JEE ADVANCED (Paper - 2)

Code - 8

MATHEMATICS

SECTION – 1 : (Only One Option Correct Type)

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE option is correct.

41. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

*42. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is

(A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$

(C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$

*43. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265

(C) 53 (D) 67

*44. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

(A) 3 (B) 6

(C) 9 (D) 15

*45. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

(A) only purely imaginary roots (B) all real roots

(C) two real and two purely imaginary roots (D) neither real nor purely imaginary roots

46. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\, dx$ is equal to

(A) $\int_0^{\log(1+\sqrt{2})} 2\left(e^u + e^{-u}\right)^{16} du$ (B) $\int_0^{\log(1+\sqrt{2})} \left(e^u + e^{-u}\right)^{17} du$

(C) $\int_0^{\log(1+\sqrt{2})} \left(e^u - e^{-u}\right)^{17} du$ (D) $\int_0^{\log(1+\sqrt{2})} 2\left(e^u - e^{-u}\right)^{16} du$

47. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$ in (–1, 1) satisfying $f(0) = 0$. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)\,dx$ is

(A) $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

48. Let $f : [0, 2] \rightarrow R$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with $f(0) = 1$. Let $F(x)=\int_0^{x^2} f(\sqrt{t})\,dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then $F(2)$ equals

(A) $e^2 - 1$ (B) $e^4 - 1$

(C) $e - 1$ (D) e^4

*49. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is

(A) 1051 (B) 1106

(C) 1113 (D) 1120

*50. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

(A) infinitely many solutions (B) three solutions

(C) one solution (D) no solution

SECTION – 2 : Comprehension Type (Only One Option Correct)

This section contains 3 paragraph, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (A), (B), (C) and (D).

Paragraph For Questions 51 and 52

Box 1 contains three cards bearing numbers 1, 2, 3 ; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 ; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.

51. The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$

(C) $\frac{57}{105}$ (D) $\frac{1}{2}$

52. The probability that x_1 , x_2 , x_3 are in an arithmetic progression, is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$

(C) $\frac{11}{105}$ (D) $\frac{7}{105}$

Paragraph For Questions 53 and 54

Let a, r, s, t be non-zero real numbers. Let $P(at^2, 2at)$, Q, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point $(2a, 0)$.

*53. The value of r is

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$

(C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

*54. If $st = 1$, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $\frac{\left(t^2+1\right)^2}{2t^3}$ (B) $\frac{a\left(t^2+1\right)^2}{2t^3}$

(C) $\frac{a\left(t^2+1\right)^2}{t^3}$ (D) $\frac{a\left(t^2+2\right)^2}{t^3}$

Paragraph For Questions 55 and 56

Given that for each $a \in (0, 1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}\left(1-t\right)^{a-1} dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$.

55. The value of $g\left(\frac{1}{2}\right)$ is

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

56. The value of $g'\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$ (B) π

(C) $-\frac{\pi}{2}$ (D) 0

SECTION – 3 : Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching list. Choices for the correct combination of elements from List-I and List-II are given as options (A), (B), (C) and (D), out of which ONE is correct.

57. Match the following:

List – I		List – II	
(P)	The number of polynomials $f(x)$ with non-negative integer coefficients of degree $\le$ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, is	(1)	8
(Q)	The number of points in the interval $\left[-\sqrt{13}, \sqrt{13}\right]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	(2)	2
(R)	$\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equals	(3)	4
(S)	$\frac{\left(\int_{-1/2}^{1/2} \cos 2x \cdot \log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_{0}^{1/2} \cos 2x \cdot \log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals	(4)	0

Codes:

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	1	4

58. Match the following:

List – I		List – II	
(P)	Let $y(x) = \cos(3\cos^{-1}x)$, $x \in [-1, 1]$, $x \ne \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2-1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals	(1)	1
(Q)	Let $A_1, A_2, \ldots, A_n (n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots n$. If $\left\|\sum_{k=1}^{n-1}(\vec{a_k} \times \vec{a_{k+1}})\right\| = \left\|\sum_{k=1}^{n-1}(\vec{a_k} \cdot \vec{a_{k+1}})\right\|$, then the minimum value of n is	(2)	2
*(R)	If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is	(3)	8
(S)	Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is	(4)	9

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

59. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : [0, \infty) \to \mathbb{R}$, $f_3 : \mathbb{R} \to \mathbb{R}$ and $f_4 : \mathbb{R} \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & if \quad x < 0 \\ e^x & if \quad x \geq 0 \end{cases}; \; f_2(x) = x^2; \; f_3(x) = \begin{cases} \sin x & if \quad x < 0 \\ x & if \quad x \geq 0 \end{cases} \text{ and } f_4(x) = \begin{cases} f_2(f_1(x)) & if \quad x < 0 \\ f_2(f_1(x)) - 1 & if \quad x \geq 0 \end{cases}$$

List – I		List – II	
(P)	f_4 is	(1)	onto but not one-one
(Q)	f_3 is	(2)	neither continuous nor one-one
(R)	f_2 o f_1 is	(3)	differentiable but not one-one
(S)	f_2 is	(4)	continuous and one-one

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

*60. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \ldots, 9$.

List – I		List – II	
(P)	For each z_k there exists a z_j such $z_k \cdot z_j = 1$	(1)	True
(Q)	There exists a $k \in \{1, 2, \ldots, 9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers	(2)	False
(R)	$\frac{\lvert 1-z_1 \rvert \lvert 1-z_2 \rvert \ldots \lvert 1-z_9 \rvert}{10}$ equals	(3)	1
(S)	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	(4)	2

Codes:

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

ANSWERS

PAPER-2 [Code – 8]
JEE(ADVANCED) 2014

MATHEMATICS

41.	**A**	42.	**B**	43.	**C**	44.	**D**
45.	**D**	46.	**A**	47.	**B**	48.	**B**
49.	**C**	50.	**D**	51.	**B**	52.	**C**
53.	**D**	54.	**B**	55.	**A**	56.	**D**
57.	**D**	58.	**A**	59.	**D**	60.	**C**

HINTS AND SOLUTIONS

MATHEMATICS

41. Either a girl will start the sequence or will be at second position and will not acquire the last position as well.

Required probability = $\frac{\left({}^3C_1 + {}^2C_1\right)}{{}^5C_2} = \frac{1}{2}$.

42. $x = a + b$
$y = ab$
$x^2 - c^2 = y$

$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2} = \cos(120°)$

$\Rightarrow \angle C = \frac{2\pi}{3}$

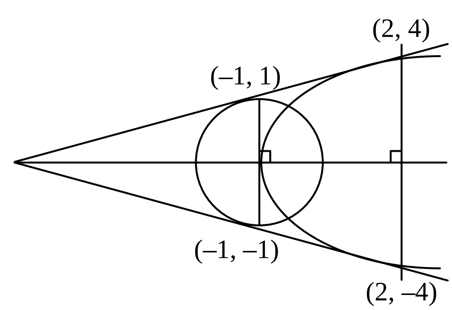
$\Rightarrow R = \frac{abc}{4\Delta}, r = \frac{\Delta}{s}$

$\Rightarrow \frac{r}{R} = \frac{4\Delta^2}{s(abc)} = \frac{4\left[\frac{1}{2}ab\sin\left(\frac{2\pi}{3}\right)\right]^2}{\frac{x+c}{2}\cdot y\cdot c}$

$\frac{r}{R} = \frac{3y}{2c(x+c)}$.

43. Number of required ways = $5! - \left\{4\cdot 4! - {}^4C_2\cdot 3! + {}^4C_3\cdot 2! - 1\right\} = 53$.

44. Area = $\left(\frac{1}{2}(1+4)\,3\right)\times 2 = 15$



45. $P(x) = ax^2 + b$ with a, b of same sign.
$P(P(x)) = a(ax^2 + b)^2 + b$
If $x \in R$ or $ix \in R$
$\Rightarrow x^2 \in R$
$\Rightarrow P(x) \in R$
$\Rightarrow P(P(x)) \neq 0$
Hence real or purely imaginary number can not satisfy $P(P(x)) = 0$.

46. $\int_{\pi/4}^{\pi/2} (2\ \text{cosec}\ x)^{17}\, dx$

Let $e^u + e^{-u} = 2$ cosec x, $x = \frac{\pi}{4} \Rightarrow u = \ln\left(1+\sqrt{2}\right)$, $x = \frac{\pi}{2} \Rightarrow u = 0$

$\Rightarrow$ cosec x + cot x = e^u and cosec x – cot x = $e^{-u} \Rightarrow \cot x = \frac{e^u - e^{-u}}{2}$

$(e^u - e^{-u})\,dx = -2 \operatorname{cosec} x \cot x\, dx$

$$\Rightarrow -\int \left(e^u + e^{-u}\right)^{17} \frac{\left(e^u - e^{-u}\right)}{2 \operatorname{cosec} x \cot x}\, du$$

$$= -2 \int_{\ln\left(1+\sqrt{2}\right)}^{0} \left(e^u + e^{-u}\right)^{16} du$$

$$= \int_{0}^{\ln\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{16} du$$

47. $\frac{dy}{dx} + \frac{x}{x^2 - 1} y = \frac{x^4 + 2x}{\sqrt{1-x^2}}$

This is a linear differential equation

I.F. = $e^{\int \frac{x}{x^2-1} dx} = e^{\frac{1}{2}\ln\left|x^2 - 1\right|} = \sqrt{1-x^2}$

$\Rightarrow$ solution is

$$y\sqrt{1-x^2} = \int \frac{x\left(x^3+2\right)}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\, dx$$

or $y\sqrt{1-x^2} = \int \left(x^4 + 2x\right) dx = \frac{x^5}{5} + x^2 + c$

f (0) = 0 $\Rightarrow$ c = 0

$\Rightarrow f(x)\sqrt{1-x^2} = \frac{x^5}{5} + x^2$

Now, $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)\,dx = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}\, dx$ (Using property)

$= 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}\, dx = 2\int_0^{\pi/3} \frac{\sin^2\theta}{\cos\theta} \cos\theta\, d\theta$ (Taking x = sin θ)

$= 2\int_0^{\pi/3} \sin^2\theta\, d\theta = 2\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/3} = 2\left(\frac{\pi}{6}\right) - 2\left(\frac{\sqrt{3}}{8}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$.

48. F(0) = 0

$F'(x) = 2x\, f(x) = f'(x)$

$f(x) = e^{x^2 + c}$

$f(x) = e^{x^2}$ ($\because$ f(0) = 1)

$F(x) = \int_0^{x^2} e^x\, dx$

$F(x) = e^{x^2} - 1$ ($\because$ F(0) = 0)

$\Rightarrow F(2) = e^4 - 1$

49. $2x_1 + 3x_2 + 4x_3 = 11$

Possibilities are (0, 1, 2); (1, 3, 0); (2, 1, 1); (4, 1, 0).
$\therefore$ Required coefficients
$= ({}^4C_0 \times {}^7C_1 \times {}^{12}C_2) + ({}^4C_1 \times {}^7C_3 \times {}^{12}C_0) + ({}^4C_2 \times {}^7C_1 \times {}^{12}C_1) + ({}^4C_4 \times {}^7C_1 \times 1)$
$= (1 \times 7 \times 66) + (4 \times 35 \times 1) + (6 \times 7 \times 12) + (1 \times 7)$
$= 462 + 140 + 504 + 7 = 1113.$

50. $\sin x + 2\sin 2x - \sin 3x = 3$
$\sin x + 4\sin x\cos x - 3\sin x + 4\sin^3 x = 3$
$\sin x\,[-2 + 4\cos x + 4(1 - \cos^2 x)] = 3$
$\sin x\,[2 - (4\cos^2 x - 4\cos x + 1) + 1] = 3$
$\sin x\,[3 - (2\cos x - 1)^2] = 3$
$\Rightarrow \sin x = 1$ and $2\cos x - 1 = 0$
$\Rightarrow x = \dfrac{\pi}{2}$ and $x = \dfrac{\pi}{3}$
which is not possible at same time
Hence, no solution

51. Case I : One odd, 2 even
Total number of ways $= 2 \times 2 \times 3 + 1 \times 3 \times 3 + 1 \times 2 \times 4 = 29$.
Case II: All 3 odd
Number of ways $= 2 \times 3 \times 4 = 24$
Favourable ways = 53
Required probability $= \dfrac{53}{3\times5\times7} = \dfrac{53}{105}$.

52. Here $2x_2 = x_1 + x_3$
$\Rightarrow x_1 + x_3 =$ even
Hence number of favourable ways $= {}^2C_1 \cdot {}^4C_2 + {}^1C_1 \cdot {}^3C_1 = 11$.

53. Slope (QR) = Slope (PK)

$$\frac{2at - 0}{at^2 - 2a} = \frac{-\frac{2a}{t} - 2ar}{\frac{a}{t^2} - ar^2}$$

$$\Rightarrow \frac{t}{t^2 - 2} = -\left(\frac{\frac{1}{t} + r}{\frac{1}{t^2} - r^2}\right) \Rightarrow r = \frac{t^2 - 1}{t}$$

54. Tangent at P: $ty = x + at^2$ or $y = \dfrac{x}{t} + at$

Normal at S: $y + \dfrac{x}{t} = \dfrac{2a}{t} + \dfrac{a}{t^3}$

Solving, $2y = at + \dfrac{2a}{t} + \dfrac{a}{t^3}$

$$y = \frac{a\left(t^2 + 1\right)^2}{2t^3}$$

55. $$g\left(\frac{1}{2}\right) = \lim_{h\to0^+} \int_h^{1-h} t^{-1/2}\left(1 - t\right)^{-1/2} dt$$

$$= \int_0^1 \frac{dt}{\sqrt{t-t^2}} = \int_0^1 \frac{dt}{\sqrt{\frac{1}{4}-\left(t-\frac{1}{2}\right)^2}} = \sin^{-1}\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right)\Bigg|_0^1$$

$= \sin^{-1} 1 - \sin^{-1}(-1) = \pi.$

56. We have g (a) = g (1 – a) and g is differentiable

Hence $g'\left(\frac{1}{2}\right) = 0.$

57. (P) $f(x) = ax^2 + bx, \int_0^1 f(x)\,dx = 1$

$\Rightarrow 2a + 3b = 6$

$\Rightarrow (a, b) \equiv (0, 2)$ and $(3, 0)$.

(Q) $f(x) = \sqrt{2}\cos\left(x^2 - \frac{\pi}{4}\right)$

For maximum value, $x^2 - \frac{\pi}{4} = 2n\pi$

$\Rightarrow x^2 = 2n\pi + \frac{\pi}{4}$

$\Rightarrow x = \pm\sqrt{\frac{\pi}{4}}, \pm\sqrt{\frac{9\pi}{4}}$ as $x \in \left[-\sqrt{3}, \sqrt{13}\right]$.

(R) $\int_0^2 \left(\frac{3x^2}{1+e^x} + \frac{3x^2}{1+e^{-x}}\right)dx = \int_0^2 3x^2\,dx = 8$.

(S) $\int_{-1/2}^{1/2} \cos 2x \ln\left(\frac{1+x}{1-x}\right)dx = 0$ as it is an odd function.

58. (P) $y = \cos(3\cos^{-1} x)$

$$y' = \frac{3\sin\left(3\cos^{-1}x\right)}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,y' = 3\sin\left(3\cos^{-1}x\right)$$

$$\Rightarrow \frac{-x}{\sqrt{1-x^2}}y' + \sqrt{1-x^2}\,y'' = 3\cos\left(3\cos^{-1}x\right).\frac{-3}{\sqrt{1-x^2}}$$

$\Rightarrow -xy' + (1-x^2)y'' = -9y$

$$\Rightarrow \frac{1}{y}\left[\left(x^2-1\right)y'' + xy'\right] = 9.$$

(Q) $(a_k \times a_{k+1}) = r^2 \sin\frac{2\pi}{n}$

$a_k \cdot a_{k+1} = r^2 \cos\frac{2\pi}{n}$

$$\Rightarrow \left|\sum_{k=1}^{n-1} \vec{a}_k \times \vec{a}_{k+1}\right| = \left|\sum_{k=1}^{n-1} a_k \cdot a_{k+1}\right|$$

$$\Rightarrow r^2(n-1)\sin\frac{2\pi}{n} = r^2(n-1)\cos\frac{2\pi}{n}$$

$\tan\frac{2\pi}{n} = 1 \Rightarrow n = \frac{8}{4k+1}$

$\Rightarrow n = 8.$

(R) $\frac{h^2}{6} + \frac{1^2}{3} = 1$, $h = \pm 2$

Tangent at (2, 1) is $\frac{2x}{6} + \frac{y}{3} = 1$ x + y = 3.

(S) $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\frac{1}{4x+1} = \tan^{-1}\frac{2}{x^2}$

$\tan^{-1}\left(\frac{3x+1}{4x^2+3x}\right) = \tan^{-1}\frac{2}{x^2}$

$\Rightarrow 3x^2 - 7x - 6 = 0$

$x = -\frac{2}{3},\ 3.$

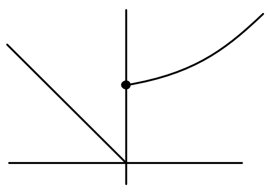
59. $f_2(f_1) = \begin{cases} x^2 & , \quad x < 0 \\ e^{2x} & , \quad x \ge 0 \end{cases}$

$f_4 : R \to [0, \infty)$

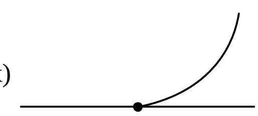
$f_4(x) = \begin{cases} f_2(f_1(x)) & , \quad x < 0 \\ f_2(f_1(x)) - 1 & , \quad x \ge 0 \end{cases}$

$= \begin{cases} x^2 & , \quad x < 0 \\ e^{2x} - 1 & , \quad x \ge 0 \end{cases}$

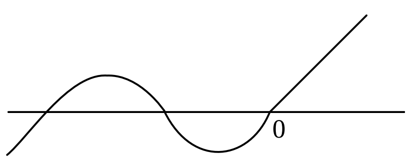
y = $f_1(x)$



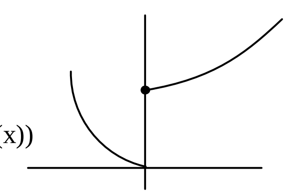
y = $f_2(x)$



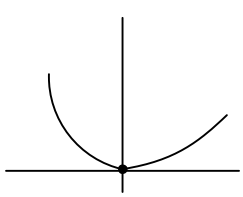
y = $f_3(x)$



y = $f_2(f_1(x))$



y = $f_4(x)$



60. (P) z_k is 10^{th} root of unity $\Rightarrow \bar{z}_k$ will also be 10^{th} root of unity. Take z_j as $\bar{z}_k$.

(Q) $z_1 \neq 0$ take $z = \frac{z_k}{z_1}$, we can always find z.

(R) $z^{10} - 1 = (z-1)(z-z_1)\ldots(z-z_9)$

$\Rightarrow (z-z_1)(z-z_2)\ldots(z-z_9) = 1 + z + z^2 + \ldots + z^9\ \forall\ z \in$ complex number.

Put z = 1

$(1 - z_1)(1 - z_2) \ldots (1 - z_9) = 10.$

(S) $1 + z_1 + z_2 + \ldots + z_9 = 0$

$\Rightarrow \text{Re}(1) + \text{Re}(z_1) + \ldots + \text{Re}(z_9) = 0$

$\Rightarrow \text{Re}(z_1) + \text{Re}(z_2) + \ldots + \text{Re}(z_9) = -1.$

$$\Rightarrow 1 - \sum_{k=1}^{9} \cos\frac{2k\pi}{10} = 2\,.$$