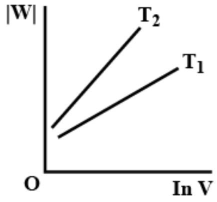
A



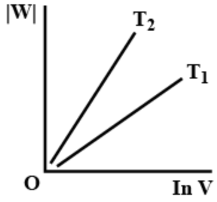
[✓ B]



C



D



Solution

$w = -nRT\ in\dfrac{V_2}{V_1}$

$w = -nRT\ in\dfrac{V_b}{V_i}$

$|w| = nRT\ in\dfrac{V_b}{V_i}$

$|w| = nRT(In\,V_b - In\,V_i)$

$|w| = nRT\ In\,V_b - nRT\,In\,V_i$

$Y = m\ x - C$

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative then

curve 1.

#1330238

The major product of following reaction is :

$R - C = N \xrightarrow[(1)\,AlH(\,i-Bu_2)]{(2)\,H_2O}$?

$\boxed{A}$ $RCHO$

B $RCOOH$

C RCH_2NH_2

D $RCONH_2$

Solution

$R - C \equiv N \xrightarrow{AlH(\,Bu_2)} R - CH = N - \xrightarrow{H_2O} R - CH = O$

#1330296

In general, the properties that decrease and increase down a group in the periodic table, respectively, are:

A electronegativity and electron gain enthalpy.

$\boxed{B}$ electronegativity and atomic radius.

C atomic radius and electronegativity

D electron gain enthalpy and electronegativity

Solution

Electronegativity decrease as we go down the group and atomic radius increase as we go down the group.

#1330347

A solution of sodium sulfate $92g$ of $Na^+\,ions$ that solution in mol kg^{-1} is:

A 16

B 8

C 4

$\boxed{D}$ 12

Solution

Here molecular weight of Na is $23g/\,mol$

$n_{Na^+} = \dfrac{92}{23} = 4$

A molality is a number of moles of solute present in per Kg of solvent.

So, molality $= 4$

#1330371

A water sample has ppm level concentration of the following metals: $Fe = 0.2;\ Mn = 5.0;\ Cu = 3.0;\ Zn = 5.0$. The metal that makes the water sample unsuitable for drinking is:

A Zn

B Fe

$\boxed{\text{C}}$ Mn

D Cu

Solution

(i) $Zn = 0.2$

(ii) $Fe = 0.2$

(iii) $Mn = 5.0$

(iv) $Cu = 3.0$

#1330546

The increasing order of pKa of the following amino acids in aqueous solution is:

Gly

Asp

Lys

Arg

A $Asp < Gly < Arg < Lys$

B $Arg < Lys < Gly < Asp$

C $Gly < Asp < Arg < Lys$

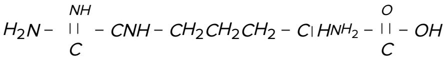
$\boxed{\text{D}}$ $Asp < Gly < Lys < Arg$

Solution

Order of acidic strength:

$HOOC-CH_2-C|HNH_2-COOH\ >\ NH_2--CH_2-COOH\ >$

Aspartic acid Glycine



$H_2N-\overset{NH}{\underset{C}{||}}-CNH-CH_2CH_2CH_2-C|HNH_2-\overset{O}{\underset{C}{||}}-OH$

Argimie

so, pK_a

$Asp < Gly < Arg < Lys$

#1330596

According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-?

A Both are unstable

B Li_2^+ is unstable and Li_2^- is stable

C Li_2^+ is unstable and Li_2^- is unstable

$\boxed{\text{D}}$ Both are stable

Solution

Both Li_2^+ and Li_2^- have bond order 0.5.

For positive bond order the molecule is stable.

Thus the given molecules are stable.

#1330801

The following results were obtained during kinetic studies of the reaction:

$2A + B \rightarrow$ Products

Experment	$[A]$ (in mol L^{-1})	$[A]$ (in mol L^{-1})	Initial Rate of reaction (in mol $L^{-1} min^{-1}$)
(I)	0.10	0.20	6.93×10^{-3}
(II)	0.10	0.25	6.93×10^{-3}
(III)	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is:

A 10

$\boxed{\text{B}}$ 5

C 100

D 1

Solution

$6.93 \times 10^{-3} = K \times (0.1)^x(0.2)y$

$6.93 \times 10^{-3} = K \times (0.1)^x(0.25)y$

so $y = 0$

and $1.386 \times 10^{-2} = K \times (0.2)^x(0.30)^y$

$\frac{1}{2} = \left(\frac{1}{2}\right)^x x = 1$

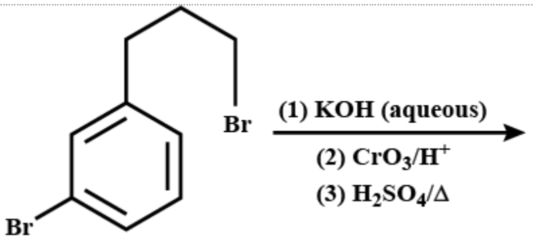
So $r = K \times (0.1) \times (0.2)^0$

$6.93 \times 10^{-3} = K \times 0.1 \times (0.2)^0$

$K = 6.93 \times 10^{-2}$

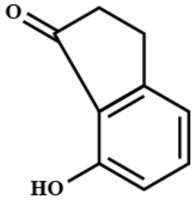
$t_{1/2} = \frac{0.693}{2k} = \frac{0.693}{0.693 \times 10^{-1} \times 2} = \frac{10}{2} = 5$

#1330848

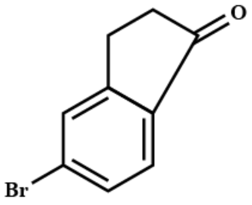


The major product of the following reaction is:

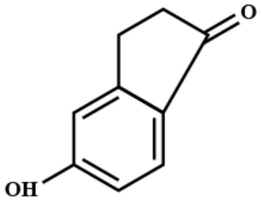
A



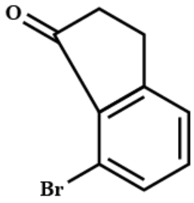
B



C



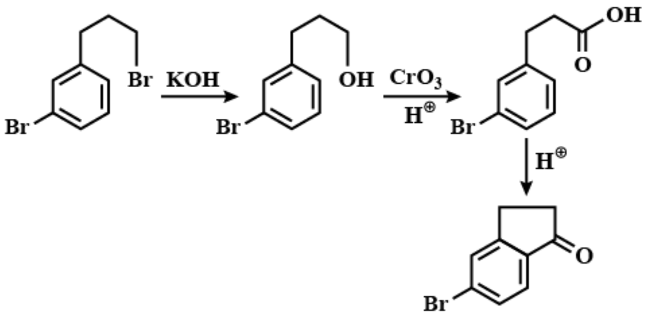
D



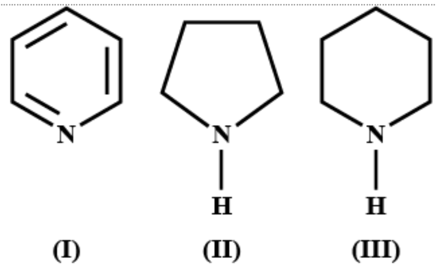
Solution

During Aromatic electrophilic substititution reaction, Br act as ortho-para directing.

The major product will be formed on less hindrance p position:



#1330892



Arrange the following amines in the decreasing order of basicity:

A *II > II > III*

B *III > II > I*

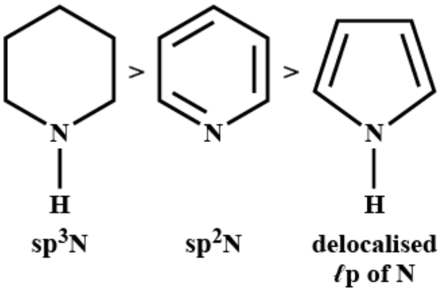
C *I > III > II*

D *III > I > II*

Solution

Basic strength increases as the electron donating capacity of nitrogen increases.

In compound III, the nitrogen has two lone pairs of electrons for donation whereas, in compound II, the lone pair of electrons are delocalized in the aromatic ring.



#1330926

Which amongst the following is the strongest acid ?

A CHI_3

B $CHCl_3$

C $CHBr_3$

[✓ **D**] $CH(CN)_3$

Solution

CN makes anino most stable so answer is $CH(CN)_3$

CN makes anino most stable so answer is $CH(CN)_3$

#1331031

The anodic half-cell of lead-acid battery is recharged unsing electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process in : (Molar mass of $PbSO_4 = 303 gmol^{-1}$)

A 22.8

[✓ **B**] 15.2

C 7.6

D 11.4

Solution

$A)\ PbSO_{4\,0.05/2\ mole} + 2OH^- \rightarrow PbO_2 + H_2SO_4 + 2e^-{}_{0.05F}$

$B)\ PbSO_{4\,0.05/2\ mole} + 2e^-{}_{0.05\ F} + 2H^+ \rightarrow Pb(s) + H_2SO_4$

$n_T(PbSO_4) = 0.05\ mole$

$m_{PbSO_4} = 0.05 \times 303 = 15.2\ gm$

#1331044

The one that is extensively used as a piezoelectric material is :

[✓ **A**] quartz

B amorphous silica

C mica

D tridymite

Solution

Quartz is used as piezoelectric material.

It produces electricity when any mechanical stress in form of pressure applied on it.

Thus it is also used in watches and oscillators.

#1331062

Aluminium is usually found in +3 oxidation state. In contarast, thallium exists in +1 and +3 oxidation states. This is due to :

A lanthanoid contraction

B lattice effect

C diagonal relationship

$\boxed{\text{D}}$ inert pair effect

Solution

Inert pair effect is the prominent character of the p-block element.

In this, the high molecular weight element of the group show lower oxidation state.

This is because on going down the group, the shielding effect increases, but d-subshell and f-subshell show poor shielding effect,

The high molecular weight members of p-block groups contain d-subshell and f-subshell.

Because of this, they show a lower oxidation state.

#1331180

The correct match between Item -I and Item- II is :

	Item - I		Item - II
(A)	Chloroxylenol	(P)	Carbylamine Test
(B)	Norethindrone	(Q)	Sodium Hydrogen carbonate Test
(C)	Sulphapyridine	(R)	Ferric chloride test
(D)	Penicillin	(s)	Bayer's test

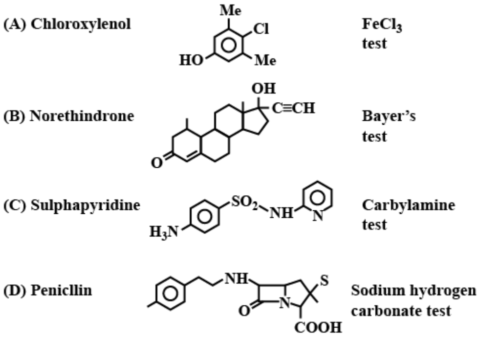
A $A \rightarrow Q;\ B \rightarrow P;\ C \rightarrow S;\ D \rightarrow R$

B $A \rightarrow R;\ B \rightarrow P;\ C \rightarrow S;\ D \rightarrow Q$

$\boxed{\text{C}}$ $A \rightarrow R;\ B \rightarrow S;\ C \rightarrow P;\ D \rightarrow Q$

D $A \rightarrow Q;\ B \rightarrow S;\ C \rightarrow P;\ D \rightarrow R$

Solution



(A) Chloroxylenol		**$FeCl_3$ test**
(B) Norethindrone		**Bayer's test**
(C) Sulphapyridine		**Carbylamine test**
(D) Penicllin		**Sodium hydrogen carbonate test**

#1331230

The ore that contains both iron and copper is:

A malachite

B dolomite

C azurite

[✓ **D**] copper pyrites

Solution

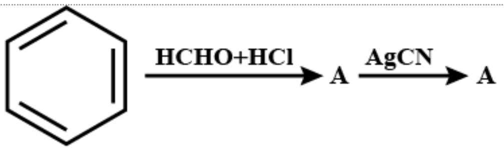
Copper pyrites : $CuFeS_2$

Malachite : $Cu(OH)_2\ .\ CuCO_3$

Azurite : $Cu(OH)_2\ .\ 2CuCO_3$

Dolomite $CaCO_3\ .\ MgCO_3$

#1331248



The compounds A and B in the following reaction are, respectively:

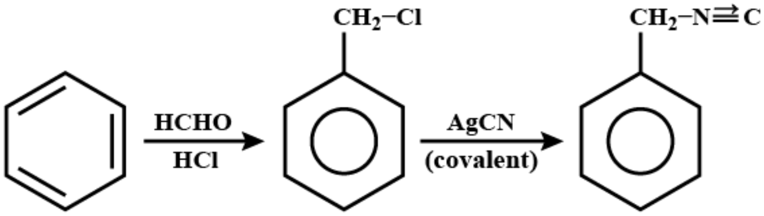
A $A =$ Benzyl alcohol, $B =$ Benzyl isocyanide

B $A =$ Benzyl alcohol, $B =$ Benzyl cyanide

C $A =$ Benzyl chloride, $B =$ Benzyl cyanide

[✓ **D**] $A =$ Benzyl chloride, $B =$ Benzyl isocyanide

Solution



#1331268

The isotopes of hydrogen are :

A Tritium and protium only

B Deuterium and tritium only

C Protium and deuterum only

[✓ **D**] Protium, deuterium and tritium

Solution

Isotopes of hydrogen is : Protium Deuterium Tritium

#1329190

Topic: Area of Bounded Regions

The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point $(2, 3)$ to it and the y-axis is

A $\frac{14}{3}$

B $\frac{56}{3}$

[✓ C] $\frac{8}{3}$

D $\frac{32}{3}$

Solution

Equation of tangent at $(2, 3)$ on

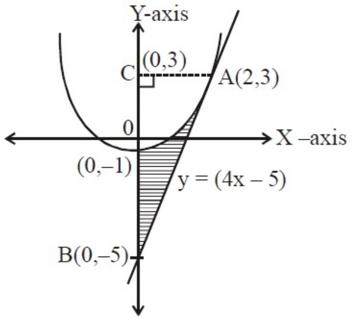
$y = x^2 - 1$, is $y = (4x - 5)......(i)$

$\therefore$ Required shaded area

$= ar(\Delta ABC) - \int_{-1}^{3}\sqrt{y+1}dy$

$= \frac{1}{2}\cdot(8)\cdot(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$

$= 8 - \frac{16}{3} = \frac{8}{3}$ (square units).



#1329238

Topic: Maxima and Minima

The maximum volume (in $cu.\ m$) of the right circular cone having slant height $3m$ is

A $3\sqrt{3}\pi$

B 6π

[✓ C] $2\sqrt{3}\pi$

D $\frac{4}{3}\pi$

Solution

$\therefore\ h = 3\cos\theta$

$r = 3\sin\theta$

Now,

$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(9\sin^2\theta)\cdot(3\cos\theta)$

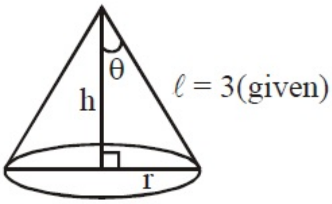
$\therefore\ \frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$

Also, $\frac{d^2V}{d\theta^2}]_{\sin\theta = \sqrt{\frac{2}{3}}} = negative$

$\Rightarrow$ Volume is maximum.

when $\sin\theta = \sqrt{\frac{2}{3}}$

$\therefore\ V_{max}\left(\sin\theta = \sqrt{\frac{2}{3}}\right) = 2\sqrt{3}\pi(in\ cu.\ m)$.



#1329322

Topic: Integration by Substitution

For $x^2 \neq n\pi + 1$, $n\epsilon N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin2(x^2-1)}{2\sin(x^2-1)+\sin2(x^2-1)}}dx$ is equal to

(where c is a constant of integration).

$\boxed{\text{A}}$ $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

B $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

C $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

D $\frac{1}{2}\log_e\left|\sec(x^2-1)\right| + c$

Solution

Put $(x^2-1) = 1$

$\Rightarrow 2xdx = dt$

$\therefore\ I = \frac{1}{2}\int\sqrt{\frac{1-\cos t}{1+\cos t}}dt$

$= \frac{1}{2}\int\tan\left(\frac{t}{2}\right)dt$

$= ln\left|\sec\frac{t}{2}\right| + c$

$I = ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$.

#1329349

Topic: Operations on Complex Numbers

Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to

A 512

B -512

[✓ C] -256

D 256

Solution

We have

$(x+1)^2 + 1 = 0$

$\Rightarrow (x+1)^2 - (i)^2 = 0$

$\Rightarrow (x+1+i)(x+1-i) = 0$

$\therefore x = -(1+i)$□□□□□$\alpha(let)$ $-(1-i)$□□□□□$\beta(let)$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7\alpha + (\beta^2)^7\beta$

$= -128(-i+1+i+1)$

$= -256\cdot$

#1329364

Topic: Linear Differential Equation

If $y = y(x)$ is the solution of the differential equation,

$x\dfrac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\dfrac{1}{2}\right)$ is equal to

A $\dfrac{7}{64}$

B $\dfrac{13}{16}$

[✓ C] $\dfrac{49}{16}$

D $\dfrac{1}{4}$

Solution

$\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = x$

$\Rightarrow I.F. = x^2$

$\therefore yx^2 = \dfrac{x^4}{4} + \dfrac{3}{4}$ (As, $y(1) = 1$)

$\therefore y\left(x = \dfrac{1}{2}\right) = \dfrac{49}{16}.$

#1329382

Topic: Tangent

Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is

A $2\sqrt{3}y = 12x + 1$

B $2\sqrt{3}y = -x - 12$

[✓ C] $\sqrt{3}y = x + 3$

D $\sqrt{3}y = 3x + 1$

Solution

Let equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$,

$\Rightarrow m^2x - ym + 1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

$\Rightarrow \frac{|3m^2+1|}{\sqrt{m^4+m^2}} = 3$

$m = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$ tangent are $x + \sqrt{3}y + 3 = 0$

and $x - \sqrt{3}y + 3 = 0$.

#1329395

Topic: Combinations of Dissimilar Things

Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is

A 200

[✓ B] 300

C 500

D 350

Solution

Require number of ways

$=$ Total number of ways $-$ When A and B are always included.

$= {}^5C_2 \cdot {}^7C_3 - {}^5C_1\, {}^5C_2 = 300$.

#1329419

Topic: Non-intersecting, Intersecting, Touching circles

Three circles of radii $a, b, c (a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then

[✓ A] $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

B a, b, c are in A.P.

C $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

D $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

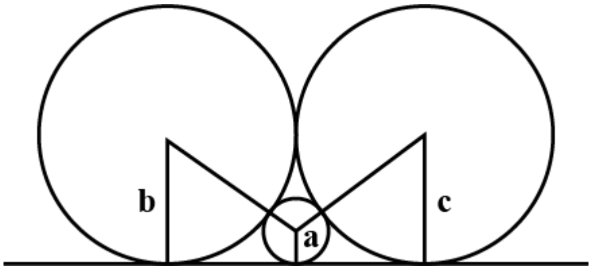
Solution

$AB = AC + CB$

$\sqrt{(b+c)^2 - (b-c)^2} = \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$

$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}}$.



#1329445

Topic: Application of Binomial Expansion

If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to

A 14

B 6

C 4

$\boxed{\text{D}}$ 8

Solution

$$\frac{2^{403}}{15} = \frac{2^3 \cdot (2^4)^{100}}{15} = \frac{8}{15}(15 + 1)^{100}$$

$$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$$

$\because$ 8λ is integer

→ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$.

#1329461

Topic: Equation of Parabola

Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

A $(4, -4)$

B $(5, 2\sqrt{6})$

$\boxed{\text{C}}$ $(8, 6)$

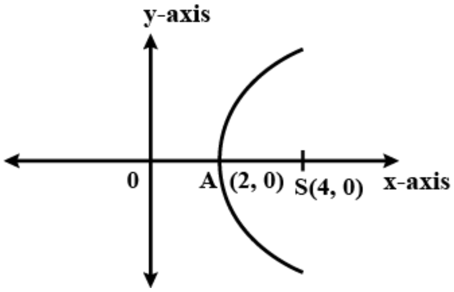
D $6, 4\sqrt{2}$

Solution

Equation of parabola is

$y^2 = 8(x - 2)$

$(8, 6)$ does not lie on parabola.



#1329485

Topic: Plane

The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point.

A $(-3, 0, 1)$

B $(3, 3, -1)$

$\boxed{\text{C}}$ $(3, 2, 1)$

D $(-3, 1, 1)$

Solution

Equation of plane

$(x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$

dr's of normal of the plane are

$1+2\lambda, 1+3\lambda, 1-\lambda$

Since plane is parallel to $y-axis, 1+3\lambda=0$

$\Rightarrow \lambda=-1/3$

So the equation of plane is

$x+4z-y=0$

Point $(3,2,1)$ satisfies this equation

Hence Answer is (3).

#1329510
Topic: Geometric Progression

If a, b and c be three distinct real numbers in G.P. and $a+b+c=xb$, then x cannot be

A 4

B -3

C -2

$\boxed{\text{D}}$ 2

Solution

$\frac{b}{r}, b, br \rightarrow G.P. (|r|\neq 1)$

given $a+b+c=xb$

$\Rightarrow b/r+b+br=xb$

$\Rightarrow b=0$ (not possible)

or $1+r+\frac{1}{r}=r \Rightarrow x-1=r+\frac{1}{r}$

$\Rightarrow x-1>2$ or $x-1<-2$

$\Rightarrow x>3$ or $x<-1$

So x can't be $'2'$.

#1329538
Topic: Various Forms of Equation of Line

Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$. Which one of the following statements is true?

A The lines are all parallel

B Each line passes through the origin

C The lines are not concurrent

$\boxed{\text{D}}$ The lines are concurrent at the point All points pass through $\left(\frac{3}{4}, \frac{1}{2}\right)$

Solution

Given set of lines $px+qy+r=0$

Given condition $3p+2q+4r=0$

$\frac{3}{4}p+\frac{1}{2}q+r=0$

So all points passes through $(\frac{3}{4}, \frac{1}{2})$

#1329627

Topic: Application of Matrices and Determinants

The system of linear equations:

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$.

A Has infinitely many solutions for $a = 4$

$\boxed{\text{B}}$ Is inconsistent when $|a| = \sqrt{3}$

C Is inconsistent when $|a| = 4$

D Has a unique solution for $|a| = \sqrt{3}$

Solution

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix} = a^2 - 3$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \\ a + 1 & 3 & a^2 - 1 \end{vmatrix} = a^2 - a + 1$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & a + 1 & a^2 - 1 \end{vmatrix} = a^2 - 3$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a + 1 \end{vmatrix} = a - 4$$

$D = 0$ at $|a| = \sqrt{3}$ but $D_3 = \pm\sqrt{3} - 4 \neq 0$

So the system is Inconsistant for $|a| = \sqrt{3}$.

#1329675

Topic: Vector Triple Product

Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to

$\boxed{\text{A}}$ $\frac{19}{2}$

B 8

C $\frac{17}{2}$

D 9

Solution

$\vec{a} \times \vec{c} = -\vec{b}$

$(\vec{a} \times \vec{c}) \times \vec{a} = -\vec{b} \times \vec{a}$

$\Rightarrow (\vec{a} \times \vec{c}) \times \vec{a} = \vec{a} \times \vec{b}$

$\Rightarrow (\vec{a} \cdot \vec{a})\vec{c} - (\vec{c} \cdot \vec{a})\vec{a} = \vec{a} \times \vec{b}$

$\Rightarrow 2\vec{c} - 4\vec{a} = \vec{a} \times \vec{b}$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + 2\hat{k}$

So, $2\vec{c} = 4\hat{i} - 4\hat{j} - \hat{i} - \hat{j} + 2\hat{k}$

$= 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\Rightarrow \vec{c} = \frac{3}{2}\hat{i} - \frac{5}{2}\hat{j} + \hat{k}$

$|\vec{c}| = \sqrt{\frac{9}{4} + \frac{25}{4} + 1} = \sqrt{\frac{38}{4}} = \sqrt{\frac{19}{2}}$

$|\vec{c}|^2 = \frac{19}{2}$.

#1329698

Topic: Arithmetic Progression

Let $a_1, a_2, \ldots.. a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to

A 57

B 47

C 42

$\boxed{\text{D}}$ 52

Solution

$S = a_1 + a_2 + \ldots.. + a_{30}$

$S = \frac{30}{2}[a_1 + a_{30}]$

$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$

$T = a_1 + a_3 + \ldots.. + a_{29}$

$= (a_1) + (a_1 + 2d) \ldots.. + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + \ldots.. + 14)$

$T = 15a_1 + 210d$

Now use $S - 2T = 75$

$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$.

#1329723

Topic: Variance and Standard Deviation

5 students of a class have an average height $150\ cm$ and variance $18\ cm^2$. A new student, whose height is $156\ cm$, joined them. The variance (in cm^2) of the height of these six students is

A 22

$\boxed{\text{B}}$ 20

C 16

D 18

Solution

Given $\bar{x} = \frac{\Sigma x_i}{5} = 150$

$\Rightarrow \sum_{i=1}^{5} x_i = 750....(i)$

$\frac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18$

$\frac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Sigma x_i^2 = 112590.....(ii)$

Given height of new student

$x_6 = 156$

Now, $\bar{x}_{new} = \frac{\sum_{i=1}^{6} x_i}{6} = \frac{750 + 156}{6} = 151$

Also, New variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - (\bar{x}_{new})^2$

$= \frac{112590 + (156)^2}{6} - (151)^2$

$= 22821 - 22801 = 20.$

#1329740

Topic: Probability Distribution

Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals

A $\frac{52}{169}$

[✓ **B**] $\frac{25}{169}$

C $\frac{49}{169}$

D $\frac{24}{169}$

Solution

Two cards are drawn successively with replacement

4 *Aces* 48 Non Aces

$P(x = 1) = \frac{^4C_1}{^{52}C_1} \times \frac{^{48}C_1}{^{52}C_1} + \frac{^{48}C_1}{^{52}C_1} \times \frac{^4C_1}{^{52}C_1} = \frac{24}{169}$

$P(x = 2) = \frac{^4C_1}{^{52}C_1} \times \frac{^4C_1}{^{52}C_1} = \frac{1}{169}$

$P(x = 1) + P(x = 2) = \frac{25}{169}.$

#1329767

Topic: Composite and Inverse Functions

For $x \epsilon R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1 - x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2^{\circ} J^{\circ} f_1)(x) = f_3(x)$ then $J(x)$ is equal to

[✓ **A**] $f_3(x)$

B $f_1(x)$

C $f_2(x)$

D $\frac{1}{x} f_3(x)$

Solution

Given $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1 - x}$

$f_2 \cdot J \cdot f_1)(x) = f_3(x)$

$f_2 \cdot (J(f_1(x))) = f_3(x)$

$f_2 \cdot \left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1 - x}$

$1 - J\left(\frac{1}{x}\right) = \frac{1}{1 - x}$

$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1 - x} = \frac{-x}{1 - x} = \frac{x}{x - 1}$

Now $x \to \frac{1}{x}$

$J(x) = \frac{\frac{1}{x}}{\frac{1}{x} - 1} = \frac{1}{1 - x} = f_3(x).$

#1329798

Topic: Operations on Complex Numbers

Let $A = \left\{0\epsilon\left(-\frac{\pi}{2}, \pi\right): \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} \text{is purely imaginary}\right\}$

Then the sum of the elements in A is

A $\frac{5\pi}{6}$

$\boxed{\textbf{B}}$ $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D π

Solution

Given $z = \frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely img

so real part becomes zero.

$z = \left(\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}\right) \times \left(\frac{1 + 2i\sin\theta}{1 + 2i\sin\theta}\right)$

$z = \frac{(3 - 4\sin^2\theta) + i(8\sin\theta)}{i + 4\sin^2\theta}$

Now $Re(z) = 0$

$\frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} = 0$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

$\because \theta\epsilon\left(-\frac{\pi}{2}, \pi\right)$

then sum of the elements in A is

$-\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}.$

#1329812

Topic: Applications on Geometrical Figures

If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

A $\frac{4}{9}$

B $\frac{7}{17}$

C $\frac{8}{17}$

$\boxed{\text{D}}$ $\frac{8}{15}$

Solution

Point of intersection is $P(2, 6)$

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$\therefore \; |\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{8}{15}.$

#1329837

Topic: Operations on Matrices

If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to

$\boxed{\text{A}}$ $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

B $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

C $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

D $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Solution

Here, $AA^T = 1$

$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Also, $A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$

$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$

#1329863

Topic: Equation of Hyperbola

Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval.

A $(2, 3]$

$\boxed{\text{B}}$ $(3, \infty)$

C $(3/2, 2]$

D $(1, 3/2]$

Solution

$e = \sqrt{1 + \tan^2\theta} = \sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \epsilon (60^\circ, 90^\circ)$

Now, $l(L.\ R) = \frac{2b^2}{a} = 2\frac{(1 - \cos^2\theta)}{\cos\theta}$

$= 2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

$l(L.\ R) \epsilon (3, \infty)$.

#1329894

Topic: Lines

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is

A $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

$\boxed{\text{B}}$ $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{3}$

D $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Solution

Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

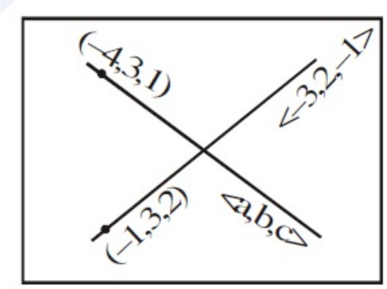
$= -2\hat{i} + 5\hat{k}$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$



#1329921

Topic: Trigonometric Functions

For any $\theta \epsilon \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals.

$\boxed{\text{A}}$ $13 - 4\cos^6\theta$

B $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

C $13 - 4\cos^2\theta + 6\cos^4\theta$

D $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

Solution

We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta) + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 13 - 4\cos^6\theta.$

#1329950

Topic: Properties of Inverse Trigonometric Functions

If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{2}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ then x is equal to

$\boxed{\text{A}}$ $\frac{\sqrt{145}}{12}$

B $\frac{\sqrt{145}}{10}$

C $\frac{\sqrt{146}}{12}$

D $\frac{\sqrt{145}}{11}$

Solution

$$\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$$

$$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\frac{2}{3x}\right)$$

$$\frac{3}{4x} = \frac{\sqrt{9x^2 - 4}}{3x}$$

$$\frac{81}{16} + 4 = 9x^2$$

$$x^2 = \frac{145}{16 \times 9} \Rightarrow x = \frac{\sqrt{145}}{12}.$$

#1329968

Topic: Definite Integrals of Special Functions

The value of $\int_0^{\pi} |\cos x|^3 dx$.

A $\frac{2}{3}$

B 0

C $\frac{-4}{3}$

[✓ D] $\frac{4}{3}$

Solution

$$\int_0^{\pi} |\cos x|^3 dx = \int_0^{\pi/2} \cos^3 x\, dx - \int_{\pi/2}^{\pi} \cos^3 x\, dx$$

$$= \int_0^{\pi/2}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3} + 3\sin x\right)_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3\sin x\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - (0 + 0) - \left\{(0 + 0) + \left(\frac{-1}{3} + 3\right)\right\}\right]$$

$$= \frac{4}{3}.$$

#1330014

Topic: Truth Tables

If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \epsilon \{\wedge, \vee\}$, then the order pair $(\oplus, \odot)$ is

[✓ A] $(\wedge, \vee)$

B $(\vee, \vee)$

C $(\wedge, \wedge)$

D $(\vee, \wedge)$

Solution

$(p \oplus q) \wedge (\sin p □ q) \equiv p \wedge q$ (given)

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$\sim p \vee q$	$\sim p \wedge q$	$(p \wedge q) \wedge (\sim p \vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table $(\oplus, □) = (\wedge, \vee)$.

#1330052

Topic: Standard Simplifications

$$\lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}.$$

$\boxed{\text{A}}$ Exists and equals $\frac{1}{4\sqrt{2}}$

B Does not exist

C Exist and equals $\frac{1}{2\sqrt{2}}$

D Exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

Solution

$$\lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

$$= \lim_{y \to 0} \frac{1+\sqrt{1+y^4}-2}{y^2(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})}$$

$$= \lim_{y \to 0} \frac{(\sqrt{1+y^4}-1)(\sqrt{1+y^4}+1)}{y^4(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})(\sqrt{1+y^4}+1)}$$

$$= \lim_{y \to 0} \frac{1+y^4-1}{y^4(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})(\sqrt{1+y^4}+1)}$$

$$= \lim_{y \to 0} \frac{1}{(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2})(\sqrt{1+y^4}+1)} = \frac{1}{4\sqrt{2}}.$$

#1330087

Topic: Continuity and Differentiability

Let $f: R \to R$ be a function defined as:

$$f(x) = \begin{cases} 5, & if \quad x \le 1 \\ a+bx, & if \quad 1 < x < 3 \\ b+5x, & if \quad 3 \le x < 5 \\ 30, & if \quad x \ge 5 \end{cases}$$

Then, f is

A Continuous if $a = 5$ and $b = 5$

B Continuous if $a = -5$ and $b = 10$

C Continuous if $a = 0$ and $b = 5$

$\boxed{\text{D}}$ Not continuous for any values of a and b

Solution

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a + bx, & \text{if} \quad 1 < x < 3 \\ b + 5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

$f(1) = 5,\ f(1^-) = 5,\ f(1^+) = a + b$

$f(3^-) = a + 3b,\ f(3) = b + 15,\ f(3^+) = b + 15$

$f(5^-) = b + 25 : f(5) = 30\ f(5^+) = 30$

From above we concluded that f is not continuous for any values of a and b.