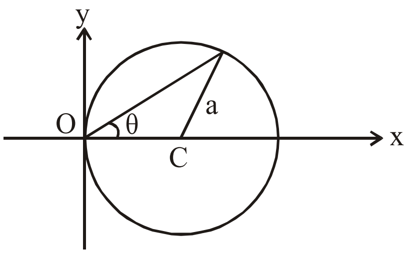
PART A – PHYSICS

1. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The center of circle is marked by 'C. The angular momentum from the origin O can be written as :-



(1) va cos 2θ
(2) va (1 + cos θ)
(3) va (1 + cos 2θ)
(4) va

Ans. (3)

Sol. (v cos θ) × 2 turns

2cos va –1 = cos 2w

2rv cos²v

va (1 + cos 2θ)

2. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly :-

(1) 4.02 V/m (2) 2.68 V/m
(3) 5.36 V/m (4) 1.34 V/m

Ans. (2)

Sol. $I = \langle u \rangle c = \dfrac{P}{4\pi r^2}$

$$\frac{\epsilon_0 \epsilon_0^2}{2} C = \frac{P}{4\pi r^2}$$

$$\epsilon_0^2 = \frac{P}{2\pi r^2 \epsilon_0 C}$$

$$\epsilon_0 = \sqrt{\frac{P}{2\pi r^2 \epsilon_0 C}} = \sqrt{\frac{3}{2\pi (5)^2 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$$

ϵ_0 = 2.68 volt/metre

3. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is :-

(1) 25°C (2) 20°C
(3) 10°C (4) 15°C

Ans. (3)

Sol. $\dfrac{10}{10} = K\,[55 - T]$

$$\frac{8}{10} = K\,[46 - T]$$

T = 10°C

for average, interval should be small.

$$\frac{60-50}{10} = K\,[55 - T]$$

$$\frac{18}{20} = K\,[51 - T]$$

$$\frac{10 \times 20}{10 \times 18} = \frac{55 - T}{51 - T}$$

510 – 101 = 55 × 9 – 91

15 = 1

4. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of some other wavelength. If wavelength of red light is 6600 Å , then wavelength of first maximum will be :-

(1) 5500 Å (2) 3300 Å
(3) 6600 Å (4) 4400 Å

Ans. (4)

Sol. $\dfrac{\lambda_R b}{a} = \dfrac{3}{2} \lambda \dfrac{b}{a}$

$$\lambda = \frac{2\lambda_R}{3} = \frac{2}{3} \times 6600$$

λ = 4400 Å

5. A 4 g bullet is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately :-

(1) 0.758 m (2) 0.19 m
(3) 0.569 m (4) 0.379 m

Ans. (4)

Sol. $\frac{4}{1000}\times 300=\frac{0.8}{10}v$

$v = \frac{3}{2} = 1.5m/s$

$v^2 - Cl^2 = 2as$

$\frac{g^3}{4} = 2 \times 3 \times 5$

$\frac{3}{8} = 5 = 0.375$ m

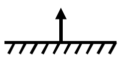
6. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground, of frequency nearly : (Speed of sound =343 m/s)

(1) 2400 Hz (2) 2500 Hz

(3) 1800 Hz (4) 2150 Hz

Ans. (2)

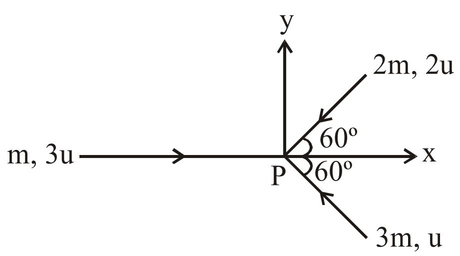
Sol. $v\downarrow$ O source $\left|\frac{343}{343+v}\right|1800 = 2150$



$343 = 1.2 \times 343 + 1.2v = 57.16$

$\left|\frac{343+57}{343}\right|\times 2150$

7. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u, and u respectively as shown in figure. The three masses collide at the same point at P and stick together. The velocity of resulting mass will be :-



(1) $\frac{u}{12}\left(-\hat{i}-\sqrt{3}\,\hat{j}\right)$ (2) $\frac{u}{12}\left(-\hat{i}+\sqrt{3}\,\hat{j}\right)$

(3) $\frac{u}{12}\left(\hat{i}-\sqrt{3}\,\hat{j}\right)$ (4) $\frac{u}{12}\left(\hat{i}+\sqrt{3}\,\hat{j}\right)$

Ans. (1)

Sol. $p_i = p_y$

$m3u\hat{i} + 3m\,[-u\cos 60°\hat{i} + u\sin 60°\hat{j}]$

$+ 2m\,[-2u\cos 60°\hat{i} - 2u\sin 60°\hat{j}]$

$3mu\hat{i} + 3m\left[-\frac{v}{2}\hat{i}+\frac{0\sqrt{3}}{2}\hat{j}\right]+2m\left[-\frac{2u}{2}\hat{i}-\frac{2u\sqrt{3}}{2}\hat{j}\right]$

$3u - \frac{3v}{2}-2u$ $\frac{3\sqrt{3}}{2}\hat{j}-2\sqrt{3}$

$-\frac{u}{2}$ $-\frac{\sqrt{3}}{2}\hat{j}$

8. For sky wave propagation, the radio waves must have a frequency range in between :-

(1) 45 MHz to 50 MHz

(2) 35 MHz to 40 MHz

(3) 1 MHz to 2 MHz

(4) 5 MHz to 25 MHz

Ans. (4)

9. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potentiometer wire. The potential difference across the ends of resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of voltmeter will be :-

(1) – 0.15 volt (2) 0.5 volt

(3) 0.15 volt (4) – 0.05 volt

Ans. (4)

Sol. Potential gradient

$\eta = \frac{1.1}{440}$

$V_{actual} = \frac{1.1}{440}\times 220$

$= 0.55$ volt

Error = 0.5 – 0.55

= – 0.05 volt

10. A piece of bone of an animal from a ruin is found to have ^{14}C activity of 12 disintegrations per minute per gm of its carbon content. The ^{14}C activity of a living animal is 16 disintegrations per minute per gm. How long ago nearly did the animal die ? (Given half life of ^{14}C is $t_{1/2}$ = 5760 years):-

(1) 3291 years (2) 1672 years

(3) 4453 year (4) 2391 years

Ans. (4)

Sol. $R = \dfrac{R_0}{2^{t/T_H}}$

$2^{t/T_H} = \dfrac{R_0}{R} = \dfrac{16}{12} = \dfrac{4}{3}$

$\dfrac{t}{T_H} = \log\dfrac{4}{3}$

$t = \dfrac{T_H}{\log 2}\ [2 \log 2 - \log 3]$

$= \dfrac{5760\text{ y }[2 \times 0.30 - 0.48]}{0.3010}$

$= \dfrac{5760\text{ y} \times 0.12}{0.3} \approx 2391\text{ y}$

11. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation :

$K(x) = K_0 + \lambda x$ (λ = a constant) The capacitance C, of this capacitor, would be related to its 'vacuum' capacitance C_0 as per the relation :-

(1) $C = \dfrac{\lambda}{d.\ell n(1 + K_0/\lambda d)}C_0$

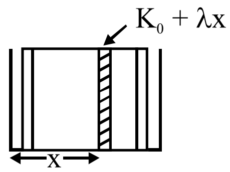
(2) $C = \dfrac{\lambda}{d.\ell n(1 + K_0\lambda d)}C_0$

(3) $C = \dfrac{\lambda d}{\ell n(1 + K_0\lambda d)}C_0$

(4) $C = \dfrac{\lambda d}{\ell n(1 + \lambda d/K_0)}C_0$

Ans. (4)

Sol.



$d_C = \dfrac{(K_0 + \lambda_x)A}{dx}$

$\int \dfrac{1}{d_C} = \int_0^{\phi} \dfrac{dx}{A \cdot (K_0 + \lambda x)}$

$C_0 = \dfrac{\varepsilon_0 A}{d} \quad \dfrac{1}{\lambda} \int_{K_0}^{K_0+\lambda d} \dfrac{dt}{At}$

$K_0 + \lambda x = t \qquad \lambda dx = dt$

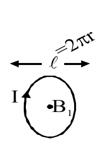
$\lambda A \qquad \dfrac{1}{\lambda A}\ell n\left[\dfrac{K_0 + \lambda d}{K_0}\right]$

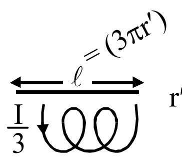
12. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current (I) passes through it. The second wire is bent into a coil with three identical loops adjacent to each other and produces magnetic field B_2 at the centre of the loops when current 1/3 passes through it. The ratio $B_1 : B_2$ is :-

(1) 1 : 9 (2) 1 : 3 (3) 9 : 1 (4) 1 : 1

Ans. (2)

Sol. $r = \dfrac{\ell}{2\pi}$





$r' = \dfrac{\ell}{6\pi}$

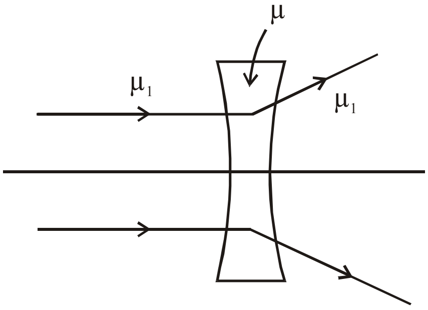
$B_1 = \dfrac{\mu_0 I}{2\pi\ell} \times 2\pi \qquad B_2 = 3 \cdot \dfrac{\mu_0 I/3}{2\pi\ell} \cdot 6\pi$

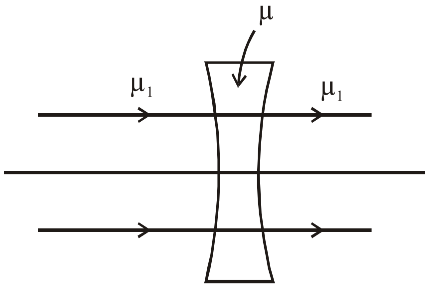
$= \dfrac{\mu_0 I}{\ell} \qquad = 3 \cdot \dfrac{\mu_0 I}{\ell} \cdot \dfrac{1}{3} \cdot 3$

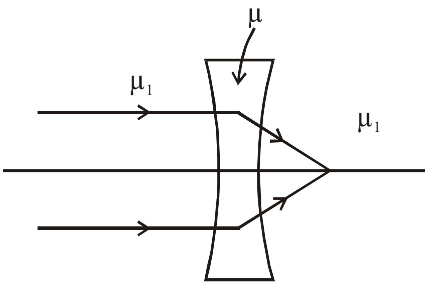
$= 3$

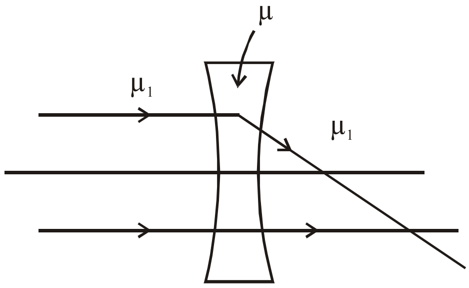
$B_1 : B_2 = 1 : 3$

13. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is :-

(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. bc = $\mu_1 > \mu$

So, option (3) is correct.

14. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator ?

(1) 27 s (2) 45 s (3) 37 s (4) 24 s

Ans. (4)

Sol. $60 = \dfrac{d}{v_{man}}$

$40 = \dfrac{d}{v_{es}}$

$t = \dfrac{d}{v_{man} + v_{es}}$

$\dfrac{1}{t} = \dfrac{v_{man}}{d} + \dfrac{v_{es}}{d} = \dfrac{1}{60} + \dfrac{1}{40}$

t = 24 sec

15. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be :-

(1) 70% and –10ºC (2) 50% and –20ºC

(3) 40% and –33ºC (4) 20% and –43ºC

Ans. (3)

Sol. $Q_1 = 1000$ J

$Q_2 = 600$

$\eta = 1 - \dfrac{Q_2}{Q_1}$

$= 40\%$

$\dfrac{T_1}{T_2} = \dfrac{Q_1}{Q_2}$

$\dfrac{400}{T_2} = \dfrac{1000}{600}$

$T_2 = 240\text{ K} - 273 = -33°C$

16. A spring of unstretched length ℓ has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is :-

(1) $\dfrac{1}{6}mv^2$ (2) mv^2 (3) $\dfrac{1}{3}mv^2$ (4) $\dfrac{1}{2}mv^2$

Ans. (1)

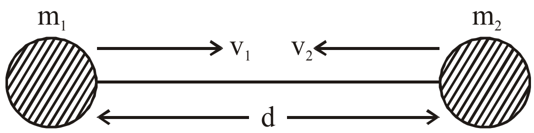
Sol. $KE = \int \frac{1}{2} dm\left(\frac{v}{\ell}x\right)^2$

$= \frac{1}{2}\int \frac{m}{\ell} dx \frac{v^2}{\ell^2} x^2$

$= \frac{mv^2}{2\ell^2}\left(\frac{x^3}{3}\right)_0^\ell$

$= \frac{mv^2}{2\ell^3} \times \frac{\ell^3}{3}$

$= \frac{mv^2}{6}$

17. 

Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd' ? (Speed of m_1 is v_1 and that of m_2 is v_2):-

(1) $v_1 = m_2\sqrt{\frac{2G}{m_1}}$

$v_2 = m_1\sqrt{\frac{2G}{m_2}}$

(2) $v_1 = v_2$

(3) $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(4) $v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

Ans. (4)

Sol. From M.E conservation

$O = -\frac{GM_1M_2}{d} + KE$

$KE = \frac{GM_1M_2}{d}$

Since momentum is constant

So $KE \propto \frac{1}{m}$

K.E. of $m_1 = \left(\frac{m_2}{m_1+m_2}\right)\frac{GM_1M_2}{d} = \frac{1}{2}m_1v_1^2$

$v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

KE of $m_2 = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{GM_1M_2}{d}\right) = \frac{1}{2}m_2v_2^2$

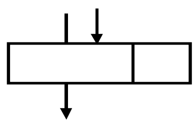
$v = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

18. Steel ruptures when a shear of 3.5×10^8 Nm^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly :-

(1) 3.3×10^4 N (2) 1.4×10^4 N

(3) 1.1×10^4 N (4) 2.7×10^4 N

Ans. (1)

Sol. $\frac{F}{A}$ = stress = 3.5×10^8 N/m^2 

$A = 2\pi rt$

$= 2\pi \times \frac{1}{2} \times 0.3$

$= 0.3\ \pi \times 10^{-4}\ m^2$

$F = A \times$ stress

$= 0.3\pi \times 10^{-4} \times 3.5 \times 10^8$

$= 1.05\ \pi \times 10^4$

$= 3.3 \times 10^4$ N

19. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is :-

(1) $\frac{e^2}{2\pi\epsilon_0 G m_e^2}$ (m_e = mass of electron)

(2) $\frac{ch}{2\pi\epsilon_0^2}$

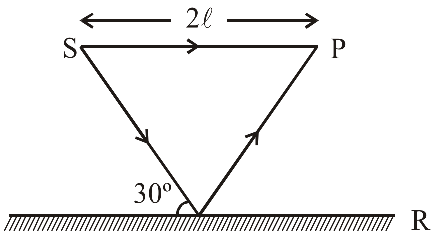
(3) $\frac{2\pi\sqrt{\mu_0\epsilon_0}}{ce^2}\frac{h}{G}$

(4) $\frac{\mu_0\epsilon_0}{c^2}\frac{G}{he^2}$

Ans. (1)

Sol. A dimensionless & unitless terms will have same value is all systems.

20. Interference pattern is observed at 'P' due to superimposition of two rays coming out from a source 'S' as shown in the figure. The value of ℓ for which maxima is obtained at 'P' is (R is perfect reflecting surface) :-



(1) $\ell = \frac{(2n-1)\lambda}{2(\sqrt{3}-1)}$ (2) $\ell = \frac{(2n-1)\lambda}{\sqrt{3}-1}$

(3) $\ell = \frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$ (4) $\ell = \frac{2n\lambda}{\sqrt{3}-1}$

Ans. (3)

Sol. $x\cos 30° = \ell$

$x\frac{\sqrt{3}}{2} = \ell$ $\qquad x = \frac{\sqrt{2\ell}}{\sqrt{3}}$

$$\frac{2\ell}{\sqrt{3}} + \frac{2\ell}{\sqrt{3}} + \frac{\lambda}{2} - 2\ell = n\lambda$$

$$n\lambda - \frac{\lambda}{2}$$

$$2\ell\left(\frac{2}{\sqrt{3}} - 1\right) = (2n-1)\frac{\lambda}{2}$$

$$2\ell\left(2-\sqrt{3}\right) = \frac{(2n-1)\lambda\sqrt{3}}{2}$$

21. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is :-

(1) F_2 (2) O_2
(3) Cl_2 (4) H_2

Ans. (4)

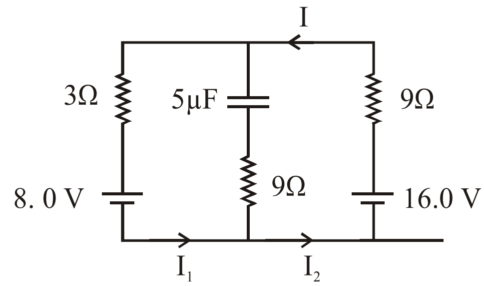
Sol. $\frac{1}{3}\rho v_{rms}^2 = p$ $\qquad pv = \frac{MRT}{M_0}$

$v_{rms} = \sqrt{\frac{3p}{\rho}}$ $\qquad \frac{p}{\rho} = \frac{RT}{M_0}$

$$1930 = \sqrt{\frac{3RT}{M_0}}$$

$$\frac{3\times 8.314\times 300}{1930\times 1930}$$

22. The circuit shown here has two batteries of 8.0V and 16.0 V and three resistors 3Ω, 9Ω and 9Ω and a capacitor 5.0 μF



How much is the current I in the circuit in steady state ?

(1) 1.6 A (2) 2.5 A (3) 0.67 A (4) 0.25 A

Ans. (3)

Sol. $\frac{8}{12}\cdot\frac{2}{3} = 0.67$

23. A beam of light has two wavelengths 4972 Å and 6216 Å with a total intensity of $3.6\text{x}10^{-3}$ Wm^{-2} equally distributed among the two wavelengths. The beam falls normally on an area of 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photo electrons liberated in 2s is approximately :-

(1) 15×10^{11} (2) 6×10^{11}
(3) 9×10^{11} (4) 11×10^{11}

Ans. (3)

Sol. $\dfrac{I\Delta t\lambda}{hc}$

$$\frac{1.8\times10^{-3}\times1\times10^{-4}}{2.5\times10^{-10}}\cdot10^{3}$$

$$\frac{18}{25}\times10^{3}\ \text{photon}$$

$$\frac{1.8\times10^{-3}\times1\times10^{-4}}{2\times10^{-10}}$$

$$\frac{12400}{6216}\cdot\frac{1.8}{2}\times10^{3}$$

24. A positive charge 'q' of mass 'm' is moving along the +x axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then :-

(1) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{2v}$

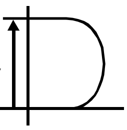
(2) $B = \dfrac{mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

(3) $B = \dfrac{2mv}{qd}$ and $\Delta t = \dfrac{\pi d}{v}$

(4) $B = \dfrac{mv}{2qd}$ and $\Delta t = \dfrac{\pi d}{2v}$

Ans. (1)

Sol. $\dfrac{1.8\times10^{-3}\times10^{-4}}{2.5\times1.6\times10^{-19}}$

R=d/2


$$\frac{mv^2}{r}=qvB \qquad \frac{18}{2.5\times1.6}10^{12}$$

$$r = \frac{mv}{qB}$$

$$\frac{d}{2}=\frac{mv}{qB} \Rightarrow B = \frac{2mv}{qd}$$

$$\frac{\pi d}{2v}$$

25. A spherically symmetric charge distribution is characterised by a charge density having the following variation

$\rho(r)=\rho_0\left(1-\dfrac{r}{R}\right)$ for $r < R$

$\rho(r)=0$ for $r \geq R$

Where r is the distance from the centre of the charge distribution and ρ_0 is a constant. The electric field at an internal point ($r < R$) is :-

(1) $\dfrac{\rho_0}{12\epsilon_0}\left(\dfrac{r}{3}-\dfrac{r^2}{4R}\right)$ (2) $\dfrac{\rho_0}{3\epsilon_0}\left(\dfrac{r}{3}-\dfrac{r^2}{4R}\right)$

(3) $\dfrac{\rho_0}{4\epsilon_0}\left(\dfrac{r}{3}-\dfrac{r^2}{4R}\right)$ (4) $\dfrac{\rho_0}{\epsilon_0}\left(\dfrac{r}{3}-\dfrac{r^2}{4R}\right)$

Ans. (4)

Sol. $\oint \varepsilon.ds=\dfrac{\Sigma a}{\varepsilon_0}$

$$\varepsilon(4\pi r^2) = \rho\cdot\int_0^r\left(1-\frac{r}{R}\right)4\pi r^2dr$$

$$4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^2}{4R}\right]_0^q = \varepsilon(4\pi r^2)$$

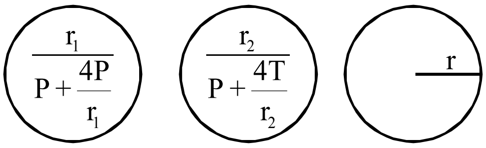
$$\varepsilon = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$$

26. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S the change in total surface area, T is the surface tension and P atmospheric pressure, which of the following relation is **correct** ?

(1) 3PV + 2ST = 0 (2) 4PV + 3ST = 0

(3) 2PV + 3ST = 0 (4) 3PV + 4ST = 0

Ans. (4)

Sol. 

$$\left(P+\frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(P+\frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3$$

$$= \left(P+\frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$PV + \frac{4T}{3}(4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2)$

$3pv + 4TS$

27. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of :-

(1) 0.9 eV to 1.6 eV (2) 0.5eVto 0.8eV

(3) 0.1 eV to 0.4 eV (4) 1.7eV to 3.0eV

Ans. (4)

Sol. $\frac{12400}{3000}$ to $\frac{12400}{7800}$ or 3.26eV to 1.6eV

28. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants ? :-

(1) $-bx$ (2) $a - bx + cx^2$

(3) $a + bx - cx^2$ (4) bx^2

Ans. (1)

29. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is :-

(1) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

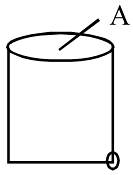
(2) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(3) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

(4) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$

Ans. (2)

Sol. $-Adh = a\cdot\sqrt{2gh}.dt$



$$\int_H^0 \frac{dh}{\sqrt{2gh}} = -\frac{q}{A}\int_0^t dt$$

$$\Rightarrow \frac{2}{\sqrt{2g}}\sqrt{h^{-1/2+1}} \qquad \frac{2}{\sqrt{2g}}\left|\sqrt{h}\right| = \frac{q}{A}t$$

$$t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$

30. A sinusoidal voltage V(t)=100 sin (500t) is applied across a pure inductance of L = 0.02 H. The current through the coil is : :-

(1) 10 cos (500t) (2) – 10 cos (500t)

(3) 10 sin (500t) (4) – 10 sin (500t)

Ans. (2)

Sol. $x_L = \omega L$

$$= \frac{2\pi\times 500}{500\times\frac{.02}{100}}$$

$i = 10 \sin[\omega t - \pi/2]$

$= -10 \cos 500t \qquad \Rightarrow 10$