

IIT-JEE-Mathematics-Mains-2005

Mains

1. A person goes to office either by car, scooter, bus or train probability of which being $1/7, 3/7, 2/7$ and $1/7$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $2/9, 1/9, 4/9$ and $1/9$ respectively. Given that he reaches office in time, then what is the probability that he travelled by a car?

2. Find the range of values of t for which

$$2 \sin t = (1-2x+5x^2)/(3x^2-2x-1), t \in [-\pi/2, \pi/2].$$

3. Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the point of contact.

4. Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $1/\sqrt{6}$ from the point $(2, 1 - 1)$.

5. If $|f(x_1) - f(x_2)| < (x_1 - x_2)^2$, for all $x_1, x_2 \in \mathbb{R}$. Find the equation of tangent to the curve $y = f(x)$ at the point $(1, 2)$.

6. If total number of runs scored in n matches is $((n+1)/4) (2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k th match are given by $k \cdot 2^{n+1-k}$, where $1 < k < n$. Find n .

7. The area of the triangle formed by the intersection of a line parallel to x -axis and passing through (h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of point P.

8. Evaluate

$$\int_0^{\pi} e^{|\cos x|} (2 \sin (1/2 \cos x) + 3 \cos (1/2 \cos x)) \sin x \, dx.$$

9. Incident ray is along the unit vector v and the reflected ray is along the unit vector w . The normal is along unit vector a outwards. Express vector w in terms of vector a and v .

10. Tangents are drawn from any point on the hyperbola $x^2/9 - y^2/4 = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact.

11. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $x^2/25 + y^2/4 = 1$. Also find the length of the intercept of the tangent between the coordinate axes.

12. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis is of length 1. Find the equation of the curve.

13. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

14. If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}i$. Find the other vertices of square.

15. If $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$. If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$.

16. If $p(x)$ be a polynomial of degree 3 satisfying $p(-1) = 10$, $p(1) = -6$ and $p(x)$ has maximum at $x = -1$ and $p'(x)$ has minima at $x = 1$. Find the distance between the local maximum and local minimum of the curve.

17. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| < 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 0$. Prove that there exists some $c \in (-3, 3)$ such that $g(c) \cdot g''(c) < 0$.

18. If

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 & + & 3a \\ 3b^2 & + & 3b \\ 3c^2 & + & 3c \end{bmatrix},$$

$f(x)$ is a quadratic function and its maximum value occurs at a point V . A is a point of intersection of $y = f(x)$ with x -axis and point B is such that chord AB subtends a right angle at V . Find the area enclosed by $f(x)$ and chord AB .