JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTIONS

(Exam Date: 20-05-2018)

PART-1 : MATHEMATICS

SECTION-1

1. For a non-zero complex number z, let arg(z) denotes the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is (are) **FALSE** ?

(A) $\arg(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition

$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line

Ans. (A,B,D)

Sol. (A) $\arg(-1 - i) = -\frac{3\pi}{4}$,

(B) $f(t) = \arg(-1 + it) = \begin{cases} \pi - \tan^{-1}(t), & t \ge 0 \\ -\pi + \tan^{-1}(t), & t < 0 \end{cases}$

Discontinuous at t = 0.

(C) $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$

$= \arg z_1 - \arg(z_2) + 2n\pi - \arg(z_1) + \arg(z_2) = 2n\pi$.

(D) $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$

$\Rightarrow \quad \dfrac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$ is real.

$\Rightarrow$ z, z_1, z_2, z_3 are concyclic.

2. In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ?

(A) $\angle QPR = 45°$

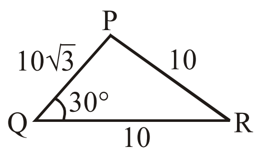
(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3}-15$

(D) The area of the circumcircle of the triangle PQR is 100π.

Ans. (B,C,D)

Sol. $\cos 30° = \dfrac{\left(10\sqrt{3}\right)^2 + (10)^2 - (PR)^2}{2\times 10\sqrt{3}\times 10}$



$\Rightarrow$ PR = 10

$\because$ QR = PR $\Rightarrow$ $\angle PQR = \angle QPR$

$\angle QPR = 30°$

(B) area of Δ PQR $= \dfrac{1}{2}\times 10\sqrt{3}\times 10\times \sin 30° = \dfrac{1}{2}\times 10\times 10\sqrt{3}\times\dfrac{1}{2}$

$= 25\sqrt{3}$

$\angle QRP = 180° - (30° + 30°) = 120°$

(C) $r = \dfrac{\Delta}{S} = \dfrac{25\sqrt{3}}{\left(\dfrac{10+10+10\sqrt{3}}{2}\right)} = \dfrac{25\sqrt{3}}{10+5\sqrt{3}}$

$= 5\sqrt{3}.\left(2-\sqrt{3}\right) = 10\sqrt{3}-15$

(D) $R = \dfrac{a}{2\sin A} = \dfrac{10}{2\sin 30°} = 10$

$\therefore$ Area $= \pi R^2 = 100\pi$

3. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ?

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_2 is $\frac{2}{\sqrt{3}}$

Ans. (C,D)

Sol. D.C. of line of intersection (a, b, c)

$\Rightarrow$ $2a + b - c = 0$

$a + 2b + c = 0$

$$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$$

$\therefore$ D.C. is (1, –1, 1)

(B) $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$

$$\Rightarrow \frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$$

$\Rightarrow$ lines are parallel.

(C) Acute angle between P_1 and P_2 = $\cos^{-1}\left(\frac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$

$$=\cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

(D) Plane is given by $(x - 4) - (y - 2) + (z + 2) = 0$

$\Rightarrow$ $x - y + z = 0$

Distance of (2, 1, 1) from plane = $\frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

4. For every twice differentiable function $f : \mathbb{R} \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ?

(A) There exist r, s $\in \mathbb{R}$, where r < s, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$

(C) $\lim_{x\to\infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \ne 0$

Ans. **(A,B,D)**

Sol. $f(x)$ can't be constant throughout the domain. Hence we can find $x \in (r, s)$ such that $f(x)$ is one-one option (A) is true.

Option (B) : $|f'(x_0)| = \left|\frac{f(0) - f(-4)}{4}\right| \le 1$ (LMVT)

Option (C) : $f(x) = \sin\left(\sqrt{85}x\right)$ satisfies given condition

but $\lim_{x\to\infty} \sin\left(\sqrt{85}\right)$ D.N.E.

$\Rightarrow$ Incorrect

Option (D) : $g(x) = f^2(x) + (f'(x))^2$

$|f'(x_1) \le 1$ (by LMVT)

$|f(x_1)| \le 2$ (given)

$\Rightarrow$ $g(x_1) \le 5$ $\exists x_1 \in (-4, 0)$

Similarly $g(x_2) \le 5$ $\exists x_2 \in (0, 4)$

$g(0) = 85$ $\Rightarrow$ $g(x)$ has maxima in (x_1, x_2) say at α.

$g'(\alpha) = 0$ & $g(\alpha) \ge 85$

$2f'(\alpha)\ (f(\alpha) + f''(\alpha)) = 0$

If $f'(\alpha) = 0$ $\Rightarrow$ $g(\alpha) = f^2(\alpha) \ge 85$ Not possible

$\Rightarrow$ $f(\alpha) + f''(\alpha) = 0$ $\exists \alpha \in (x_1, x_2) \in (-4, 4)$

option (D) correct.

5. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{(f(x) - g(x))})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Ans. **(B,C)**

Sol. $f'(x) = e^{(f(x) - g(x))}\ g'(x)\ \forall\ x \in \mathbb{R}$

$\Rightarrow$ $e^{-f(x)}.\ f'(x) - e^{-g(x)}g'(x) = 0$

$\Rightarrow$ $\int\left(e^{-f(x)} f'(x) - e^{-g(x)}.g'(x)\right)dx = C$

$\Rightarrow$ $-e^{-f(x)} + e^{-g(x)} = C$

$\Rightarrow$ $-e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$

$$\Rightarrow \quad -\frac{1}{e}+e^{-g(1)}=-e^{-f(2)}+\frac{1}{e}$$

$$\Rightarrow \quad e^{-f(2)}+e^{-g(1)}=\frac{2}{e}$$

$$\therefore \quad e^{-f(2)}<\frac{2}{e} \text{ and } e^{-g(1)}<\frac{2}{e}$$

$\Rightarrow$ $-f(2) < \ln 2 - 1$ and $-g(1) < \ln 2 - 1$

$\Rightarrow$ $f(2) > 1 - \ln 2$ and $g(1) > 1 - \ln 2$

6. Let $f : [0, \infty) \to \mathbb{R}$ be a continuous function such that $f(x)=1-2x+\int_0^x e^{x-t} f(t)dt$ for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE ?

(A) The curve $y = f(x)$ passes through the point $(1, 2)$

(B) The curve $y = f(x)$ passes through the point $(2, -1)$

(C) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi-1}{4}$

Ans. (B,C)

Sol. $f(x) = 1-2x+\int_0^x e^{x-t} f(t)dt$

$$\Rightarrow \quad e^{-x} f(x)=e^{-x}(1-2x)+\int_0^x e^{-t} f(t)dt$$

Differentiate w.r.t. x.

$-e^{-x} f(x)+e^{-x} f'(x)=-e^{-x}(1-2x)+e^{-x}(-2)+e^{-x} f(x)$

$\Rightarrow$ $-f(x) + f'(x) = -(1 - 2x) - 2 + f(x)$.

$\Rightarrow$ $f'(x) - 2f(x) = 2x - 3$

Integrating factor $= e^{-2x}$.

$f(x).e^{-2x} = \int e^{-2x}(2x-3)dx$

$$=(2x-3)\int e^{-2x}dx-\int\left((2)\int e^{-2x}dx\right)dx$$

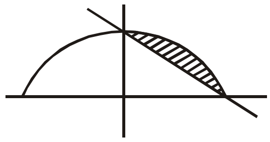
$$=\frac{(2x-3)e^{-2x}}{-2}-\frac{e^{-2x}}{2}+c$$

$$f(x)=\frac{2x-3}{-2}-\frac{1}{2}+ce^{2x}$$

$f(0)=\frac{3}{2}-\frac{1}{2}+c=1\Rightarrow c=0$

$\therefore \quad f(x) = 1 - x$

Area $=\frac{\pi}{4}-\frac{1}{2}=\frac{\pi-2}{4}$



SECTION-2

7. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}}\times\left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is ——

Ans. (8)

Sol. $\log_2 9^{\frac{2}{\log_2(\log_2 9)}}\times 7^{\frac{1/2}{\log_4 7}}$

$=\left(\log_2 9\right)^{2\log^2_{\log_2 9}}\times 7^{\frac{1}{2}\log_7 4}$

$= 4 \times 2 = 8$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is ——

Ans. (625)

Sol. Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits

$= 5 \times 5 \times 5 \times 5 = 625$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ——

Ans. (3748)

Sol. X : 1, 6, 11, ..., 10086

Y : 9, 16, 23, ..., 14128

$X \cap Y$: 16, 51, 86, ...

Let $m = n(X \cap Y)$

$\therefore \quad 16 + (m - 1) \times 35 \leq 10086$

$\Rightarrow \quad m \leq 288.71$

$\Rightarrow \quad m = 288$

$\therefore \quad n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2018 + 2018 - 288 = 3748$

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is ——

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume value in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

Ans. **(2)**

Sol. $\sum_{i=1}^{\infty} x^{i+1} = \frac{x^2}{1-x}$

$\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i = \frac{x}{2-x}$

$\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i = \frac{-x}{2+x}$

$\sum_{i=1}^{\infty}(-x)^i = \frac{-x}{1+x}$

To have real solutions

$$\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i = \sum_{i=1}^{\infty}\left(\frac{-x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i$$

$$\frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{-x}{2+x} + \frac{x}{1+x}$$

$x(x^3 + 2x^2 + 5x - 2) = 0$

$\therefore$ $x = 0$ and let $f(x) = x^3 + 2x^2 + 5x - 2$

$$f\left(\frac{1}{2}\right) . f\left(-\frac{1}{2}\right) < 0$$

Hence two solutions exist

11. For each positive integer n, let

$$y_n = \frac{1}{n}(n+1)(n+2)...(n+n)^{1/n}$$

For $x \in \mathbb{R}$, let [x] be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of [L] is ——

Ans. (1)

Sol. $y_n = \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$

$y_n = \prod_{r=1}^{n}\left(1+\frac{r}{n}\right)^{1/n}$

$\log y_n = \frac{1}{n}\sum_{r=1}^{n}\ell n\left(1+\frac{r}{n}\right)$

$\Rightarrow \lim_{n\to\infty}\log y_n = \lim_{x\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ell n\left(1+\frac{r}{n}\right)$

$\Rightarrow \log L = \int_0^1 \ell n(1+x)dx$

$\Rightarrow \log L = \log\frac{4}{e}$

$\Rightarrow L = \frac{4}{e}$

$\Rightarrow [L] = 1$

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b}=0$. For some x, y $\in \mathbb{R}$, let $\vec{c} = x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ——

Ans. (3)

Sol. $\vec{c} = x\vec{a}+y\vec{b}+\vec{a}\times\vec{b}$

$\vec{c}.\vec{a} = x$ and $x = 2\cos\alpha$

$\vec{c}.\vec{b} = y$ and $y = 2\cos\alpha$

Also, $|\vec{a}\times\vec{b}| = 1$

$\therefore \quad \vec{c} = 2\cos(\vec{a}+\vec{b})+\vec{a}\times\vec{b}$

$\vec{c}^2 = 4\cos^2\alpha(\vec{a}+\vec{b})^2+(\vec{a}\times\vec{b})^2 + 2\cos\alpha(\vec{a}+\vec{b}).(\vec{a}\times\vec{b})$

$4 = 8\cos^2\alpha + 1$

$8\cos^2\alpha = 3$

13. Let a, b, c be three non-zero real numbers such that the equation

$$\sqrt{3}a\cos x + 2b\sin x = c, \quad x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of $\frac{b}{a}$ is ——

Ans. (0.5)

Sol. $\sqrt{3}\cos x+\frac{2b}{a}\sin x=\frac{c}{a}$

Now, $\sqrt{3}\cos\alpha+\frac{2b}{a}\sin\alpha=\frac{c}{a}$ ..... (1)

$\sqrt{3}\cos\beta+\frac{2b}{a}\sin\beta=\frac{c}{a}$ ..... (2)

$\sqrt{3}[\cos\alpha-\cos\beta]+\frac{2b}{a}(\sin\alpha-\sin\beta)=0$

$\sqrt{3}\left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]+\frac{2b}{a}\left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]=0$

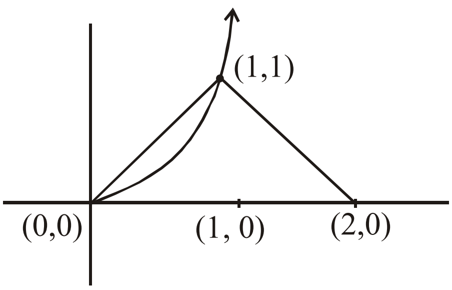
$-\sqrt{3}+2\sqrt{3}.\frac{b}{a}=0$

$\frac{b}{a}=\frac{1}{2}=0.5$

14. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is ——

Ans. (4)

Sol.



Area $=\int_0^1 (x-x^n)dx=\frac{3}{10}$

$\left[\frac{x^2}{2}-\frac{x^{n+1}}{n+1}\right]_0^1=\frac{3}{10}$

$\frac{1}{2}-\frac{1}{n+1}=\frac{3}{10}$ $\quad\therefore$ n + 1 = 5

$\Rightarrow$ n = 4

SECTION-3

Paragraph "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two question based on Paragraph "X", the question given below is one of them*)

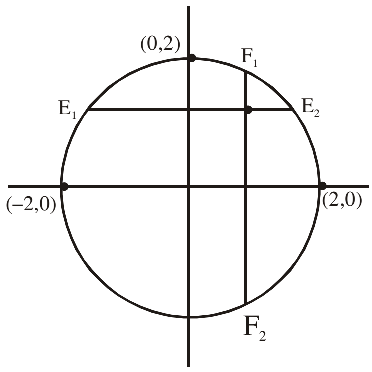
15. Let E_1E_2 and F_1F_2 be the chord of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slop -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents of S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3 and G_3 lie on the curve

(A) $x + y = 4$ (B) $(x - 4)^2 + (y - 4)^2 = 16$

(C) $(x - 4)(y - 4) = 4$ (D) $xy = 4$

Ans. (A)

Sol.



co-ordinates of E_1 and E_2 are obtained by solving $y = 1$ and $x^2 + y^2 = 4$

$\therefore \quad E_1\left(-\sqrt{3},1\right)$ and $E_2\left(\sqrt{3},1\right)$

co-ordinates of F_1 and F_2 are obtained by solving

$x = 1$ and $x^2 + y^2 = 4$

$F_1\left(1,\sqrt{3}\right)$ and $F_2\left(1,-\sqrt{3}\right)$

Tangent at E_1 : $-\sqrt{3}x + y = 4$

Tangent at E_2 : $\sqrt{3}x + y = 4$

$\therefore \quad E_3\ (0, 4)$

Tangent at F_1 : $x + \sqrt{3}y = 4$

Tangent at F_2 : $x - \sqrt{3}y = 4$

$\therefore \quad F_3(4, 0)$

and similarly $G_3(2, 2)$

(0, 4), (4, 0) and (2, 2) lies on $x + y = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$

(*There are two questions based on Paragraph "X", the question given below is one of them*)

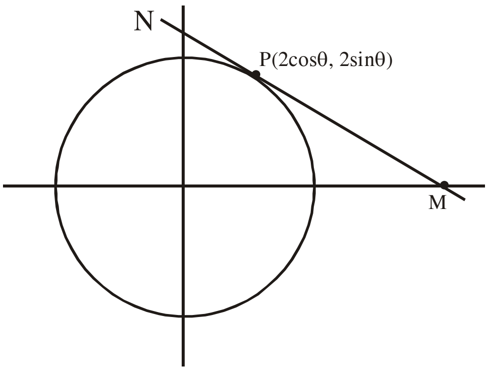
16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve -

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

Ans. (D)

Sol.



Tangent at $P(2\cos\theta, 2\sin\theta)$ is $x\cos\theta + y\sin\theta = 2$

$M(2\sec\theta, 0)$ and $N(0, 2\operatorname{cosec}\theta)$

Let midpoint be (h, k)

$h = \sec\theta, \quad k = \operatorname{cosec}\theta$

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

PARAGRAPH "A"

There are five students S_1, S_2, S_4 and S_5 in a music class and for them there are five sets R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A". the question given below is one of them*)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and **NONE** of the remaining students gets the seat previously allotted to him/her is -

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

Ans. (A)

Sol. Required probability = $\dfrac{4!\left(\dfrac{1}{2!}-\dfrac{1}{3!}+\dfrac{1}{4!}\right)}{5!} = \dfrac{9}{120} = \dfrac{3}{40}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A", the question given below is one of them*)

18. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

(A) $\dfrac{1}{15}$ (B) $\dfrac{1}{10}$ (C) $\dfrac{7}{60}$ (D) $\dfrac{1}{5}$

Ans. (C)

Sol. $n(T_1 \cap T_2 \cap T_3 \cap T_4)$ = Total $-\ n\left(\overline{T}_1 \cup \overline{T}_2 \cup \overline{T}_3 \cup \overline{T}_4\right)$

$= 5! - \left({}^4C_1 4!2! - \left({}^3C_1.3!2! + {}^3C_1 3!2!2!\right) + \left({}^2C_1 2!2! + {}^4C_1.2.2!\right) - 2\right)$

$= 14$

Probability $= \dfrac{14}{5!} = \dfrac{7}{60}$