JEE ADVANCED - 2013

Paper - 2

PHYSICS

SECTION – 1 (One or more options correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

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*1. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant.

Sol. (**B**)

$$\frac{-2GMm}{L}+\frac{1}{2}mv^2=0$$

$$\Rightarrow v=2\sqrt{\frac{GM}{L}}$$

Note: *The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each) – which is the potential energy of the system.*

*2. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0. It collides elastically with a rigid wall. After this collision,

(A) the speed of the particle when it returns to its equilibrium position is u_0.

(B) the time at which the particle passes through the equilibrium position for the first time is $t=\pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t=\frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes through the equilibrium position for the second time is $t=\frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

Sol. **(A, D)**

$v=u_0\sin\omega t$ (suppose t_1 is the time of collision) $\frac{u_0}{2}=u_0\cos\omega t_1 \Rightarrow t_1=\frac{\pi}{3\omega}$

Now the particle returns to equilibrium position at time $t_2 = 2t_1$ i.e. $\frac{2\pi}{3\omega}$ with the same mechanical energy i.e. its speed will u_0.

Let t_3 is the time at which the particle passes through the equilibrium position for the second time.

$\therefore\ t_3 = \frac{T}{2} + 2t_1$

$= \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega}$

$= \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

Energy of particle and spring remains conserved.

3. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)

(A) In the region $0 < r < R$, the magnetic field is non-zero

(B) In the region $R < r < 2R$, the magnetic field is along the common axis.

(C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis.

(D) In the region $r > 2R$, the magnetic field is non-zero.

Sol. **(A, D)**

Due to field of solenoid is non zero in region $0 < r < R$ and non zero in region $r>2R$ due to conductor.

*4. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

(A) If the wind blows from the observer to the source, $f_2 > f_1$.

(B) If the wind blows from the source to the observer, $f_2 > f_1$.

(C) If the wind blows from observer to the source, $f_2 < f_1$.

(D) If the wind blows from the source to the observer $f_2 < f_1$.

Sol. **(A, B)**

If wind blows from source to observer

$f_2 = f_1\left(\frac{V + w + u}{V + w - u}\right)$

When wind blows from observer towards source

$f_2 = f_1\left(\frac{V - w + u}{V - w - u}\right)$

In both cases, $f_2 > f_1$.

*5. Using the expression $2d \sin\theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range θ to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°,

(A) the absolute error in d remains constant.

(B) the absolute error in d increases

(C) the fractional error in d remains constant.

(D) the fractional error in d decreases.

Sol. **(D)**

$d = \frac{\lambda}{2\sin\theta}$

$\ln d = \ln\left(\frac{\lambda}{2}\right) - \ln \sin\theta$

$$\frac{\Delta d}{d} = 0 - \frac{\cos\theta d\theta}{\sin\theta}$$

$$\left(\frac{\Delta d}{d}\right)_{max} = \pm\cot\theta\Delta\theta$$

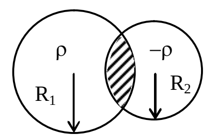
Also $(\Delta d)_{max} = d\cot\theta\Delta\theta$

$$\frac{\lambda}{2\sin\theta}\cot\theta\Delta\theta$$

$$= \frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta$$

As θ increases $\cot\theta$ decreases and $\frac{\cos\theta}{\sin^2\theta}$ also decreases.

6. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



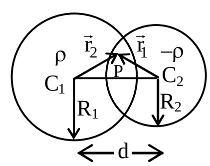
(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction

Sol. **(C, D)**

In triangle PC_1C_2

$\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is



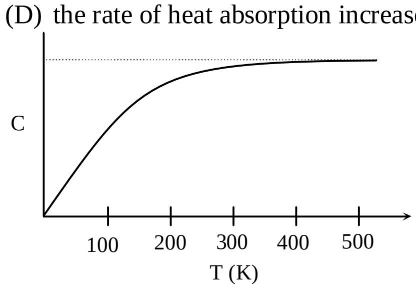
$$\vec{E} = \frac{K\left(\rho\frac{4}{3}\pi R_1^3\right)\vec{r}_2}{R_1^3} + \frac{K\left(\rho\frac{4}{3}\pi R_2^3\right)(-\vec{r}_1)}{R_2^3}$$

$$\vec{E} = K\rho\frac{4}{3}\pi(\vec{r}_2 - \vec{r}_1)$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d}$$

***7.** The figure shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.

(A) the rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.
(B) heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400 – 500 K.
(C) there is no change in the rate of heat absorption in range 400 – 500 K.
(D) the rate of heat absorption increases in the range 200 – 300 K.



Sol. **(A, B, C, D)**

Option (A) is correct because the graph between (0 – 100 K) appears to be a straight line upto a reasonable approximation.

Option (B) is correct because area under the curve in the temperature range (0 – 100 K) is less than in range (400 – 500 K.)

Option (C) is correct because the graph of C versus T is constant in the temperature range (400 – 500 K)

Option (D) is correct because in the temperature range (200 – 300 K) specific heat capacity increases with temperature.

8. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0 where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck's constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$ (C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

Sol. **(A, C)**

Given data

$$4.5a_0 = a_0 \frac{n^2}{Z} \quad \ldots(i)$$

$$\frac{nh}{2\pi} = \frac{3h}{2\pi} \quad \ldots(ii)$$

So n = 3 and z = 2

So possible wavelength are

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{1}{3R}$$

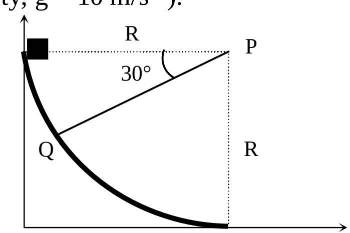
$$\frac{1}{\lambda_3} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_3 = \frac{9}{5R}$$

SECTION – 2 : (Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, date etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of paragraph has **only one correct answer** along the four choice (A), (B), (C) and (D).

Paragraph for Questions 9 to 10

A small block of mass 1 kg is released from rest at the top of a rough track. The track is circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, below, is 150 J. (Take the acceleration due to gravity, g = 10 m/s^{-2}).



*9. The speed of the block when it reaches the point Q is

(A) 5 ms^{-1} (B) 10 ms^{-1} (C) $10\sqrt{3}$ ms^{-1} (D) 20 ms^{-1}

Sol. **(B)**

Using work energy theorem

$$mg\,R\sin 30° + W_f = \frac{1}{2}mv^2$$

$$200 - 150 = \frac{v^2}{2}$$

v = 10 m/s

*10. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

Sol. **(A)**

$$N - mg\cos 60° = \frac{mv^2}{R}$$

$$N = 5 + \frac{5}{2} = 7.5 \text{ Newton.}$$

Paragraph for Questions 11 to 12

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with the power factor unity. All the currents and voltage mentioned are rms values.

11. If the direct transmission method with a cable of resistance 0.4 $\Omega\ km^{-1}$ is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30 (C) 40 (D) 50

Sol. **(B)**

For direct transmission

$$P = i^2R = (150)^2(0.4\times 20) = 1.8\times 10^5 \text{ w}$$

$$\text{fraction(in \%)} = \frac{1.8\times 10^5}{6\times 10^5}\times 100 = 30\%$$

12. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

Sol. **(A)**

$$\frac{40000}{200} = 200$$

Paragraph for Questions 13 to 14

A point Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

13. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change, is

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

Sol. **(B)**

$$E(2\pi R) = \pi R^2 \frac{dB}{dt}$$

$$E = \frac{RB}{2}$$

14. The change in the magnetic dipole moment associated with the orbit, at the end of time interval of the magnetic field change, is

(A) $-\gamma BQR^2$ (B) $-\gamma \frac{BQR^2}{2}$ (C) $\gamma \frac{BQR^2}{2}$ (D) γBQR^2

Sol. **(B)**

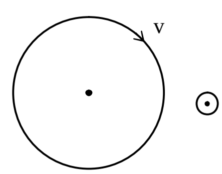
$$\Delta L = \int \tau dt$$

$$= Q\left(\frac{R}{2}B\right)R(1)$$

$$= \frac{QR^2B}{2}, \text{ in magnitude}$$

$$\Delta\mu = \gamma \Delta L$$

$$= -\gamma \frac{BQR^2}{2} \text{ (taking into account the direction)}$$



Paragraph for Questions 15 to 16

The mass of nucleus $^A_Z X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of mass m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M′ only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

^1_1H	1.007825 u	^2_1H	2.014102 u	^3_1H	3.016050 u	^4_2He	4.002603 u
^6_3Li	6.015123 u	^7_3Li	7.016004 u	$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u	$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

15. The correct statement is

(A) The nucleus ${}_3^6\text{Li}$ can emit an alpha particle

(B) The nucleus ${}_{84}^{210}\text{Po}$ can emit a proton.

(C) Deuteron and alpha particle can undergo complete fusion.

(D) The nuclei ${}_{30}^{70}\text{Zn}$ and ${}_{34}^{82}\text{Se}$ can undergo complete fusion.

Sol. **(C)**

$${}_3^6\text{Li} \rightarrow {}_2^4\text{He} + {}_1^2\text{H}$$

$$\frac{Q}{C^2} = 6.015123 - 4.002603 - 2.014102$$

$$0 = -0.001582 < 0$$

So no α-decay is possible

$${}_{84}^{210}\text{P}_0 \rightarrow {}_1^1\text{H} + {}_{83}^{209}\text{Bi}$$

$$\frac{Q}{C^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$$

So, this reaction is not possible

$${}_1^2\text{H} + {}_2^4\text{He} \rightarrow {}_3^6\text{Li}$$

$$\frac{Q}{C^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$$

So, this reaction is possible

$${}_{30}^{70}\text{Zn} + {}_{34}^{82}\text{Se} \rightarrow {}_{64}^{152}\text{Gd}$$

$$\frac{Q}{C^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$$

So this reaction is not possible

16. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}_{84}^{210}\text{Po}$ at rest undergoes alpha decay, is

(A) 5319 (B) 5422 (C) 5707 (D) 5818

Sol. **(A)**

$${}_{84}^{210}\text{Po} \rightarrow {}_2^4\text{He} + {}_{82}^{206}\text{Pb}$$

Q = (209.982876 – 4.002603 – 205.97455)C^2

= 5.422 MeV

from conservation of momentum

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$4K_1 = 206K_2$

$$\therefore K_1 = \frac{103}{2}K_2$$

$K_1 + K_2 = 5.422$

$$K_1 + \frac{2}{103}K_1 = 5.422$$

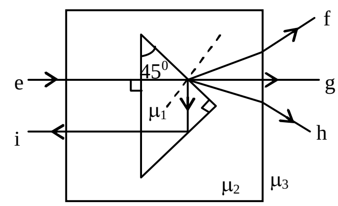
$$\Rightarrow \frac{105}{103}K_1 = 5.422$$

$\therefore$ K_1 = 5.319 MeV = 5319 KeV

SECTION – 3 (Matching List Type)

This section contains **4 multiple choice questions.** Each question has matching lists. The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

17. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh', or 'ei'.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	$e \rightarrow f$	1.	$\mu_1 > \sqrt{2}\,\mu_2$
Q.	$e \rightarrow g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \rightarrow h$	3.	$\mu_1 = \mu_2$
S.	$e \rightarrow i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	Q	R	S
(A)	2	3	1	4
(B)	1	2	4	3
(C)	4	1	2	3
(D)	2	3	4	1

Sol. **(D)**

P. → (2) ; Q. → (3); R. → (4); S. → (1)

P. $\mu_2 > \mu_1 \ldots$ (towards normal)

$\mu_2 > \mu_3 \ldots$ (away from normal)

Q. $\mu_1 = \mu_2 \ldots$ (No change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

R. $\mu_1 > \mu_2 \ldots$ (Away from the normal)

$\mu_2 > \mu_3 \ldots$ (Away from the normal)

$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\mu_2}$. Since $\sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

S. For TIR : $45^0 > C \Rightarrow \sin 45^0 > \sin C \Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$

*18. Match List I with List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	Boltzmann Constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Plank Constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

Sol. **(C)**

P. $\rightarrow$ (4) ; Q. $\rightarrow$ (2); R. $\rightarrow$ (1); S. $\rightarrow$ (3)

P. $KE = \frac{3}{2}K'T \Rightarrow [ML^2T^{-2}] = K'[K] \Rightarrow K' = [ML^2T^{-2}K^{-1}]$

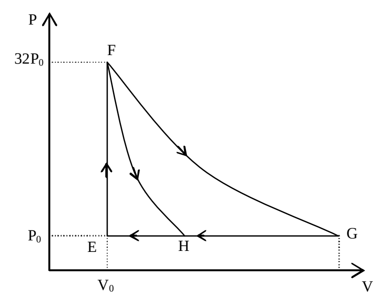
Q. $F = 6\pi\eta rv \Rightarrow [MLT^{-2}] = \eta[L][LT^{-1}] \Rightarrow \eta = [ML^{-1}T^{-1}]$

R. $E = hf \Rightarrow [ML^2T^{-2}] = \frac{h}{[T]} \Rightarrow h = [ML^2T^{-1}]$

S. $\frac{dQ}{dt} = \frac{K'A(\Delta T)}{\Delta x} \Rightarrow \frac{[ML^2T^{-2}]}{[T]} = \frac{k[L^2][K']}{[L]}$

$K' = [MLT^{-3}K^{-1}]$

*19. One mole of mono-atomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	G $\rightarrow$ E	1.	160 P_0V_0 ln2
Q.	G $\rightarrow$ H	2.	36 P_0V_0
R.	F $\rightarrow$ H	3.	24 P_0V_0
S.	F $\rightarrow$ G	4.	31 P_0 V_0

Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

Sol. **(A)**

P. $\rightarrow$ (4) ; Q. $\rightarrow$ (3); R. $\rightarrow$ (2); S. $\rightarrow$ (1)

Apply $PV^{1+2/3}$ = constant for F to H.

$(32P_0)V_0^{5/3} = P_0V_H^{5/3} \Rightarrow V_H = 8V_0$

For path FG PV = constant

$\Rightarrow (32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$

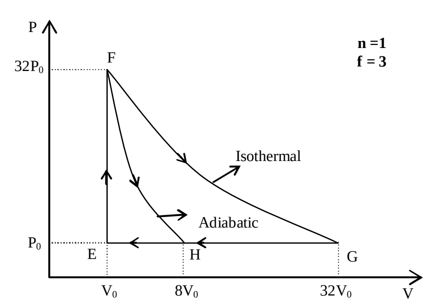
Work done in GE = 31 P_0V_0

Work done in GH = 24 P_0V_0

Work done in FH = $\frac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in FG = $RT \ln\left(\frac{V_G}{V_F}\right)$

$= 160P_0V_0 \ln 2.$



20. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

	List I		List II
P.	Alpha decay	1.	${}^{15}_{8}O \rightarrow {}^{15}_{7}N + ...$
Q.	β+ decay	2.	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + ...$
R.	Fission	3.	${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + ...$
S.	Proton emission	4.	${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

Sol. **(C)**

P. → (2) ; Q. → (1); R. → (4); S. → (3)

${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{1}\beta$ (Beta decay)

${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$ (Alpha decay)

${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + {}^{1}_{1}H$ (Proton emission)

${}^{239}_{94}Ph \rightarrow {}^{140}_{57}La + {}^{99}_{37}Rb$ (fission)