# JEE(ADVANCED) 2013

## PAPER-1 [Code – 5]

## PHYSICS

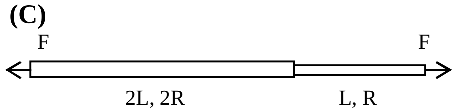
### SECTION – 1: (Only one option correct Type)

This section contains **10 multiple choice questions.** Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

* 1. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50
(C) 2.00 (D) 4.00

***Sol.*** **(C)**




$$k_1 = \frac{\pi 4R^2 x}{2L}, k_2 = \frac{\pi R^2 y}{L}$$

$$F = k_1 x = k_2 y \Rightarrow \frac{y}{x} = \frac{k_1}{k_2} = \mathbf{2}$$

* 2. The work done on a particle of mass m by a force $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions, when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$
(C) $\frac{K\pi}{2a}$ (D) 0

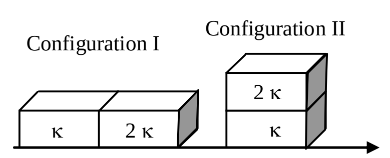
***Sol.*** **(D)**

$$dw = \vec{F}\cdot d\vec{r} = \vec{F}\cdot(dx\hat{i} + dy\hat{j}) = K\int \frac{xdx}{(x^2+y^2)^{3/2}} + \frac{ydy}{(x^2+y^2)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{K}{a^3}\int_a^0 xdx + \int_0^a ydy = \frac{K}{a^3}\left(\frac{-a^2}{2} + \frac{a^2}{2}\right) = \mathbf{0.}$$

* 3. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity $\kappa$ and the other $2\kappa$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 *s* to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is




(A) 2.0 s (B) 3.0 s
(C) 4.5 s (D) 6.0 s

***Sol.*** **(A)**

$$R_1 = \frac{L}{\kappa A} + \frac{L}{2\kappa A} = \frac{3L}{2\kappa A}$$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{\kappa A}\right)} + \frac{1}{\left(\frac{L}{2\kappa A}\right)} = \frac{3\kappa A}{L}$$

$$R_2 = \frac{L}{3\kappa A}$$

$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T}{R_1} t_1 = \frac{\Delta T}{R_2} t_2$$

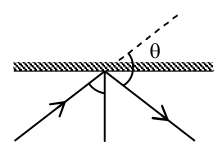
$$\Rightarrow t_2 = \frac{R_2}{R_1} t_1 = \mathbf{2\ sec.}$$

4. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(A) 30° (B) 45°
(C) 60° (D) 75°

***Sol.*** **(A)**

Let angle between the directions of incident ray and reflected ray be $\theta$




$$\cos\theta = \frac{1}{2}(\hat{i} + \sqrt{3}\hat{j}) \cdot \frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120°$$

* 5. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24$^{th}$ division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm
(C) 5.136 cm (D) 5.148 cm

***Sol.*** **(B)**

Main scale division (s) = .05 cm

Vernier scale division (v) = $\frac{49}{100}$ = .049

Least count = .05 – .049 = .001 cm

Diameter: 5.10 + 24 × .001

**= 5.124 cm**

* 6. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

***Sol.*** **(D)**

$PV = nRT = \frac{m}{M}RT$

$\Rightarrow PM = \rho RT$

$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right)\times\left(\frac{M_1}{M_2}\right) = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$

Here $\rho_1$ and $\rho_2$ are the densities of gases in the vessel containing the mixture.

7. In the Young's double slit experiment using a monochromatic light of wavelength $\lambda$, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n+1)\frac{\lambda}{2}$ (B) $(2n+1)\frac{\lambda}{4}$ (C) $(2n+1)\frac{\lambda}{8}$ (D) $(2n+1)\frac{\lambda}{16}$

***Sol.*** **(B)**

$\frac{I_{max}}{2} = I_m \cos^2\left(\frac{\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\phi}{2} = \frac{\pi}{4}$

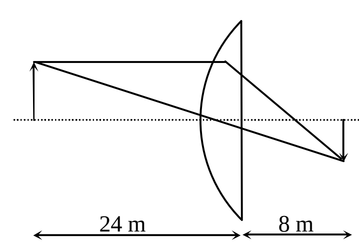
$\Rightarrow \phi = \frac{\pi}{2}(2n+1)$

$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{2}(2n+1) = \frac{\lambda}{4}(2n+1)$

8. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

(A) 1 m (B) 2 m (C) 3 m (D) 4 m

***Sol.*** **(C)**




8. $\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$

$\Rightarrow \frac{1}{f} = \frac{\mu-1}{R} = \frac{1}{2R}$

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R}$

$\Rightarrow \frac{3+1}{24} = \frac{1}{2R}$

$\Rightarrow R = 3$ m

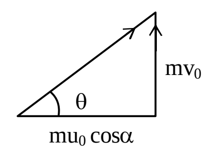
* 9. A particle of mass m is projected from the ground with an initial speed $u_0$ at an angle $\alpha$ with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed $u_0$. The angle that the composite system makes with the horizontal immediately after the collision is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$ (C) $\frac{\pi}{4}-\alpha$ (D) $\frac{\pi}{2}$

***Sol.*** **(A)**

Velocity of particle performing projectile motion at highest point

$= v_1 = v_0 \cos \alpha$




Velocity of particle thrown vertically upwards at the position of collision

$$= v_2^2 = u_0^2 - 2g\frac{u^2 \sin^2 \alpha}{2g} = v_0 \cos\alpha$$

So, from conservation of momentum

$$\tan\theta = \frac{mv_0 \cos\alpha}{mu_0 \cos\alpha} = 1$$

$\Rightarrow \quad \theta = \pi/4$

10. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is $3 \times 10^8$ m/s. The final momentum of the object is

(A) $0.3 \times 10^{-17}$ kg ms$^{-1}$ (B) $1.0 \times 10^{-17}$ kg ms$^{-1}$

(C) $3.0 \times 10^{-17}$ kg ms$^{-1}$ (B) $9.0 \times 10^{-17}$ kg ms$^{-1}$

***Sol.*** **(B)**

$t = 100\times10^{-9}$ sec, $P = 30\times10^{-3}$ Watt, $C = C\times10^{8}$ m/s

$$\text{Momentum} = \frac{Pt}{C} = \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}} = 1.0 \times 10^{-17} \text{ kg ms}^{-1}$$

## SECTION – 2 : (One or more options correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

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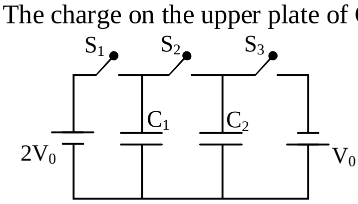
11. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch $S_1$ is pressed first to fully charge the capacitor $C_1$ and then released. The switch $S_2$ is then pressed to charge the capacitor $C_2$. After some time, $S_2$ is released and then $S_3$ is pressed. After some time,

(A) the charge on the upper plate of $C_1$ is $2CV_0$.

(B) the charge on the upper plate of $C_1$ is $CV_0$.

(C) the charge on the upper plate of $C_1$ is 0.

(D) The charge on the upper plate of $C_2$ is $-CV_0$.




***Sol.*** **(B, D)**

After switch $S_1$ is closed, $C_1$ is charged by $2CV_0$, when switch $S_2$ is closed, $C_1$ and $C_2$ both have upper plate charge $CV_0$.

When $S_3$ is closed, then upper plate of $C_2$ becomes charged by $-CV_0$ and lower plate by $+CV_0$.

12. A particle of mass M and positive charge Q, moving with a constant velocity $u_1 = 4\hat{i}ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$ m/s$^{-1}$. The correct statement(s) is (are)

(A) The direction of the magnetic field is –z direction.
(B) The direction of the magnetic field is +z direction.
(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.
(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

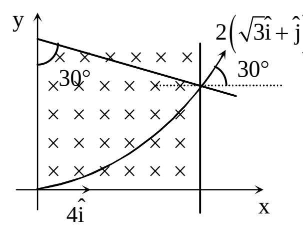
***Sol.*** **(A, C)**

So magnetic field is along –ve, z-direction.

Time taken in the magnetic field = $10\times10^{-3} = \frac{\pi M}{6QB}$

$$B = \frac{\pi M}{60\times10^{-3}Q} = \frac{1000\pi M}{60Q}$$

$$= \frac{50\pi M}{3Q}$$

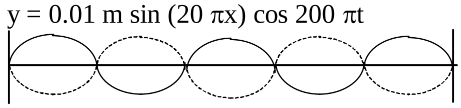



* 13. A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation $y(x, t) = 0.01m \sin [(62.8m^{-1})x] \cos[(628s^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are)

(A) The number of nodes is 5.
(B) the length of the string is 0.25 m.
(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.
(D) The fundamental frequency is 100 Hz.

***Sol.*** **(B, C)**

$y = 0.01$ m $\sin (20\pi x) \cos 200\pi t$



no. of nodes is 6

$20\pi = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{1}{10}$ m $= 0.1$ m

length of the spring = $0.5\times\frac{1}{2} = 0.25$

Mid point is the antinode

Frequency at this mode is $f = \frac{200\pi}{2\pi} = 100$Hz

$\therefore \quad$ Fundamental frequency = $\frac{100}{5} = 20$Hz.

* 14. A solid sphere of radius R and density $\rho$ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density $3\rho$. The complete arrangement is placed in a liquid of density $2\rho$ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

***Sol.*** **(A, D)**

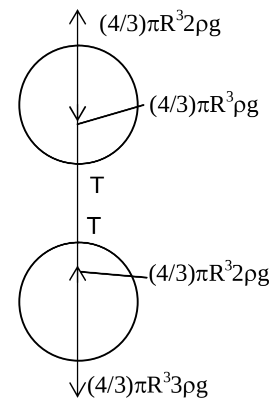
At equilibrium,

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$T = \frac{4}{3}\pi R^3 \rho g$$

$$\therefore \ \Delta\ell = \frac{4}{3k}\pi R^3 \rho g$$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.




15. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities $\rho_1$ and $\rho_2$ respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centre of the spheres is zero. The ratio $\rho_1/\rho_2$ can be

(A) – 4

(B) $-\frac{32}{25}$

(C) $\frac{32}{25}$

(D) 4

***Sol.*** **(B, D)**

At point $P_1$, $\frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$

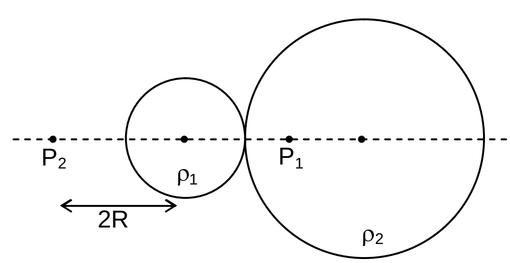
$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3}$$

$$\frac{\rho_1}{\rho_2} = 4$$

At point $P_2$,

$$\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$$

$$\therefore \ \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$



## SECTION – 3 : (Integer value correct Type)

This section contains **5 questions**. The answer to each question is **a single digit integer,** ranging from 0 to 9 (*both inclusive*).

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* 16. A bob of mass m , suspended by a string of length $l_1$ is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length $l_2$, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $l_1/l_2$ is

***Sol.*** **(5)**

The initial speed of 1st bob (suspended by a string of length $l_1$)is $\sqrt{5gl_1}$ .

The speed of this bob at highest point will be $\sqrt{gl_1}$ .

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

* 17. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in m/s) of the particle is zero, the speed (in m/s) after 5 s is

***Sol.*** **(5)**

$$\text{Power} = \frac{dW}{dt} \quad \Rightarrow W = 0.5 \times 5 = 2.5 = KE_f - KE_i$$

$$2.5 = \frac{M}{2}(v_f^2 - v_i^2)$$

$$\Rightarrow v_f = 5$$

18. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

***Sol.*** **(1)**

Slope of graph is h/e = constant

$\Rightarrow$ 1

19. A freshly prepared sample of a radioisotope of half-life 1386 s has activity $10^3$ disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

***Sol.*** **(4)**

$f = (1 - e^{-\lambda t}) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$

$f = 0.04$

Hence % decay $\approx 4$ %

* 20. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad $s^{-1}$ about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad $s^{-1}$) of the system is

***Sol.*** **(8)**

Conservation of angular momentum about vertical axis of disc

$$\frac{50(0.4)^2}{2}\times 10 = \left[\frac{50(0.4)^2}{2} + 4(6.25)(0.2)^2\right]\omega$$

$\omega$ = 8 rad/sec