IIT - JEE ADVANCED - 2012

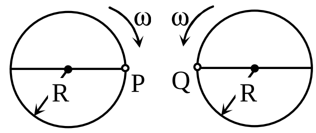
PAPER-2 [Code – 8]

PART - I: PHYSICS

SECTION I : Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

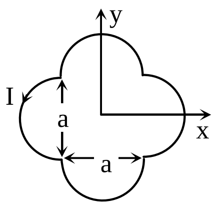
1. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



Sol. **(A)**

In each rotation relative speed becomes zero twice and becomes maximum twice.

2. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

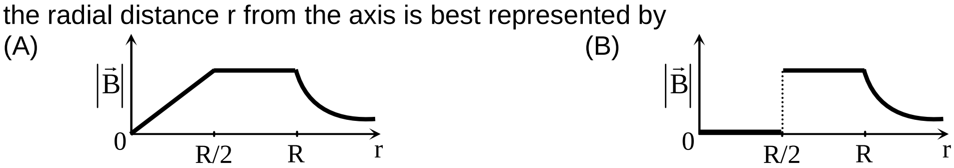


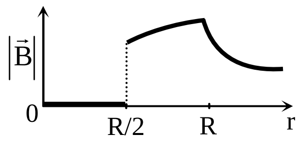
(A) $a^2 I\,\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\,\hat{k}$ (D) $(2\pi+1)a^2 I\,\hat{k}$

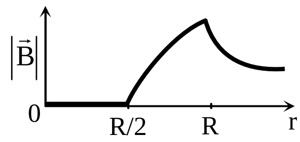
Sol. **(B)**

Magnetic moment, $\vec{M} = I\vec{A} = I\left(\frac{\pi}{2}+1\right)a^2\hat{K}$

3. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by



(C) 

(D) 

Sol. **(D)**

Inside the cavity, B = 0

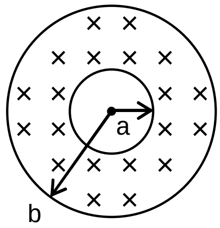
Outside the cylinder, $B = \frac{\mu_0 I}{2\pi r}$

In the shaded region

$$B = \frac{\mu_0 I}{2\pi r(b^2 - a^2)}\left(r - \frac{a^2}{r}\right)$$

at r = a, B = 0

at r = b, $B = \frac{\mu_0 I}{2\pi b}$



4. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half-filled if ρ_C is less than 0.5.
(B) more than half-filled if ρ_C is more than 1.0.
(C) half-filled if ρ_C is more than 0.5.
(D) less than half-filled if ρ_C is less than 0.5.

Sol. **(A)**

$$\frac{V_m + V_a + V_w}{2}\rho_w g = V_m \rho_c \rho_w g + V_w \rho_w g$$

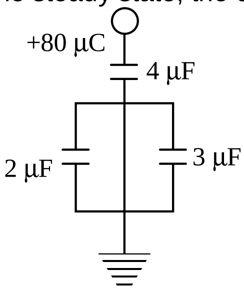
$$V_w = V_m(1 - 2\rho_C) + V_a$$

if $\rho_C > \frac{1}{2} \Rightarrow V_w < V_a$

if $\rho_C < \frac{1}{2} \Rightarrow V_w > V_a$,

where, V_w = volume occupied by water in the shell
V_a = volume occupied by air in the shell
V_m = volume of the material in the shell

5. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(A) +32 μC
(B) +40 μC
(C) +48 μC
(D) +80 μC

Sol. **(C)**

Let 'q' be the final charge on 3μ F capacitor then

$$\frac{80 - q}{2} = \frac{q}{3} \Rightarrow q = 48\mu C$$

6. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the

balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

(A) 62 J (B) 104 J (C) 124 J (D) 208 J

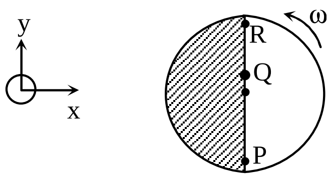
Sol. **(D)**

$\Delta Q = nC_p\Delta T$ (Isobaric process)

$= 2\times\frac{5}{2}R\times(35-30)$

$= 208\,J$

7. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

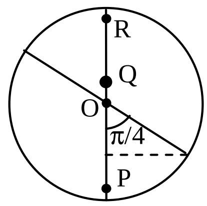
Sol. **(C)**

At t = $\frac{1}{8}\times\frac{2\pi}{\omega} = \frac{\pi}{4\omega}$

x –coordinate of P = $\omega R\left(\frac{\pi}{4\omega}\right)$

= $\frac{\pi R}{4} > R\cos 45°$

$\therefore$ Both particles P and Q land in unshaded region.



8. A student is performing the experiment of resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

Sol. **(B)**

L + e = $\frac{\lambda}{4}$

$\Rightarrow$ L = $\frac{\lambda}{4} - e$

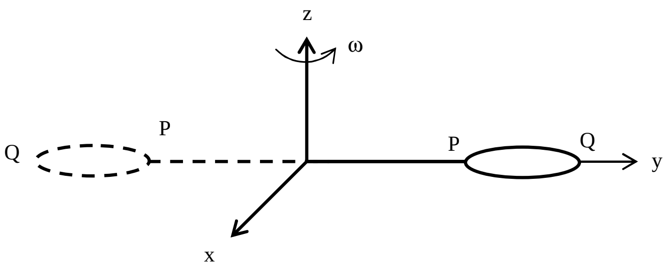
= 16.4 – 1.2 = 15.2 cm

SECTION II : Paragraph Type

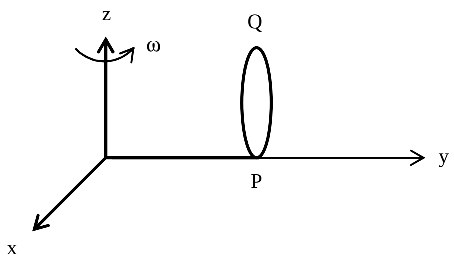
This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Paragraph for Questions 9 and 10

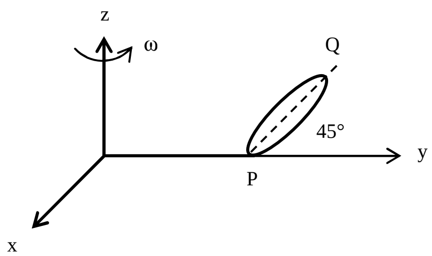
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case



Now consider two similar systems as shown in the figure: Case(a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

9. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
   (A) It is vertical for both the cases (a) and (b)
   (B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).
   (C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).
   (D) It is vertical for case (a); and is 45° to the x-z plane and is normal to the plane of the disc for case (b).

Sol. **(A)**

10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?
   (A) It is $\sqrt{2}\omega$ for both the cases.
   (B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).
   (C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).
   (D) It is ω for both the cases.

Sol. **(D)**

Paragraph for Questions 11 and 12

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

11. If the anti-neutrino had a mass of $3 eV/c^2$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?
(A) $0 \le K \le 0.8 \times 10^6 \, eV$
(B) $3.0 \, eV \le K \le 0.8 \times 10^6 \, eV$
(C) $3.0 \, eV \le K < 0.8 \times 10^6 \, eV$
(D) $0 \le K < 0.8 \times 10^6 \, eV$

Sol. **(D)**

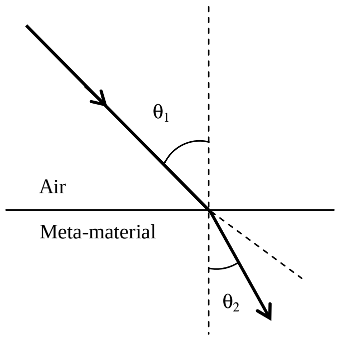
12. What is the maximum energy of the anti-neutrino?
(A) Zero
(B) Much less than $0.8 \times 10^6 \, eV$.
(C) Nearly $0.8 \times 10^6 \, eV$
(D) Much larger than $0.8 \times 10^6 \, eV$

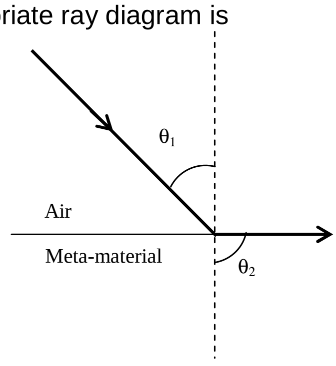
Sol. (C)

Paragraph for Questions 13 and 14

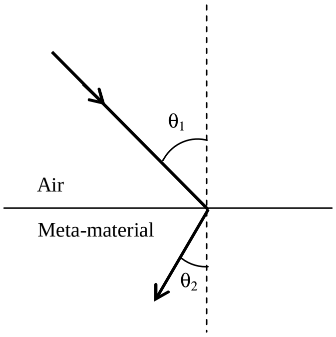
Most materials have the refractive index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r, are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

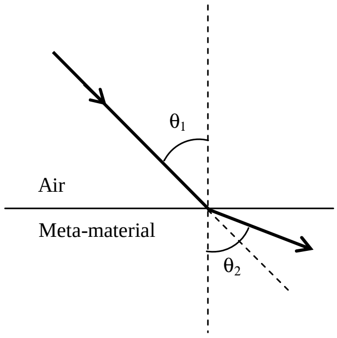
13. For light incident from air on a meta-material, the appropriate ray diagram is
(A)


(B)


(C)



(D)



Sol. **(C)**

14. Choose the correct statement.

(A) The speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is wavelength of the light in air.

Sol. **(B)**

SECTION III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

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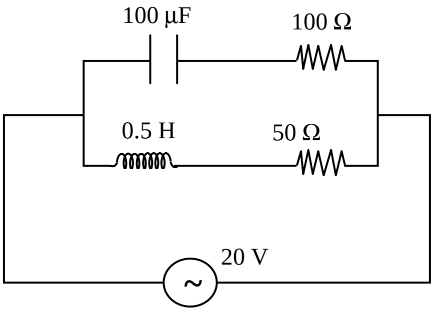
15. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are)



(A) The current through the circuit, I is 0.3 A.

(B) The current through the circuit, I is $0.3\sqrt{2}$ A.

(C) The voltage across 100 Ω resistor = $10\sqrt{2}$ V.

(D) The voltage across 50 Ω resistor = 10 V.

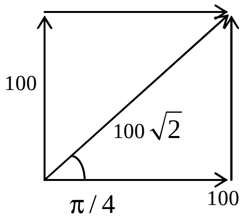
Sol. **(A, C)**

$I_{upper} = \frac{20}{100\sqrt{2}}$; $+\frac{\pi}{4}$ ahead of voltage

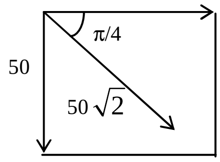
$I_{lower} = \frac{20}{50\sqrt{2}}$; $-\frac{\pi}{4}$ behind voltage

$I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{10}} \approx 0.3A$

$V_{100\,\Omega} = \frac{20}{100\sqrt{2}} \times 100 = 10\sqrt{2}$.

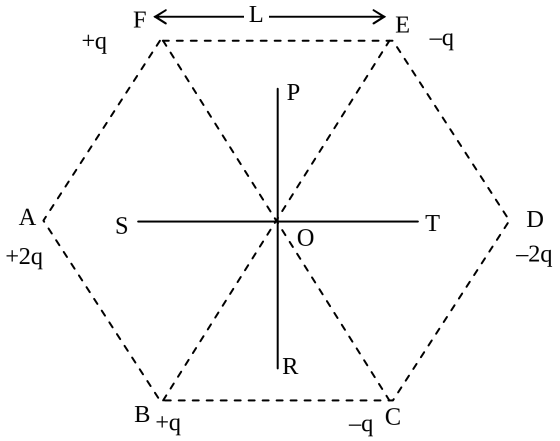


Capacitive circuit



Inductive circuit

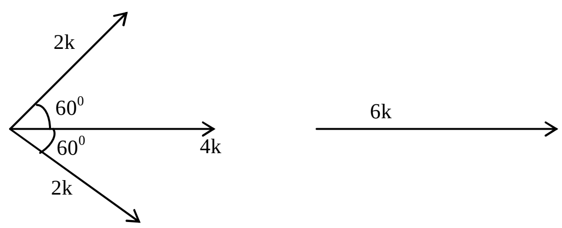
16. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct?



(A) The electric field at O is 6K along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

Sol. **(A, B, C)**
Line PR is perpendicular bisector of all the dipoles.

At point O: $\frac{1}{4\pi\varepsilon_0}\Sigma\frac{Q_i}{r_i} = 0$



17. Two spherical planets P and Q have the same uniform density ρ, masses M_p and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then

(A) $V_Q > V_R > V_P$
(B) $V_R > V_Q > V_P$
(C) $V_R / V_P = 3$
(D) $V_P / V_Q = \frac{1}{2}$

Sol. **(B, D)**
By calculation, if Mass of P = m
and Radius of P = R
Then Mass of Q = 8M
and radius of Q = 2R
and Mass of R = 9M
and radius of R = $9^{1/3}$R

$$V_P\sqrt{\frac{2GM}{R}}$$

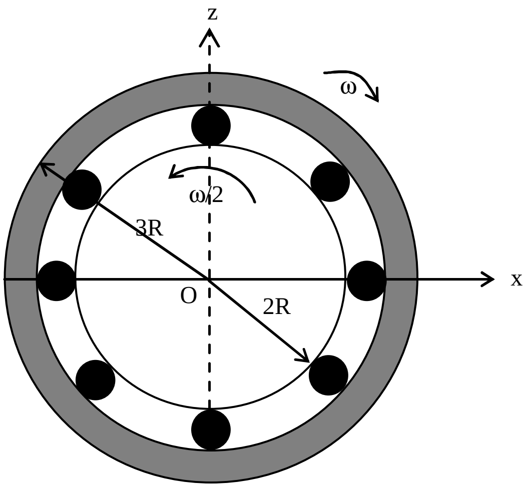
$$V_Q = \sqrt{\frac{2G8M}{2R}} = 2V_p$$

$$V_R = \sqrt{\frac{2G8M}{9^{1/3}R}} = 9^{1/3}V_p$$

$$\therefore \quad V_R > V_Q = V_p$$

$$\frac{V_Q}{V_P} = 2$$

18. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30^0 with the horizontal. Then with respect to the horizontal surface,



(A) the point O has linear velocity $3R\omega\hat{i}$

(B) the point P has linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$.

(C) the point P has linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

Sol. **(A, B)**

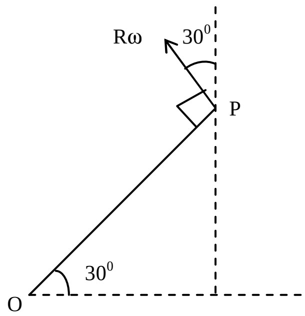
$\vec{V}_0(3R)\omega\hat{i} = 0$

$\therefore\ \vec{v}_0 = 3R\omega\hat{i}$

$\vec{v}_{P,O} = \frac{-R\omega}{4}\hat{i} + \frac{R\omega\sqrt{3}}{4}\hat{j}$

$\therefore\ \vec{v}_P = \vec{v}_{P,O} + \vec{v}_O$

$= \frac{11}{4}R\omega\hat{i} + R\omega\frac{\sqrt{3}}{4}\hat{j}$



19. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?
(A) Both cylinders P and Q reach the ground at the same time.
(B) Cylinders P has larger linear acceleration than cylinder Q.
(C) Both cylinders reach the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

Sol. **(D)**

a = $\frac{Mg\sin\theta}{M + \frac{I}{R^2}}$

$a_P = \dfrac{Mg\sin\theta}{M + \dfrac{MR^2}{R^2}} \approx \dfrac{g}{2}$

$a_Q = g\sin\theta$ as $I_Q \sim 0$

$\therefore\ \omega_P = \dfrac{\sqrt{2.\frac{g}{2}.\ell}}{R}$

$\omega_Q = \dfrac{\sqrt{2.g.\ell}}{R}$

$\therefore\ \omega_Q > \omega_P$

20. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement(s) is(are)
(A) The emf induced in the loop is zero if the current is constant.
(B) The emf induced in the loop is finite if the current is constant.
(C) The emf induced in the loop is zero if the current decreases at a steady rate.
(D) The emf induced in the loop is infinite if the current decreases at a steady rate.

Sol. **(A, C)**

ϕ = zero

$\therefore\ \dfrac{d\phi}{dt} = \text{zero}$

$\therefore$ A, C are correct.