IIT - JEE ADVENCED - 2012

PAPER-1 [Code – 8]

PART - III: MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) a = 1, b = 4 (B) a = 1, b = –4
(C) a = 2, b = – 3 (D) a = 2, b = 3

Sol. **(B)**

Given $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$

$\Rightarrow \lim_{x\to\infty}\frac{x^2+x+1-ax^2-ax-bx-b}{(x+1)}=4 \Rightarrow \lim_{x\to\infty}\frac{(1-a)x^2+(1-a-b)x+(1-b)}{(x+1)}=4$

$\Rightarrow$ 1 – a = 0 and 1 – a – b = 4 $\Rightarrow$ b = –4, a = 1.

42. Let P = [a_{ij}] be a 3 × 3 matrix and let Q = [b_{ij}], where $b_{ij} = 2^{i+j}a_{ij}$ for 1 ≤ i, j ≤ 3. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11}
(C) 2^{12} (D) 2^{13}

Sol. **(D)**

$$|Q|=\begin{vmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{vmatrix} \Rightarrow |Q|=2^2\cdot 2^3\cdot 2^4\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ 2a_{21} & 2a_{22} & 2a_{23}\\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33}\end{vmatrix}$$

$$|Q|=2^9\cdot 2\cdot 2^2\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

$\Rightarrow |Q| = 2^{12}|P|$

$|Q| = 2^{13}$.

43. The locus of the mid–point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2+y^2)-36x+45y=0$ (B) $20(x^2+y^2)+36x-45y=0$
(C) $36(x^2+y^2)-20x+45y=0$ (D) $36(x^2+y^2)+20x-45y=0$

Sol. **(A)**

Equation of the chord bisected at P (h, k)

$hx + ky = h^2 + k^2$ …(i)

Let any point on line be $\left(\alpha,\ \frac{4}{5}\alpha-4\right)$

Equation of the chord of contact is

$\Rightarrow\ \alpha x + \left(\frac{4}{5}\alpha - 4\right) y = 9$ …(ii)

Comparing (i) and (ii)

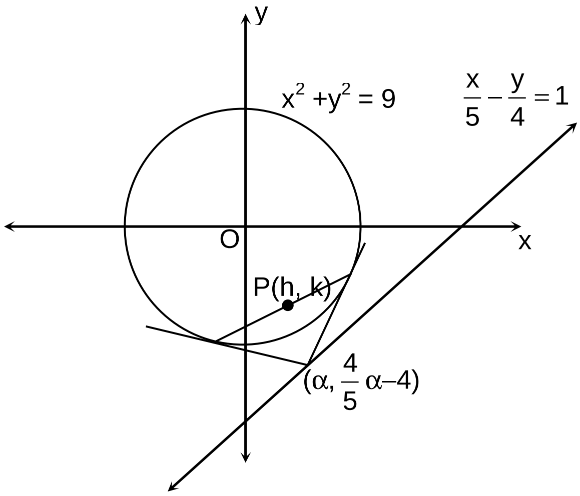
$$\frac{h}{\alpha} = \frac{k}{\frac{4}{5}\alpha - 4} = \frac{h^2 + k^2}{9}$$

$$\alpha = \frac{20h}{4h - 5k}$$

Now, $\frac{h(4h - 5k)}{20h} = \frac{h^2 + k^2}{9}$

$20\ (h^2 + k^2) = 9\ (4h - 5k)$

$20\ (x^2 + y^2) - 36x + 45y = 0.$



44. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75
(B) 150
(C) 210
(D) 243

Sol. **(B)**

Number of ways

$= 3^5 - {}^3C_1.2^5 + {}^3C_2 1^5$

$= 243 - 96 + 3 = 150.$

45. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Sol. **(C)**

$$I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}\, dx$$

Let sec x + tan x = t

$\Rightarrow$ sec x – tan x = 1/t

Now (sec x tan x + $\sec^2$ x) dx = dt

sec x (sec x + tan x) dx = dt

sec x dx = $\frac{dt}{t}$, $\frac{1}{2}\left(t + \frac{1}{t}\right)$ = sec x

$$I = \frac{1}{2}\int \frac{\left(t + \frac{1}{t}\right)}{t^{9/2}} \frac{dt}{t}$$

$$= \frac{1}{2}\int (t^{-9/2} + t^{-13/2}) dt$$

$$= \frac{1}{2}\left[\frac{t^{-9/2+1}}{-\frac{9}{2}+1} + \frac{t^{-13/2+1}}{-\frac{13}{2}+1}\right]$$

$= \frac{1}{2}\left[\frac{t^{-7/2}}{-\frac{7}{2}} + \frac{t^{-11/2}}{-\frac{11}{2}}\right]$

$= -\frac{1}{7}t^{-7/2} - \frac{1}{11}t^{-11/2}$

$= -\frac{1}{7}\frac{1}{t^{7/2}} - \frac{1}{11}\frac{1}{t^{11/2}}$

$= -\frac{1}{t^{11/2}}\left(\frac{1}{11} + \frac{t^2}{7}\right) = -\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + k$

46. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2, 1, 4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 2
(D) $2\sqrt{2}$

Sol. **(A)**

D. R. of QR is 1, 4, 1

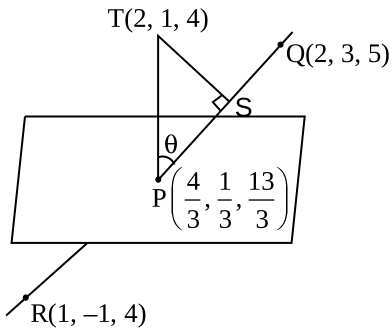
Coordinate of P ≡ $\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

D. R. of PT is 2, 2, –1

Angle between QR and PT is 45º

And PT = 1

$\Rightarrow$ PS = TS = $\frac{1}{\sqrt{2}}$



47. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

(A) differentiable both at x = 0 and at x = 2
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

Sol. **(B)**

$f'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0}\frac{h^2\left|\cos\frac{\pi}{h}\right| - 0}{h}$

$= \lim_{h\to 0} h\cos\left(\frac{\pi}{h}\right) = 0$

so, f(x) is differentiable at x = 0

$f'(2^+) = \lim_{h\to 0}\frac{f(2+h) - f(2)}{h}$

$= \lim_{h\to 0}\frac{(2+h)^2\left|\cos\frac{\pi}{2+h}\right| - 0}{h}$

$$= \lim_{h\to 0} \frac{(2+h)^2 \cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$f'(2^+) = \lim_{h\to 0} \frac{(2+h)^2}{h} \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)$$

$$= \lim_{h\to 0} \frac{(2+h)^2}{h} \sin\left[\frac{\pi \cdot h}{2(2+h)}\right]$$

$$= \lim_{h\to 0} \frac{(2+h)^2}{\frac{\pi h}{2(2+h)}} \sin \frac{\pi h}{2(2+h)} \times \frac{\pi}{2(2+h)} = \pi$$

Again, $f'(2^-) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$

$$= \lim_{h\to 0} \frac{(2-h)^2 \left|\cos\left(\frac{\pi}{2-h}\right)\right|}{-h}$$

$$= \lim_{h\to 0} \frac{-(2-h)^2 \cos\left(\frac{\pi}{2-h}\right)}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2 \sin\left[\frac{\pi}{2} - \frac{\pi}{2-h}\right]}{h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2}{h} \cdot \sin\left[\frac{-\pi h}{2(2-h)}\right]$$

$$= -\lim_{h\to 0} \frac{(2-h)^2}{\frac{\pi h}{2(2-h)}} \cdot \sin \frac{\pi h}{2(2-h)} \times \frac{\pi}{2(2-h)} = -\pi$$

48. Let z be a complex number such that the imaginary part of z is nonzero and a = $z^2 + z + 1$ is real. Then a cannot take the value

(A) –1
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$

Sol. **(D)**
Given equation is $z^2 + z + 1 - a = 0$
Clearly this equation do not have real roots if
D < 0
$\Rightarrow 1 - 4(1 - a) < 0$
$\Rightarrow 4a < 3$
$a < \frac{3}{4}$.

49. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$
(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

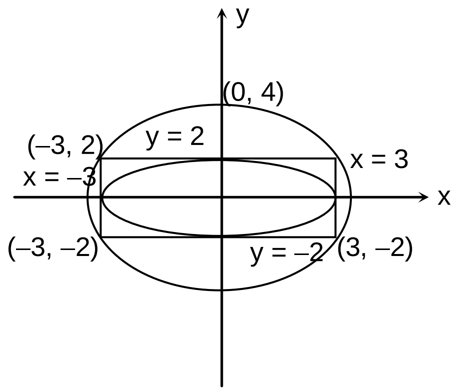
Sol. **(C)**

Equation of ellipse is $(y + 2)(y - 2) + \lambda(x + 3)(x - 3) = 0$

It passes through (0, 4) $\Rightarrow \lambda = \frac{4}{3}$

Equation of ellipse is $\frac{x^2}{12} + \frac{y^2}{16} = 1$

$e = \frac{1}{2}$.



Alternate

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as it is passing through (0, 4) and (3, 2).

So, $b^2 = 16$ and $\frac{9}{a^2} + \frac{4}{16} = 1$

$\Rightarrow a^2 = 12$

So, $12 = 16(1 - e^2)$

$\Rightarrow e = 1/2$.

50. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one–one and onto (B) onto but not one–one
(C) one–one but not onto (D) neither one–one nor onto

Sol. **(B)**

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

f (x) is increasing in [0, 2] and decreasing in [2, 3]

f (x) is many one

f (0) = 1

f (2) = 29

f (3) = 28

Range is [1, 29]

Hence, f (x) is many-one-onto

SECTION II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

51. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

Sol. **(A, B)**

Slope of tangent = 2

The tangents are y = 2x ± $\sqrt{9\times4-4}$
i.e., 2x − y = ± $4\sqrt{2}$

$\Rightarrow \frac{x}{2\sqrt{2}}-\frac{y}{4\sqrt{2}}=1$ and $-\frac{x}{2\sqrt{2}}+\frac{y}{4\sqrt{2}}=1$

Comparing it with $\frac{xx_1}{9}-\frac{yy_1}{4}=1$

We get point of contact as $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Alternate:

Equation of tangent at P (θ) is $\left(\frac{\sec\theta}{3}\right)x-\left(\frac{\tan\theta}{2}\right)y=1$

$\Rightarrow$ Slope = $\frac{2\sec\theta}{3\tan\theta}=2$

$\Rightarrow \sin\theta=\frac{1}{3}$

$\Rightarrow$ points are $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

52. Let θ, φ ∈ [0, 2π] be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1$, $\tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$. Then φ **cannot** satisfy

(A) $0<\varphi<\frac{\pi}{2}$
(B) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$
(C) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$
(D) $\frac{3\pi}{2}<\varphi<2\pi$

Sol. **(A, C, D)**

2 cos θ (1 − sin φ) = $\frac{2\sin^2\theta}{\sin\theta}\cos\varphi-1=2\sin\theta\cos\varphi-1$

2 cos θ − 2 cos θ sin φ = 2 sin θ cos φ − 1
2 cos θ + 1 = 2 sin (θ + φ)

$\tan(2\pi-\theta)>0$ ⇒ tan θ < 0 and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$

$\Rightarrow \theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)$

$\frac{1}{2}$ < sin (θ + φ) < 1

$\Rightarrow 2\pi+\frac{\pi}{6}<\theta+\varphi<\frac{5\pi}{6}+2\pi$

$2\pi+\frac{\pi}{6}-\theta_{max}<\varphi<2\pi+\frac{5\pi}{6}-\theta_{min}$

$\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$.

53. If y(x) satisfies the differential equation y′ − ytanx = 2x secx and y(0) = 0, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$
(B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Sol. **(A, D)**

$\frac{dy}{dx}-y\tan x=2x\sec x$

$\cos x\frac{dy}{dx}+(-\sin x)y=2x$

$\frac{d}{dx}(y\cos x)=2x$

y (x) cos x = x^2 + c, where c = 0 since y (0) = 0

when x = $\frac{\pi}{4}$, $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$, when x = $\frac{\pi}{3}$, $y\left(\frac{\pi}{3}\right)=\frac{2\pi^2}{9}$

when x = $\frac{\pi}{4}$, $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}+\frac{\pi}{\sqrt{2}}$

when x = $\frac{\pi}{3}$, $y'\left(\frac{\pi}{3}\right)=\frac{2\pi^2}{3\sqrt{3}}+\frac{4\pi}{3}$

54. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2},\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true ?

(A) $P\left[X_1^c\middle|X\right]=\frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(C) $P\left[X\middle|X_2\right]=\frac{5}{16}$ (D) $P\left[X\middle|X_1\right]=\frac{7}{16}$

Sol. **(B, D)**

$P(X_1)=\frac{1}{2}$, $P(X_2)=\frac{1}{4}$, $P(X_3)=\frac{1}{4}$

P(X) = $P(X_1\cap X_2\cap X_3^C)+P(X_1\cap X_2^C\cap X_3)+P(X_1^C\cap X_2\cap X_3)+P(X_1\cap X_2\cap X_3)=\frac{1}{4}$

(A) $P(X_1^C/X)=\frac{P(X\cap X_1^C)}{P(X)}=\frac{\frac{1}{32}}{\frac{1}{4}}=\frac{1}{8}$

(B) P [exactly two engines of the ship are functioning | X] = $\frac{\frac{7}{32}}{\frac{1}{4}}=\frac{7}{8}$

(C) $P\left(\frac{X}{X_2}\right)=\frac{\frac{5}{32}}{\frac{1}{4}}=\frac{5}{8}$

(D) $P\left(\frac{X}{X_1}\right)=\frac{\frac{7}{32}}{\frac{1}{2}}=\frac{7}{16}$

55. Let S be the area of the region enclosed by $y = e^{-x^2}$, y = 0, x = 0 and x = 1. Then

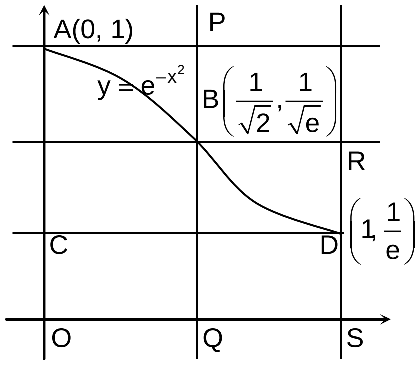
(A) $S \geq \frac{1}{e}$

(B) $S \geq 1 - \frac{1}{e}$

(C) $S \leq \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$

(D) $S \leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Sol. **(A, B, D)**



$S > \frac{1}{e}$ (As area of rectangle OCDS = 1/e)

Since $e^{-x^2} \geq e^{-x}$ $\forall$ x $\in$ [0, 1]

$\Rightarrow S > \int_0^1 e^{-x} dx = \left(1-\frac{1}{e}\right)$

Area of rectangle OAPQ + Area of rectangle QBRS > S

$S < \frac{1}{\sqrt{2}}(1) + \left(1-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{e}}\right)$.

Since $\frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right) < 1 - \frac{1}{e}$

Hence, (C) is incorrect.

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

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56. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{b}-\vec{c}\right|^2 + \left|\vec{c}-\vec{a}\right|^2 = 9$, then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

Sol. **(3)**

As, $\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{b}-\vec{c}\right|^2 + \left|\vec{c}-\vec{a}\right|^2 = 3\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2\right) - \left|\vec{a}+\vec{b}+\vec{c}\right|^2$

$\Rightarrow 3 \times 3 - \left|\vec{a}+\vec{b}+\vec{c}\right|^2 = 9$

$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right| = 0 \Rightarrow \vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow \vec{b}+\vec{c} = -\vec{a}$

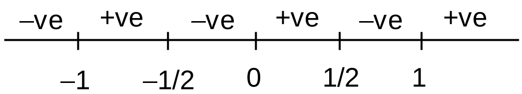
$\Rightarrow \left|2\vec{a}+5\left(\vec{b}+\vec{c}\right)\right| = \left|-3\vec{a}\right| = 3\left|\vec{a}\right| = 3$.

57. Let f : IR $\to$ IR be defined as f(x) = $|x| + \left|x^2-1\right|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. **(5)**

$f'(x) = \frac{|x|}{x} + \frac{\left|x^2-1\right|}{x^2-1}\cdot(2x)$

$$= \begin{cases} 2x-1 & , \quad x < -1 \\ -(2x+1) & , \quad -1 < x < 0 \\ 1-2x & , \quad 0 < x < 1 \\ 2x+1 & , \quad x > 1 \end{cases}$$



So, $f'(x)$ changes sign at points

$x = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

so, total number of points of local maximum or minimum is 5.

58. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Sol. **(4)**

The parabola is $x = 2t^2$, $y = 4t$

Solving it with the circle we get :

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$\Rightarrow t^4 + 3t^2 - 4t = 0 \Rightarrow t = 0, 1$

so, the points P and Q are (0, 0) and (2, 4) which are also diametrically opposite points on the circle. The focus is $S \equiv (2, 0)$.

The area of $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$.

59. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then $p'(0)$ is

Sol. **(9)**

Let $p'(x) = k(x - 1)(x - 3)$

$\Rightarrow p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

Now, $p(1) = 6 \Rightarrow \frac{4}{3}k + c = 6$

also, $p(3) = 2 \Rightarrow c = 2$

so, $k = 3$, so, $p'(0) = 3k = 9$.

60. The value of $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$ is

Sol. **(4)**

Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}} = y$

So, $4 - \frac{1}{3\sqrt{2}}y = y^2$ (y > 0)

$\Rightarrow y^2 + \frac{1}{3\sqrt{2}}y - 4 = 0 \Rightarrow y = \frac{8}{3\sqrt{2}}$

so, the required value is $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right)$

$= 6 + \log_{3/2}\frac{4}{9} = 6 - 2 = 4$.