IIT-JEE-Mathematics-Paper1-2007

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1. Let α, β be the roots of the equation x^2 - px + r = 0 and α/2, 2β be the roots of the equation x^2 - qx + r = 0. Then the value of r is

(A) 2/9 (p - q)(2q - p)
(B) 2/9 (q – p)(2p – q)
(C) 2/9 (q – 2p)(2q – p)
(D) 2/9 (2p - q)(2q - p)

2. Let f(x) be differentiable on the interval (0, ∞) such that f(1) = 1, and

$$\lim_{t\to\infty}(t^2 f(x)-x^2 f(t))/(t-x) - 1$$

for each x > 0. Then f(x) is

(A) $1/3x + (2x^2)/3$
(B) $(-1)/3x + (4x^2)/3$
(C) $(-1)/x + 2/x^2$
(D) 1/x

3. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to this wife is

(A) 1/2
(B) 1/3
(C) 2/5
(D) 1/5

4. The tangent to the curve y = ex drawn at the point (e, e^e) intersects the line joining the points $(e - 1, e^{e-1})$ and $(e+1, e^{e+1})$

(A) on the left of x = e
(B) on the right of x = e
(C) at no points
(D) at all points

5. $\lim_{x\to\pi/4} \int_2^{\sec^2 x} f(t)\,dt \,/\, (x^2 - \pi^2/16)$ equals

(A) 8/π f(2)
(B) 2/π f(2)

(C) 2/π f(1/2)
(D) 4f(2)

6. A hyperbola, having the transverse axis of length 2 sin, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

(A) x^2 cosex^2 θ - y^2 sec^2 θ = 1
(B) x^2 sec^2 θ - y^2 cosec^2 θ = 1
(C) x^2 sin^2 θ - y^2 cos^2 θ = 1
(D) x^2 cos^2 θ - y^2 sin^2 θ = 1

7. The number of distinct real values of λ, for which the vectors -λ2î + ĵ + k̂, î - λ2ĵ + k̂ and î + ĵ - λ2k̂ are coplanar, is

(A) zero
(B) one
(C) two
(D) three

8. A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4} + 4i$
(B) $(3 - 4i)e^{i\pi/4}$
(C) $(4 + 3i)e^{i\pi/4}$
(D) $(3 + 4i)e^{i\pi/4}$

9. The number of solutions of the pair of equations

$$2\sin^2 q - \cos 2q = 0$$

$$2\cos^2 q - 3 \sin q - 0$$

in the interval [0, 2p] is

(A) zero

(B) one

(C) two

(D) four

10. Let H_1, H_2, ..., H_n be mutually exclusive and exhaustive events with $P(H_i) > 0$, i = 1, 2, ..., n. Let E be any other event with 0 < P (E) < 1.

STATEMENT-1

P(Hi|E) > P(E|Hi) • P(Hi) for i = 1, 2, ... n.

Because

STATEMENT-2

$\sum^{n}_{i=1} P(H_i) = 1$.

(A) Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

11. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

STATEMENT-1

The tangents are mutually perpendicular

Because

STATEMENT-2

The locus of the points form which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

12. Let the vectors PQ, QR, RS, ST, TU and UP represent the sides of a regular hexagon.

STATEMENT-1

$PQ^{->} \times (RS^{->} + ST^{->}) \neq 0^{->}$.

Because

STATEMENT-2

$PQ^{->} \times RS^{->} = 0^{->}$ and $PQ^{->} \times ST^{->} \neq 0^{->}$.

(A) Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

13. Let F(x) be an indefinite integral of $\sin^2 x$.

STATEMENT-1

The function F(x) satisfies F(x + p) - F(x) for all real x.

STATEMTN-2

$\sin^2(x + p) = \sin^2 x$ for all real x.

(A) Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Paragraph

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r - 1). Let

$$T_r = V_{r+1} - V_r - 2 \text{ and } Q_r = T_{r+1} - T_r \text{ for } r = 1, 2, \ldots\ldots$$

14. The sum of $V_1 + V_2 + \ldots\ldots + V_n$ is

(A) $1/12\ n(n + 1)(3n^2 - n + 1)$

(B) $1/12\ n(n + 1)(3n^2 + n + 1)$

(C) $1/12\ n(2n^2 - n + 1)$

(D) $1/12\ n(2n^2 - 2n + 3)$

15. T_r is always

(A) an odd number

(B) an even number

(C) a prime number

(D) a composite number

16. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 6

(C) $Q_1, Q_2, Q_3, \ldots\ldots$ are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots\ldots$

Paragraph

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

17. The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$

(B) $1 : 2$

(C) $1 : 4$

(D) $1 : 8$

18. The radius of the circumcircle of the triangle PRS is

(A) 5

(B) $3\sqrt{3}$

(C) $3\sqrt{2}$

(D) $2\sqrt{3}$

19. The radius of the incircle of the triangle PQR is

(A) 4

(B) 3

(C) 8/3

(D) 2

20. Consider the following linear equations

$ax + by + cz = 0$

$bx + cy + az = 0$

$cx + ay + bz = 0$

Match the conditions/expressions in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I		Column II	
(A)	a + b + c ¹ 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(p)	the equations represent planes meeting only at a single point.
(B)	a + b + c = 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(q)	the equations represent the line x = y = z.
(C)	a + b + c ¹ 0 and $a^2 + b^2 + c^2$ ¹ ab + bc + ca	(r)	the equations represent identical planes.
(D)	a + b + c = 0 and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of the three dimensional space.

21. In the following [x] denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties in Column II.

Column I		Column II	
(A)	x|x|	(p)	continuous in (-1,1)
(B)	√|x|	(q)	differentiable in (-1,1)
(C)	x + |x|	(r)	strictly increasing in (-1,1)
(D)	|x - 1| + |x + 1|	(s)	not differentiable at least at one point in (-1, 1)

22. Match the integrals in Column I with the values in Column II.

Column I		Column II	
(A)	$\int_{-1}^{1} dx/(1+x^2)$	(p)	1/2 log(2/3)
(B)	$\int_{0}^{1} dx/\sqrt{(1-x^2)}$	(q)	2log(2/3)

(C)	$\int_2^3 dx/(1-x^2)$	(r)	$\pi/3$
(D)	$\int_1^2 dx/(x\sqrt{(x^2-1)})$	(s)	$-\pi/2$