

# JEE(Advanced) – 2018 TEST PAPER - 1 WITH SOLUTIONS

(Exam Date: 21-05-2018)

## PART-1 : MATHEMATICS

### SECTION-1

1. For a non-zero complex number  $z$ , let  $\arg(z)$  denotes the principal argument with  $-\pi < \arg(z) \leq \pi$ . Then, which of the following statement(s) is (are) **FALSE** ?

(A)  $\arg(-1 - i) = \frac{\pi}{4}$ , where  $i = \sqrt{-1}$

(B) The function  $f : \mathbb{R} \rightarrow (-\pi, \pi]$ , defined by  $f(t) = \arg(-1 + it)$  for all  $t \in \mathbb{R}$ , is continuous at all points of  $\mathbb{R}$ , where  $i = \sqrt{-1}$

(C) For any two non-zero complex numbers  $z_1$  and  $z_2$ ,  $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$  is an integer multiple of  $2\pi$

(D) For any three given distinct complex numbers  $z_1, z_2$  and  $z_3$ , the locus of the point  $z$  satisfying the condition

$$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi, \text{ lies on a straight line}$$

**Ans. (A,B,D)**

**Sol.** (A)  $\arg(-1 - i) = -\frac{3\pi}{4}$ ,

$$(B) f(t) = \arg(-1 + it) = \begin{cases} \pi - \tan^{-1}(t), & t \geq 0 \\ -\pi + \tan^{-1}(t), & t < 0 \end{cases}$$

Discontinuous at  $t = 0$ .

$$(C) \arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$$

$$= \arg z_1 - \arg(z_2) + 2n\pi - \arg(z_1) + \arg(z_2) = 2n\pi.$$

$$(D) \arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$$

$$\Rightarrow \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \text{ is real.}$$

$\Rightarrow z, z_1, z_2, z_3$  are concyclic.

2. In a triangle PQR, let  $\angle PQR = 30^\circ$  and the sides PQ and QR have lengths  $10\sqrt{3}$  and 10, respectively.

Then, which of the following statement(s) is (are) TRUE ?

(A)  $\angle QPR = 45^\circ$

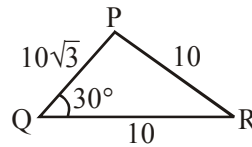
(B) The area of the triangle PQR is  $25\sqrt{3}$  and  $\angle QRP = 120^\circ$

(C) The radius of the incircle of the triangle PQR is  $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is  $100\pi$ .

Ans. (B,C,D)

Sol.  $\cos 30^\circ = \frac{(10\sqrt{3})^2 + (10)^2 - (PR)^2}{2 \times 10\sqrt{3} \times 10}$



$$\Rightarrow PR = 10$$

$$\because QR = PR \Rightarrow \angle PQR = \angle QPR$$

$$\angle QPR = 30^\circ$$

$$(B) \text{ area of } \Delta PQR = \frac{1}{2} \times 10\sqrt{3} \times 10 \times \sin 30^\circ = \frac{1}{2} \times 10 \times 10\sqrt{3} \times \frac{1}{2}$$

$$= 25\sqrt{3}$$

$$\angle QRP = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$(C) r = \frac{\Delta}{S} = \frac{25\sqrt{3}}{\left(\frac{10+10+10\sqrt{3}}{2}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}}$$

$$= 5\sqrt{3} \cdot (2 - \sqrt{3}) = 10\sqrt{3} - 15$$

$$(D) R = \frac{a}{2 \sin A} = \frac{10}{2 \sin 30^\circ} = 10$$

$$\therefore \text{Area} = \pi R^2 = 100\pi$$

3. Let  $P_1 : 2x + y - z = 3$  and  $P_2 : x + 2y + z = 2$  be two planes. Then, which of the following statement(s) is (are) TRUE ?

(A) The line of intersection of  $P_1$  and  $P_2$  has direction ratios 1, 2, -1

(B) The line  $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$  is perpendicular to the line of intersection of  $P_1$  and  $P_2$

(C) The acute angle between  $P_1$  and  $P_2$  is  $60^\circ$

(D) If  $P_3$  is the plane passing through the point (4, 2, -2) and perpendicular to the line of intersection of  $P_1$  and  $P_2$ , then the distance of the point (2, 1, 1) from the plane  $P_3$  is  $\frac{2}{\sqrt{3}}$

**Ans. (C,D)**

**Sol.** D.C. of line of intersection (a, b, c)

$$\Rightarrow 2a + b - c = 0$$

$$a + 2b + c = 0$$

$$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$$

$\therefore$  D.C. is (1, -1, 1)

(B)  $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$

$$\Rightarrow \frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$$

$\Rightarrow$  lines are parallel.

(C) Acute angle between  $P_1$  and  $P_2 = \cos^{-1}\left(\frac{2 \times 1 + 1 \times 2 - 1 \times 1}{\sqrt{6}\sqrt{6}}\right)$

$$= \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

(D) Plane is given by  $(x - 4) - (y - 2) + (z + 2) = 0$

$$\Rightarrow x - y + z = 0$$

$$\text{Distance of (2, 1, 1) from plane} = \frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

4. For every twice differentiable function  $f : \mathbb{R} \rightarrow [-2, 2]$  with  $(f(0))^2 + (f'(0))^2 = 85$ , which of the following statement(s) is (are) TRUE ?

(A) There exist  $r, s \in \mathbb{R}$ , where  $r < s$ , such that  $f$  is one-one on the open interval  $(r, s)$

(B) There exists  $x_0 \in (-4, 0)$  such that  $|f'(x_0)| \leq 1$

(C)  $\lim_{x \rightarrow \infty} f(x) = 1$

(D) There exists  $\alpha \in (-4, 4)$  such that  $f(\alpha) + f''(\alpha) = 0$  and  $f'(\alpha) \neq 0$

**Ans. (A,B,D)**

**Sol.**  $f(x)$  can't be constant throughout the domain. Hence we can find  $x \in (r, s)$  such that  $f(x)$  is one-one  
option (A) is true.

$$\text{Option (B): } |f'(x_0)| = \left| \frac{f(0) - f(-4)}{4} \right| \leq 1 \quad (\text{LMVT})$$

Option (C):  $f(x) = \sin(\sqrt{85x})$  satisfies given condition

$$\text{but } \lim_{x \rightarrow \infty} \sin(\sqrt{85x}) \text{ D.N.E.}$$

$\Rightarrow$  Incorrect

$$\text{Option (D): } g(x) = f^2(x) + (f'(x))^2$$

$$|f'(x_1)| \leq 1 \quad (\text{by LMVT})$$

$$|f(x_1)| \leq 2 \quad (\text{given})$$

$$\Rightarrow g(x_1) \leq 5 \quad \exists x_1 \in (-4, 0)$$

$$\text{Similarly } g(x_2) \leq 5 \quad \exists x_2 \in (0, 4)$$

$$g(0) = 85 \quad \Rightarrow g(x) \text{ has maxima in } (x_1, x_2) \text{ say at } \alpha.$$

$$g'(\alpha) = 0 \quad \& \quad g(\alpha) \geq 85$$

$$2f'(\alpha) (f(\alpha) + f''(\alpha)) = 0$$

$$\text{If } f'(\alpha) = 0 \Rightarrow g(\alpha) = f^2(\alpha) \geq 85 \text{ Not possible}$$

$$\Rightarrow f(\alpha) + f''(\alpha) = 0 \quad \exists \alpha \in (x_1, x_2) \in (-4, 4)$$

option (D) correct.

**5.** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be two non-constant differentiable functions. If  $f'(x) = (e^{(f(x)-g(x))})g'(x)$  for all  $x \in \mathbb{R}$ , and  $f(1) = g(2) = 1$ , then which of the following statement(s) is (are) TRUE ?

$$(A) f(2) < 1 - \log_e 2$$

$$(B) f(2) > 1 - \log_e 2$$

$$(C) g(1) > 1 - \log_e 2$$

$$(D) g(1) < 1 - \log_e 2$$

**Ans. (B,C)**

$$\text{Sol. } f'(x) = e^{(f(x)-g(x))} g'(x) \quad \forall x \in \mathbb{R}$$

$$\Rightarrow e^{-f(x)} \cdot f'(x) - e^{-g(x)} g'(x) = 0$$

$$\Rightarrow \int (e^{-f(x)} f'(x) - e^{-g(x)} g'(x)) dx = C$$

$$\Rightarrow -e^{-f(x)} + e^{-g(x)} = C$$

$$\Rightarrow -e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$$

$$\Rightarrow -\frac{1}{e} + e^{-g(1)} = -e^{-f(2)} + \frac{1}{e}$$

$$\Rightarrow e^{-f(2)} + e^{-g(1)} = \frac{2}{e}$$

$$\therefore e^{-f(2)} < \frac{2}{e} \text{ and } e^{-g(1)} < \frac{2}{e}$$

$$\Rightarrow -f(2) < \ln 2 - 1 \text{ and } -g(1) < \ln 2 - 1$$

$$\Rightarrow f(2) > 1 - \ln 2 \text{ and } g(1) > 1 - \ln 2$$

6. Let  $f : [0, \infty) \rightarrow \mathbb{R}$  be a continuous function such that  $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$

for all  $x \in [0, \infty)$ . Then, which of the following statement(s) is (are) TRUE ?

(A) The curve  $y = f(x)$  passes through the point (1, 2)

(B) The curve  $y = f(x)$  passes through the point (2, -1)

(C) The area of the region  $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$  is  $\frac{\pi-2}{4}$

(D) The area of the region  $\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1-x^2}\}$  is  $\frac{\pi-1}{4}$

**Ans. (B,C)**

**Sol.**  $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t) dt$

$$\Rightarrow e^{-x} f(x) = e^{-x} (1 - 2x) + \int_0^x e^{-t} f(t) dt$$

Differentiate w.r.t. x.

$$-e^{-x} f(x) + e^{-x} f'(x) = -e^{-x} (1 - 2x) + e^{-x} (-2) + e^{-x} f(x)$$

$$\Rightarrow -f(x) + f'(x) = -(1 - 2x) - 2 + f(x).$$

$$\Rightarrow f'(x) - 2f(x) = 2x - 3$$

Integrating factor =  $e^{-2x}$ .

$$f(x) \cdot e^{-2x} = \int e^{-2x} (2x - 3) dx$$

$$= (2x - 3) \int e^{-2x} dx - \int \left( (2) \int e^{-2x} dx \right) dx$$

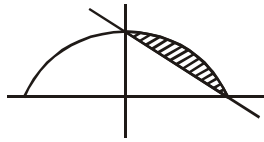
$$= \frac{(2x - 3)e^{-2x}}{-2} - \frac{e^{-2x}}{2} + c$$

$$f(x) = \frac{2x - 3}{-2} - \frac{1}{2} + ce^{2x}$$

$$f(0) = \frac{3}{2} - \frac{1}{2} + c = 1 \Rightarrow c = 0$$

$$\therefore f(x) = 1 - x$$

$$\text{Area} = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$$



## SECTION-2

7. The value of  $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$  is —

**Ans. (8)**

$$\text{Sol. } \log_2 9^{\frac{2}{\log_2(\log_2 9)}} \times 7^{\frac{1/2}{\log_4 7}}$$

$$= (\log_2 9)^{2 \log_2^2 9} \times 7^{\frac{1}{2} \log_7 4}$$

$$= 4 \times 2 = 8$$

8. The number of 5 digit numbers which are divisible by 4, with digits from the set  $\{1, 2, 3, 4, 5\}$  and the repetition of digits is allowed, is —

**Ans. (625)**

**Sol.** Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$  Total No. of digits

$$= 5 \times 5 \times 5 \times 5 = 625$$

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ....., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ....., Then, the number of elements in the set  $X \cup Y$  is —

**Ans. (3748)**

**Sol.** X : 1, 6, 11, ....., 10086

Y : 9, 16, 23, ....., 14128

$X \cap Y$  : 16, 51, 86, .....

Let  $m = n(X \cap Y)$

$$\therefore 16 + (m - 1) \times 35 \leq 10086$$

$$\Rightarrow m \leq 288.71$$

$$\Rightarrow m = 288$$

$$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$= 2018 + 2018 - 288 = 3748$$

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i\right)$$

lying in the interval  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  is —

(Here, the inverse trigonometric functions  $\sin^{-1}x$  and  $\cos^{-1}x$  assume value in  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and  $[0, \pi]$ , respectively.)

Ans. (2)

Sol. 
$$\sum_{i=1}^{\infty} x^{i+1} = \frac{x^2}{1-x}$$

$$\sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i = \frac{x}{2-x}$$

$$\sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i = \frac{-x}{2+x}$$

$$\sum_{i=1}^{\infty} (-x)^i = \frac{-x}{1+x}$$

To have real solutions

$$\sum_{i=1}^{\infty} x^{i+1} - x \sum_{i=1}^{\infty} \left(\frac{x}{2}\right)^i = \sum_{i=1}^{\infty} \left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty} (-x)^i$$

$$\frac{x^2}{1-x} - \frac{x^2}{2-x} = \frac{-x}{2+x} + \frac{x}{1+x}$$

$$x(x^3 + 2x^2 + 5x - 2) = 0$$

$$\therefore x = 0 \text{ and let } f(x) = x^3 + 2x^2 + 5x - 2$$

$$f\left(\frac{1}{2}\right) \cdot f\left(-\frac{1}{2}\right) < 0$$

Hence two solutions exist

11. For each positive integer  $n$ , let

$$y_n = \frac{1}{n}(n+1)(n+2)\dots(n+n)^{1/n}$$

For  $x \in \mathbb{R}$ , let  $[x]$  be the greatest integer less than or equal to  $x$ . If  $\lim_{n \rightarrow \infty} y_n = L$ , then the value of  $[L]$

is —

**Ans. (1)**

**Sol.**  $y_n = \left\{ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \dots \left(1 + \frac{n}{n}\right) \right\}^{\frac{1}{n}}$

$$y_n = \prod_{r=1}^n \left(1 + \frac{r}{n}\right)^{1/n}$$

$$\log y_n = \frac{1}{n} \sum_{r=1}^n \ell n \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \log y_n = \lim_{x \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \ell n \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \log L = \int_0^1 \ell n(1+x) dx$$

$$\Rightarrow \log L = \log \frac{4}{e}$$

$$\Rightarrow L = \frac{4}{e}$$

$$\Rightarrow [L] = 1$$

**12.** Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors such that  $\vec{a} \cdot \vec{b} = 0$ . For some  $x, y \in \mathbb{R}$ , let  $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$ . If  $|\vec{c}| = 2$  and the vector  $\vec{c}$  is inclined at the same angle  $\alpha$  to both  $\vec{a}$  and  $\vec{b}$ , then the value of  $8\cos^2 \alpha$  is —

**Ans. (3)**

**Sol.**  $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$

$$\vec{c} \cdot \vec{a} = x \text{ and } x = 2\cos\alpha$$

$$\vec{c} \cdot \vec{b} = y \text{ and } y = 2\cos\alpha$$

$$\text{Also, } |\vec{a} \times \vec{b}| = 1$$

$$\therefore \vec{c} = 2\cos(\vec{a} + \vec{b}) + \vec{a} \times \vec{b}$$

$$\vec{c}^2 = 4\cos^2 \alpha (\vec{a} + \vec{b})^2 + (\vec{a} \times \vec{b})^2 + 2\cos\alpha (\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$$

$$4 = 8\cos^2 \alpha + 1$$

$$8\cos^2 \alpha = 3$$

**13.** Let  $a, b, c$  be three non-zero real numbers such that the equation

$$\sqrt{3}a \cos x + 2b \sin x = c, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

has two distinct real roots  $\alpha$  and  $\beta$  with  $\alpha + \beta = \frac{\pi}{3}$ . Then the value of  $\frac{b}{a}$  is —



**Ans. (0.5)**

**Sol.**  $\sqrt{3} \cos x + \frac{2b}{a} \sin x = \frac{c}{a}$

Now,  $\sqrt{3} \cos \alpha + \frac{2b}{a} \sin \alpha = \frac{c}{a}$  ..... (1)

$$\sqrt{3} \cos \beta + \frac{2b}{a} \sin \beta = \frac{c}{a} \quad \dots (2)$$

$$\sqrt{3} [\cos \alpha - \cos \beta] + \frac{2b}{a} (\sin \alpha - \sin \beta) = 0$$

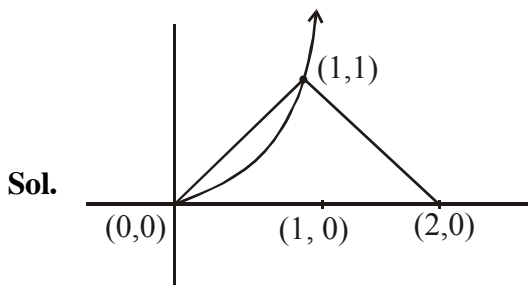
$$\sqrt{3} \left[ -2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) \right] + \frac{2b}{a} \left[ 2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) \right] = 0$$

$$-\sqrt{3} + 2\sqrt{3} \cdot \frac{b}{a} = 0$$

$$\frac{b}{a} = \frac{1}{2} = 0.5$$

- 14.** A farmer  $F_1$  has a land in the shape of a triangle with vertices at  $P(0, 0)$ ,  $Q(1, 1)$  and  $R(2, 0)$ . From this land, a neighbouring farmer  $F_2$  takes away the region which lies between the side  $PQ$  and a curve of the form  $y = x^n$  ( $n > 1$ ). If the area of the region taken away by the farmer  $F_2$  is exactly 30% of the area of  $\Delta PQR$ , then the value of  $n$  is —

**Ans. (4)**



$$\text{Area} = \int_0^1 (x - x^n) dx = \frac{3}{10}$$

$$\left[ \frac{x^2}{2} - \frac{x^{n+1}}{n+1} \right]_0^1 = \frac{3}{10}$$

$$\frac{1}{2} - \frac{1}{n+1} = \frac{3}{10} \quad \therefore n + 1 = 5$$

$$\Rightarrow n = 4$$

### SECTION-3

#### Paragraph "X"

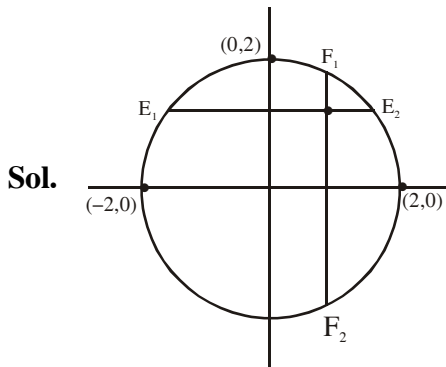
Let  $S$  be the circle in the  $xy$ -plane defined by the equation  $x^2 + y^2 = 4$ .

(There are two question based on Paragraph "X", the question given below is one of them)

- 15.** Let  $E_1E_2$  and  $F_1F_2$  be the chord of  $S$  passing through the point  $P_0(1, 1)$  and parallel to the  $x$ -axis and the  $y$ -axis, respectively. Let  $G_1G_2$  be the chord of  $S$  passing through  $P_0$  and having slop  $-1$ . Let the tangents to  $S$  at  $E_1$  and  $E_2$  meet at  $E_3$ , the tangents of  $S$  at  $F_1$  and  $F_2$  meet at  $F_3$ , and the tangents to  $S$  at  $G_1$  and  $G_2$  meet at  $G_3$ . Then, the points  $E_3$ ,  $F_3$  and  $G_3$  lie on the curve

- (A)  $x + y = 4$  (B)  $(x - 4)^2 + (y - 4)^2 = 16$   
 (C)  $(x - 4)(y - 4) = 4$  (D)  $xy = 4$

**Ans. (A)**



co-ordinates of  $E_1$  and  $E_2$  are obtained by solving  $y = 1$  and  $x^2 + y^2 = 4$

$$\therefore E_1(-\sqrt{3}, 1) \text{ and } E_2(\sqrt{3}, 1)$$

co-ordinates of  $F_1$  and  $F_2$  are obtained by solving

$$x = 1 \text{ and } x^2 + y^2 = 4$$

$$F_1(1, \sqrt{3}) \text{ and } F_2(1, -\sqrt{3})$$

Tangent at  $E_1$  :  $-\sqrt{3}x + y = 4$

Tangent at  $E_2$  :  $\sqrt{3}x + y = 4$

$$\therefore E_3(0, 4)$$

Tangent at  $F_1$  :  $x + \sqrt{3}y = 4$

Tangent at  $F_2$  :  $x - \sqrt{3}y = 4$

$$\therefore F_3(4, 0)$$

and similarly  $G_3(2, 2)$

$(0, 4)$ ,  $(4, 0)$  and  $(2, 2)$  lies on  $x + y = 4$

**PARAGRAPH "X"**

Let S be the circle in the xy-plane defined by the equation  $x^2 + y^2 = 4$

*(There are two questions based on Paragraph "X", the question given below is one of them)*

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve -

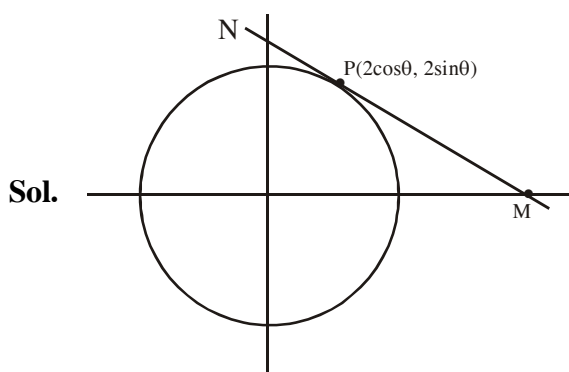
(A)  $(x + y)^2 = 3xy$

(B)  $x^{2/3} + y^{2/3} = 2^{4/3}$

(C)  $x^2 + y^2 = 2xy$

(D)  $x^2 + y^2 = x^2y^2$

**Ans. (D)**



Tangent at  $P(2\cos\theta, 2\sin\theta)$  is  $x\cos\theta + y\sin\theta = 2$

$M(2\sec\theta, 0)$  and  $N(0, 2\csc\theta)$

Let midpoint be  $(h, k)$

$h = \sec\theta, k = \csc\theta$

$$\frac{1}{h^2} + \frac{1}{k^2} = 1$$

$$\frac{1}{x^2} + \frac{1}{y^2} = 1$$

**PARAGRAPH "A"**

There are five students  $S_1, S_2, S_3$  and  $S_4$  and  $S_5$  in a music class and for them there are five sets  $R_1, R_2, R_3, R_4$  and  $R_5$  arranged in a row, where initially the seat  $R_i$  is allotted to the student  $S_i, i = 1, 2, 3, 4, 5$ . But, on the examination day, the five students are randomly allotted the five seats.

*(There are two questions based on Paragraph "A". the question given below is one of them)*

17. The probability that, on the examination day, the student  $S_1$  gets the previously allotted seat  $R_1$  and **NONE** of the remaining students gets the seat previously allotted to him/her is -

(A)  $\frac{3}{40}$

(B)  $\frac{1}{8}$

(C)  $\frac{7}{40}$

(D)  $\frac{1}{5}$

**Ans. (A)**

**Sol.** Required probability =  $\frac{4! \left( \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right)}{5!} = \frac{9}{120} = \frac{3}{40}$

**PARAGRAPH "A"**

There are five students  $S_1, S_2, S_3, S_4$  and  $S_5$  in a music class and for them there are five seats  $R_1, R_2, R_3, R_4$  and  $R_5$  arranged in a row, where initially the seat  $R_i$  is allotted to the student  $S_i, i = 1, 2, 3, 4, 5$ . But, on the examination day, the five students are randomly allotted the five seats.

*(There are two questions based on Paragraph "A", the question given below is one of them)*

- 18.** For  $i = 1, 2, 3, 4$ , let  $T_i$  denote the event that the students  $S_i$  and  $S_{i+1}$  do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event  $T_1 \cap T_2 \cap T_3 \cap T_4$  is-

- (A)  $\frac{1}{15}$                       (B)  $\frac{1}{10}$                       (C)  $\frac{7}{60}$                       (D)  $\frac{1}{5}$

**Ans. (C)**

**Sol.**  $n(T_1 \cap T_2 \cap T_3 \cap T_4) = \text{Total} - n(\bar{T}_1 \cup \bar{T}_2 \cup \bar{T}_3 \cup \bar{T}_4)$

$$= 5! - \left( {}^4C_1 4! 2! - \left( {}^3C_1 \cdot 3! 2! + {}^3C_1 3! 2! 2! \right) + \left( {}^2C_1 2! 2! + {}^4C_1 \cdot 2 \cdot 2! \right) - 2 \right)$$

$$= 14$$

$$\text{Probability} = \frac{14}{5!} = \frac{7}{60}$$