IIT-JEE-2008-Paper1

PAPER - I
SECTION - I
Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

1. If $0 < x < 1$ then $\sqrt{1+x^2}\,[\{x \cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2}$
(1) $x/\sqrt{1+x^2}$
(2) x
(3) $x\sqrt{1+x^2}$
(4) $\sqrt{1+x^2}$

2. Consider the two curves
$C_1 : y^2 = 4x$
$C_2 : x^2 + y^2 - 6x + 1 = 0$
Then,
(1) C_1 and C_2 touch each other only at one point
(2) C_1 and C_2 touch each other exactly at two points
(3) C_1 and C_2 intersect (but do not touch) at exactly two points
(4) C_1 and C_2 neither intersect nor touch each other

3. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors a, b, c such that a.b = b.c = c.a = 1/2
Then, the volume of the parallelopiped is
(1) $1/\sqrt{2}$
(2) $1/2\sqrt{2}$
(3) $\sqrt{3} / 2$
(4) $1/\sqrt{3}$

4. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents
(1) four straight lines, when c = 0 and a, b are of the same sign
(2) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(3) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(4) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

5. The total number of local maxima and minima of the function

$$f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases} \text{ is}$$

(1) 0
(2) 1
(3) 2
(4) 3

6.

$$\text{Let } g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}; \; 0 < x < 2,$$

m and n are integers $m \neq 0$, $n > 0$ and let p be the left hand derivative of

$|x - 1|$ at $x = 1$. If $\lim\limits_{x \to 1+} g(x) = p$, then

(1) $n = 1, m = 1$
(2) $n = 1, m = -1$
(3) $n = 2, m = 2$
(4) $n > 2, m = n$

SECTION II

Multiple Correct Answers Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONE OR MORE is/are correct.

7. Let P (x_1, y_1) and Q(x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(1) $x^2 + 2\sqrt{3}\, y = 3 + \sqrt{3}$
(2) $x^2 - 2\sqrt{3}\, y = 3 + \sqrt{3}$
(3) $x^2 + 2\sqrt{3}\, y = 3 - \sqrt{3}$
(4) $x^2 - 2\sqrt{3}\, y = 3 - \sqrt{3}$

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(1) $1/PS + 1/ST < 2/\sqrt{(QS * SR)}$

(2) $1/PS + 1/ST > 2/\sqrt{(QS * SR)}$

(3) $1/PS + 1/ST < 4/ QR$

(4) $1/PS + 1/ST > 4/ QR$

9. Let f(x) be a non-constant twice differentiable function defined

$(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'\left(\frac{1}{4}\right) = 0$

Then,

(1) $f''(x)$ vanishes at least twice on [0, 1]

(2) $f'\left(\frac{1}{2}\right) = 0$

(3) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right) \sin x \, dx = 0$

(4) $\int_{0}^{1/2} f(t) e^{\sin \pi t} \, dt = \int_{1/2}^{1} f(1-t) \, e^{\sin \pi t} \, dt$

10.

Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for n = 1, 2, 3, ... then ,

(1) $S_n < \Pi/3\sqrt{3}$

(2) $S_n > \Pi/3\sqrt{3}$

(3) $T_n < \Pi/3\sqrt{3}$

(4) $T_n > \Pi/3\sqrt{3}$

SECTION - III

Assertion - Reason Type

This section contains 4 reasoning type questions. Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

11. Consider the system of equations
$x - 2y + 3z = -1$
$-x + y - 2z = k$
$x - 3y + 4z = 1$

STATEMENT-1: The system of equations has no solutions for $k \neq 3$
and

STATEMENT-2:

The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$

(1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

12. Consider the system of equations $ax + by = 0$, $cx + dy = 0$, where a, b, c, dÎ{0, 1}

STATEMENT-1: The probability that the system of equations has a unique solution is 3/8
and

STATEMENT-2: The probability that the system of equations has a solution is 1.

(1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

13. Let f and g be real valued functions defined on interval (-1, 1) such that g''(x) is continuous $g(0) \neq 0$, $g''(0) = 0$

STATEMENT-1: $g''(0) \neq 0$, and $f(x) = g(x) \sin x$
$\lim_{x\to 0} [g(x) \cot x - g(x) \operatorname{cosec} x] = f''(0)$.
and

STATEMENT-2: $f''(0) = g(0)$

(1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

14. Consider three planes $P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2$
Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and $P_1$and P_2, respectively

STATEMENT-1: At least two of the lines L_1, L_2 and L_3 are non-parallel
and

STATEMENT-2: The three planes do not have a common point

(1) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(2) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(3) STATEMENT-1 is True, STATEMENT-2 is False
(4) STATEMENT-1 is False, STATEMENT-2 is True

SECTION - IV

Linked Comprehension Type

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

Paragraph for Questions Nos. 15 to 17

Let A, B, C be three sets of complex numbers as defined below

$A = \{ z: \text{Im} z > 1\}$

$B = \{ z: |z-2-i| = 3\}$

$C = \{ z: \text{Re}((1-i)z) = \sqrt{2}\}$

15. The number of element in the set $A \cap B \cap C$ is

(1) 0

(2) 1

(3) 2

(4) ∞

16. Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

(1) 25 and 29

(2) 30 and 34

(3) 35 and 39

(4) 40 and 44

17. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between

(1) -6 and 3

(2) -3 and 6

(3) -6 and 6

(4) -3 and 9

Paragraph for Questions Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D,E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $(3\sqrt{3}/2, 3/2)$ Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(1) $(x-2\sqrt{3})^2 + (y-1)^2 = 1$

(2) $(x-2\sqrt{3})^2 + (y-1/2)^2 = 1$

(3) $(x-\sqrt{3})^2 + (y+1)^2 = 1$

(4) $(x-\sqrt{3})^2 + (y-1)^2 = 1$

19. Points E and F are given by

(1) $(\sqrt{3}/2, 3/2)$ $(\sqrt{3}, 0)$

(2) $(\sqrt{3}/2, 1/2)$ $(\sqrt{3}, 0)$
(3) $(\sqrt{3}/2, 3/2)$ $(\sqrt{3}/2, 1/2)$
(4) $(3/2, \sqrt{3}/2)$ $(\sqrt{3}/2, 1/2)$

20. Equations of the sides QR, RP are
(1) $y = (2/\sqrt{3})x + 1$, $y = -(2/\sqrt{3})x - 1$
(2) $y = (1/\sqrt{3})x$ $y = 0$
(3) $y = (\sqrt{3}/2)x + 1$, $y = -(\sqrt{3}/2)x - 1$
(4) $y = (\sqrt{3})x$, $y = 0$

Paragraph for Questions Nos. 21 to 23

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line. If x Î (-∞,2) U (2,∞)the equation implicitly defines a unique real valued differentiable function y = f(x).
If x Î (-2, 2), the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g (0) = 0.

21. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$
(1) $4\sqrt{2} / 7^3\, 3^2$
(2) $-4\sqrt{2} / 7^3\, 3^2$
(3) $4\sqrt{2} / 7^3\, 3$
(4) $-4\sqrt{2} / 7^3\, 3$

22. The area of the region bounded by the curve y = f(x), the x-axis, and the lines x = a and x = b, where -∞ < a < b < -2, is

(1) $\int_a^b \frac{x}{3[((f(x))^2 - 1]} dx + bf(b) - af(a)$

(2) $-\int_a^b \frac{x}{3((f(x))^2 - 1)} dx + bf(b) - af(a)$

(3) $\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$

(4) $-\int_a^b \frac{x}{3((f(x))^2 - 1)} dx - bf(b) + af(a)$

23. $\int_{1}^{-1} g'(x)dx =$

(1) 2g(-1)
(2) 0
(3) -2g (1)
(4) 2g(1)