PART C – MATHEMATICS

11th

1. If X has a binomial distribution, B(n, p) with parameters n and p such that P(X = 2) = P(X = 3), then E(X), the mean of variable X, is

(1) 3– p (2) $\frac{p}{3}$

(3) $\frac{p}{2}$ (4) 2 – p

Ans. (1)

Sol. P(x = 2) = P (x = 3)

$${}^{n}C_{2}\ p^{2}(1-p)^{n-2} = {}^{n}C_{3}\ p^{3}(1-p)^{n-3}$$

$$\frac{(1-p)}{n-2} = \frac{p}{3} \Rightarrow n\,p = 3 - p$$

2. The integral $\int x\cos^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right)dx\,(x>0)$ is equal to

(1) – x + (1 + x^2) $\tan^{-1}$x + c

(2) x – (1 + x^2) $\tan^{-1}$x + c

(3) x – (1 + x^2) $\cot^{-1}$x + c

(4) – x + (1 + x^2) $\cot^{-1}$x + c

Ans. (1)

Sol. put $x = \tan\theta$ $\cos^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right) = \cos^{-1}(\cos 2\theta) = 2\theta$

$$\int \tan\theta\ (2\theta)\ \sec^{2}\theta\ d\theta$$

$$= 2\theta\ .\ \int \tan\theta\ \sec^{2}\theta\ d\theta\ -\ 2\int\left(\frac{d\theta}{d\theta}.\int \tan\theta \sec^{2}\theta\ d\theta\right)d\theta$$

$$= 2\theta\ .\frac{\tan^{2}\theta}{2} - 2\int \frac{\tan 2\theta}{2}\ d\theta$$

$$= \theta\ \tan^{2}\theta - \int(\sec^{2}\theta - 1)\ d\theta$$

$$= \theta\ \tan^{2}\theta - \tan\theta + \theta + C$$

$$= \tan^{-1}x\ .\ x^{2} - x + \tan^{-1}x + C$$

$$= -\,x + (1 + x^{2})\tan^{-1}x + C$$

3. Left f be an odd function defined on the set of real numbers such that for x ≥ 0,
f(x) = 3sinx + 4cosx

Then f(x) at $x = -\frac{11\pi}{6}$ is equal to

(1) $\frac{3}{2} - 2\sqrt{3}$ (2) $-\frac{3}{2} - 2\sqrt{3}$

(3) $-\frac{3}{2} + 2\sqrt{3}$ (4) $\frac{3}{2} + 2\sqrt{3}$

Ans. (1)

Sol. f(–x) = – f(x) as f(x) is odd function

$$f\left(\frac{-11\pi}{6}\right) = -\left[3\sin\left(\frac{+11\pi}{6}\right) + 4\cos\left(\frac{+11\pi}{6}\right)\right]$$

$$= -\left[3\sin\left(\frac{11\pi}{6}\right) + 4\cos\left(\frac{11\pi}{6}\right)\right]$$

$$= -\left[3\sin\left(2\pi - \frac{\pi}{6}\right) + 4\cos\left(2\pi - \frac{\pi}{6}\right)\right]$$

$$= +\ 3\ \sin \pi/6 - 4\ \cos\frac{\pi}{6}$$

$$= 3\times\frac{1}{2} - \frac{4\sqrt{3}}{2} = \frac{3}{2} - 2\sqrt{3}$$

4. The plane containing the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and parallel to the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{4}$ passes through the point

(1) (1, 0, 5)

(2) (0, 3, –5)

(3) (–1, – 3, 0)

(4) (1, – 2, 5)

Ans. (1)

Sol. Normal vector = $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 1 & 4 \end{vmatrix} = 5i - \hat{j} - \hat{k}$

point (1,2,3) lies in plane so equation of plane = 5 (x – 1) – 1 (y – 2) – 1 (z – 3) = 0

5x – y – z = 0

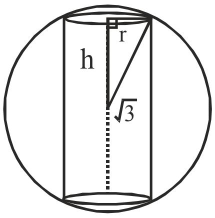
so option [1] is correct

5. The volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius $=\sqrt{3}$ is

(1) 4π (2) $\frac{4}{3}\sqrt{3}\pi$ (3) $\frac{8}{3}\sqrt{3}\pi$ (4) 2π

Ans. (1)

Sol. $h^2 + r^2 = 3$



$r^2 = 3 - h^2$ ...(1)

$\therefore$ $V = \pi r^2 . 2h$

$= 2\pi (r^2 h)$

$V = 2\pi (3h - h^3)$

$\frac{dr}{dh} = 0 \Rightarrow h^2 = 1 \Rightarrow h = 1$

$\therefore$ $r^2 = 3 - h^2$

$r^2 = 3 - 1 = 2$

So $V_{max} = 2\pi (2 \times 1)$

$= 4\pi$

6. The proposition ~(pv~q)v~(p∨q) is logically equivalent to

(1) ~p (2) ~q (3) p (4) q

Ans. (1)

p	q	~ q	PV(~ q)	pvq	~ (PV ~ q)	~ (pvq)	AvB
T	T	F	T	T	F	F	F
F	F	T	T	F	F	T	T
T	F	T	T	T	F	F	F
F	T	F	F	T	T	F	T

Same as ~ p

7. If the general solution of the differential equation $y' = \frac{y}{x} + \Phi\left(\frac{x}{y}\right)$, for some function Φ, is given by y ln |cx| = x, where c is an arbitrary constant, then $\Phi(2)$ is equal to

(1) 4 (2) – 4

(3) $\frac{1}{4}$ (4) $-\frac{1}{4}$

Ans. (4)

Sol. $y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ ...(1) is solution of

y ln |cx| = x ...(2)

d.w.r. to x

$\frac{y}{|cx|}.\frac{|cx|}{cx}.c + \ln|cx|y' = 1$

$\frac{y}{x} + \frac{x}{y}y' = 1$ (use ln |cx| = $\frac{x}{y}$)

$y' = \left(1 - \frac{y}{x}\right)\frac{y}{x}$

use y' in equation (1)

$\frac{y}{x}\left(1 - \frac{y}{x}\right) = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$

put $\left(\frac{x}{y}\right) = 2 \Rightarrow \left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{2} + \phi(2)$

$= \frac{1}{4} = \frac{1}{2} + \phi(2)$

$\phi(2) = -\frac{1}{4}$

8. For the curve y = 3 sinθ cosθ, x = e^θsinθ, $0 \le \theta \le \pi$, the tangent is parallel to x-axis when θ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Ans. (3)

Sol. $\frac{dy}{dx} = 0 \Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = 0$

$\Rightarrow \frac{3\left[-\sin^2\theta + \cos^2\theta\right]}{e^\theta\cos\theta + \sin\theta e^\theta} = 0$

$\Rightarrow \frac{3\cos 2\theta}{e^\theta(\cos\theta + \sin\theta)}$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Reject (3π/4) because at $\theta = \frac{3\pi}{4}$

Denomentor cos θ + sin θ = 0

So $\theta = \frac{\pi}{4}$ ans

9. If for $n \ge 1$, $P_n = \int_1^e (\log x)^n dx$, then $P_{10} - 90P_8$ is equal to

(1) 10e (2) 10

(3) – 9 (4) – 9e

Ans. (4)

Sol. $P_n = \int_1^e (\log x)^n .1dx$

Integrate by parts

$$P_n = \left(x(\log x)^n\right)_1^e - \int_1^e x\, n(\log x)^{n-1}.\frac{1}{x}dx$$

$P_n = e - n\ P_{n-1} \Rightarrow p_n + n\ P_{n-1} = e$

put n = 10 $P_{10} + 10P_9 = e$ ...(1)

n = 9 $P_5 + 9P_8 = e$ ...(2)

use (2) in (1) $P_{10} + 10\ (e - 9P_8) = e$

$P_{10} - 90\ P_8 = e - 10\ e$

$= -9e$

10. Let f(x) = x|x|, g(x) = sinx and h(x) = (gof)(x). Then

(1) h′(x) is differentiable at x = 0

(2) h′(x) is continuous at x = 0 but is not differentiable at x = 0

(3) h(x) is differentiable at x = 0 but h′(x) is not continuous at x = 0

(4) h(x) is not differentiable at x = 0

Ans. (2)

Sol. $h(x) = \begin{Bmatrix} \sin x^2 & x \ge 0 \\ -\sin x^2 & x < 0 \end{Bmatrix}$

$h'(x) = \begin{Bmatrix} 2x\cos x^2 & x \ge 0 \\ -2x\cos x^2 & x < 0 \end{Bmatrix}$

$h'(0) = h'(0^+) = h'[0^-)$

so h'(x) is continuous at x = 0

$h''(x) = \begin{Bmatrix} 2[\cos x^2 - 2x^2 \sin x^2] & x \ge 0 \\ -2[\cos x^2 - 2x^2 \sin x^2] & x < 0 \end{Bmatrix}$

$h''(0^+) \ne h''(0^-)$ so h"(x) is not continuous at x = 0

so h'(x) is not differentiable at x = 0

11. A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is

(1) $\frac{64}{127}$ (2) $\frac{63}{128}$ (3) $\frac{1}{2}$ (4) $\frac{31}{128}$

Ans. (1)

Sol. Total non empty subsects = $2^7 - 1 = 127$

Let $x \in S$ also present in A

So no. of A's containg x = 2^6

Probability = $\frac{2^6}{127}$

12. If $\lim_{x\to 2} \frac{\tan(x-2)\{x^2 + (k-2)x - 2k\}}{x^2 - 4x + 4} = 5$ then k is equal to

(1) 3 (2) 1 (3) 0 (4) 2

Ans. (1)

Sol. $\lim_{x\to 2} \frac{\tan(x-2)\left[x^2 + kx - 2k - 2x\right)}{(x-2)^2} = 5$

$\lim_{x\to 2} \left(\frac{\tan(x-2)}{(x-2)}\right)\frac{(x+k)(x-2)}{(x-2)} = 5$

1. (2 + k) = 5

K = 3

13. Let P(3secθ, 2tanθ) and Q(3secϕ, 2tanϕ) where $\theta + \phi = \frac{\pi}{2}$, be two distinct points on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. Then the ordinate of the point of intersection of the normals at P and Q is

(1) $\frac{-11}{3}$ (2) $\frac{-13}{2}$ (3) $\frac{13}{2}$ (4) $\frac{11}{3}$

Ans. (2)

Sol. p (3 sec θ, 2 tan θ) Q = (3 sec ϕ , 2 tanϕ)

$\theta + \phi = \frac{\pi}{2}$ Q = (3 cosec θ , 2 cotθ)

Equation of normal at p =

= 3x cos θ + 2y cot θ = 13

= 3x sin θ cos θ + 2y cos θ = 13 sinθ ...(1)

equation of normal at Q ⇒

= 3x sin θ + 2y tan θ = 13

= 3x sin θ cos θ + 2y sin θ = 13 cosθ ...(2)

(1)-(2) ⇒

2y (cos θ – sin θ) = 13 (sin θ – cos θ)

$2y = -13 \Rightarrow y = \frac{-13}{2}$

14. In a geometric progression, if the ratio of the sume of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35. Then the first term of this geometric progression is

(1) 42 (2) 28
(3) 21 (4) 7

Ans. (2)

Sol. Let first term is a & C.R = r

given $\dfrac{(a+ar+ar^2+ar^3+ar^4)}{\left(\dfrac{1}{a}+\dfrac{1}{ar}+\dfrac{1}{ar^2}+\dfrac{1}{ar^3}+\dfrac{1}{ar^4}\right)}=49$

$a^2 r^4 = 49 \Rightarrow ar^2 = 7, -7$

also given that $a + ar^2 = 35$

if $ar^2 = 7 \Rightarrow a = 35 - 7 = 28$

if $ar^2 = -7 \Rightarrow a = 35 + 7 = 42$

but if a = 42 then $r^2 = -\dfrac{7}{42}$

which is not possible so

a = 28

15. Let A {2, 3, 5}, B (– 1, 3, 2) and C(λ, 5, μ) be the vertices of a ΔABC. If the median through A is equally inclined to the coordinate axes, then

(1) $8\lambda - 5\mu = 0$ (2) $10\lambda - 7\mu = 0$
(3) $5\lambda - 8\mu = 0$ (4) $7\lambda - 10\mu = 0$

Ans. (2)

Sol. Mid point of B & C is $\left(\dfrac{\lambda-1}{2}, 4, \dfrac{\mu+2}{2}\right)$

Let say $D = \left(\dfrac{\lambda-1}{2}, 4, \dfrac{\mu+2}{2}\right)$

A = (2, 3, 5)

DR's of AD = $\dfrac{\lambda-5}{2}$, 1 , $\dfrac{\mu-8}{2}$

$\lambda = 7$ & $\mu = 10$

$\Rightarrow \lambda = \dfrac{\lambda}{10} = \dfrac{\mu}{10} \Rightarrow 10\lambda - 7\mu = 0$

16. The set of all real values of λ for which exactly two common tangents can be drawn to the circles

$x^2 + y^2 - 4x - 4y + 6 = 0$ and

$x^2 + y^2 - 10x - 10y + \lambda = 0$ is the interval

(1) (18, 48) (2) (12, 24)
(3) (18, 42) (4) (12, 32)

Ans. (3)

Sol. C_1 (2, 2)C_2 (5, 5)

$r_1 = \sqrt{2}$ $r_2 = \sqrt{50}-1$

$|r_1 - r_2| < c_1 c_2 < r_1 + r_2$

$\left|\sqrt{50-\lambda}-\sqrt{2}\right| < \sqrt{9+9} < \sqrt{50-\lambda}+\sqrt{2}$

$-18 < \left[\sqrt{50-\lambda}-\sqrt{2}\right] < 18$ | $\sqrt{18}-\sqrt{2} < \sqrt{50-\lambda}$

$\lambda > 18$ | $20 - 12 < 50 - \lambda$

$\lambda < 42$

$\lambda \in (18, 42)$

17. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then

$\arg\left(\dfrac{z_1}{z_4}\right)+\arg\left(\dfrac{z_2}{z_3}\right)$ equals

(1) 0 (2) $\dfrac{\pi}{2}$ (3) $\dfrac{3\pi}{2}$ (4) π

Ans. (1)

Sol. $Z_2 = \bar{Z}_1$ & $Z_4 = \bar{Z}_3$

$\arg\left(\dfrac{Z_1}{Z_4}\right) + \arg\left(\dfrac{Z_2}{Z_3}\right)$

$= \arg Z_1 - \arg Z_4 + \arg Z_2 - \arg Z_3$

$= \arg Z_1 - \arg \bar{Z}_3 + \arg \bar{Z}_1 - \arg Z_3$

$= \arg Z_1 + \arg Z_3 - \arg Z_1 - \arg Z_3 = 0$

18. If $2\cos\theta + \sin\theta = 1 \left(\theta \neq \dfrac{\pi}{2}\right)$,

then $7\cos\theta + 6\sin\theta$ is equal to

(1) $\dfrac{1}{2}$ (2) $\dfrac{46}{5}$ (3) 2 (4) $\dfrac{11}{2}$

Ans. (2)

Sol. $2\cos\theta + \sin\theta = 1$ ...(1)

$7\cos\theta + 6\sin\theta = k$ (let) ...(2)

from (1) & (2) $\boxed{\cos\theta = \dfrac{6-k}{5}}$ $\boxed{\sin\theta = \dfrac{2k-7}{5}}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow (6 - K)^2 + (2k - 7)^2 = 25$

$\Rightarrow K = 2$

19. An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is :

(1) 18 (7!) (2) 40 (7!)

(3) 36 (7!) (4) 72 (7!)

Ans. (3)

Sol. Eight digit no divisible by 9 i.e. sum of digits divisible by 9

(i) Total no formed by 1,2,3,4,5,6,7,8 = 81

(ii) Total no formed by 0,2,3,4,5,6,7,9 = 7×7!

(iii) Total no formed by 1,0,3,4,5,6,9,8 = 7×7!

(iv) Total no formed by 1,2,0,4,5,9,7,8 = 7×7!

(v) Total no formed by 1,2,3,0,5,6,7,8 = 7×7!

8! + 28 × 7 !

= 36 × 7 !

20. The coefficient of x^{50} in the binomial expansion of

$(1 + x)^{1000} + x(1 + x)^{999} + x^2(1 + x)^{998} + ... + x^{1000}$ is

(1) $\frac{(1000)!}{(50)!(950)!}$ (2) $\frac{(1001)!}{(50)!(951)!}$

(3) $\frac{(1000)!}{(49)!(951)!}$ (4) $\frac{(1001)!}{(51)!(950)!}$

Ans. (2)

Sol. Coefficient of x^{50} e^n

$$= (1+x)^{1000}\frac{\left[1-\left(\frac{x}{1+x}\right)^{100}\right]}{\left[1-\frac{x}{1+x}\right]}$$

$= (1 + x)^{1001} - x^{1001}$

coeficient or $x^{50} = {}^{1001}C_{50} = \frac{(1001)!}{(50)!\,(951)!}$

21. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus rectum of $y^2 = 8x$ then

(1) $L_1 < L_2$ (2) $L_1 > L_2$

(3) $\frac{L_1}{L_2} = \sqrt{2}$ (4) $L_1 = L_2$

Ans. (1)

Sol. $x^2 + y^2 = 9$ & $y^2 = 8x$

L_2 = L.R. of $y^2 = 8x \Rightarrow L_2 = 8$

Solve $x^2 + 8x = 9 \Rightarrow x = 1, -9$

$x = -9$ reject

$\therefore$ $y^2 = 8x$ so $y^2 = 8$

$y = \pm\sqrt{8}$

Point of intersection are $(1,\sqrt{8})\,(1,-\sqrt{8})$

So $L_1 = 2\sqrt{8}$

$$\frac{L_1}{L_2} = \frac{2\sqrt{8}}{8} = \frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}} < 1$$

$L_1 < L_2$

22. The angle of elevation of the top of a vertical tower from a point P on the horizontal ground was observed to be α. After moving a distance 2 metres from P towards the foot of the tower, the angle of elevation changes to β. Then the height (in metres) of the tower is

(1) $\frac{\cos(\beta-\alpha)}{\sin\alpha\sin\beta}$

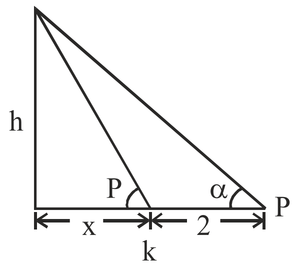
(2) $\frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$

(3) $\frac{2\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

(4) $\frac{\sin\alpha\sin\beta}{\cos(\beta-\alpha)}$

Ans. (2)

Sol. From figure



$\tan\alpha = \frac{h}{x+2}$ & $\tan\beta = \frac{h}{x}$

$x\tan\alpha + 2\tan\alpha = h$

$$h\frac{\tan\alpha}{\tan\beta} + 2\tan\alpha = h$$

$$h = \frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$$

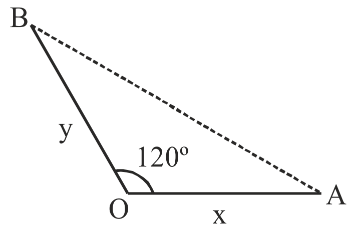
23. Two ships A and B are sailing straight away from a fixed point O along routes such that $\angle AOB$ is always 120°. At a certain instance, OA = 8 km, OB = 6 km and the ship A is sailing at the rate of 20 km/hr while the ship B sailing at the rate of 30 km/hr. Then the distance between A and B is changing at the rate (in km/hr)

(1) $\frac{260}{37}$ (2) $\frac{80}{37}$

(3) $\frac{80}{\sqrt{37}}$ (4) $\frac{260}{\sqrt{37}}$

Ans. (4)

Sol.



Let at any time t

OA = x OB = y

$\frac{dx}{dt} = 20$ $\frac{dy}{dt} = 30$

$\cos(120°) = \frac{x^2 + y^2 - AB^2}{2xy}$

$AB^2 = x^2 + y^2 + xy$ ...(1)

D.w.R. To . t

$2(AB)\frac{d}{dt}(AB) = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2\frac{dy}{dt} + y\frac{dx}{dt}$ ...(2)

when x = 8 y = 6 then $AB = \sqrt{148}$ from (1)

So $\frac{d}{dt}(AB) = \frac{\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt} + \frac{xdy}{dt} + y\frac{dx}{dt}\right)}{2AB}$

use x = 8 y = 6 $AB = \sqrt{148}$

$\frac{d}{dt}(AB) = 260 / \sqrt{37}$

24. If α and β are roots of the equation $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to

(1) $248\sqrt{2}$ (2) $280\sqrt{2}$

(3) $-32\sqrt{2}$ (4) $-280\sqrt{2}$

Ans. (2)

Sol. $x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$

$\alpha + \beta = 4\sqrt{2}\,k$

$\alpha\beta = 2k^4 - 1$

$\Rightarrow \alpha^2 + \beta^2 = 66$

$(\alpha + \beta)^2 - 2\alpha\beta = 66$

$32k^2 - 2(2k^4 - 1) = 66$

$2(2k^4) - 32k^2 + 64 = 0$

$4(k^2 - 4)^2 = 0 \Rightarrow k^2 = 4 \Rightarrow k = 2$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (8\sqrt{2})(66 - 31) = 280\sqrt{2}$

25. Let for i = 1, 2, 3 $p_i(x)$ be a polynomial of degree 2 in x, $p_i'(x)$ and $p_i''(x)$ be the first and second order derivatives of $p_i(x)$ respectively.

Let, $A(x) = \begin{bmatrix} p_1(x) & p_1'(x) & p_1''(x) \\ p_2(x) & p_2'(x) & p_2''(x) \\ p_3(x) & p_3'(x) & p_3''(x) \end{bmatrix}$

and $B(x) = [A(x)]^T A(x)$. Then determinant of B(x)

(1) Does not depend on x

(2) Is a polynomial of degree 6 in x

(3) Is a polynomial of degree 3 in x

(4) Is a polynomial of degree 2 in x

Ans. (1)

Sol. Let $P_i = a_i x^2 + b_i x + c_i$ $a_i \neq 0$

$b_i, c_i \in R$

$A(x) = \begin{bmatrix} a_1x^2 + b_1x + c_1 & 2a_1x + b_1 & 2a_1 \\ a_2x^2 + b_2x + c_2 & 2a_2x + b_2 & 2a_2 \\ a_3x^2 + b_3x + c_3 & 2a_3x + b_3 & 2a_3 \end{bmatrix}$

use (i) $C_2 \to C_2 - x\,C_3$

then use (ii) $C_1 \to C_1 - x\,C_2 - \frac{x^2}{2}C_3$

$A(x) = \begin{bmatrix} c_1 & b_1 & 2a_1 \\ c_2 & b_2 & 2a_2 \\ c_3 & b_3 & 2a_3 \end{bmatrix} \Rightarrow |A| = \text{constant}$

So $|B| = |A^T|\,|A| = |A|^2$ = constant independent from n

26. The sum of the first 20 terms common between the series 3 + 7 +11 +15 +.....and

1 + 6 + 11 +16 +....., is :

(1) 4220 (2) 4020

(3) 4000 (4) 4200

Ans. (2)

Sol. from x

$A.P_1 = 3, 7, 11, 15$ ...... $d_1 = 4$

$A.P_2 = 1, 6, 11, 16$ ...... $d_2 = 5$

I[st] common term = 11 $d = LCM(d_1, d_2)$

$d = 20$

New A.P of common terms having

a = 11 as Ist term & d = 20

sum of 20 term $\Rightarrow \frac{20}{2}[2 \times 11 + 19 \times 20]$

$= 4020$

27. If $|\vec{c}|^2 = 60$ and $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$, then a value of $\vec{c}.(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

(1) $4\sqrt{2}$ (2) 24 (3) $12\sqrt{2}$ (4) 12

Ans. (3)

Sol. $\vec{C} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = 0$

$\vec{C} = \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$

$|C| = \lambda\sqrt{30} \Rightarrow \lambda^2(30) = |c|^2 = 60$

$\lambda = \pm\sqrt{2}$

$\Rightarrow \vec{C}.(-7\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda\ (i + 2j + 5k).(-7i + 2j + 3k)$

$\Rightarrow \lambda\ (-7 + 4 + 15) = 12\lambda$

$= 12\sqrt{2}$ or $-12\sqrt{2}$

28. Let A be a 3 × 3 matrix such that

$$A\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Then A^{-1} is

(1) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

Ans. (2)

Sol. $\therefore AA^{-1} = I$

given $A\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

use column transformation and make RHS as I

(i) $C_1 \leftrightarrow C_3$ $A\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(ii) $C_2 \leftrightarrow C_3$ $A\begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

29. A stair-case of length l rests against a vertical wall and a floor of a room,. Let P be a point on the stair-case, nearer to its end on the wall, that divides its length in the ratio 1 : 2. If the stair-case begins to slide on the floor, then the locus of P is :

(1) An ellipse of eccentricity $\frac{\sqrt{3}}{2}$

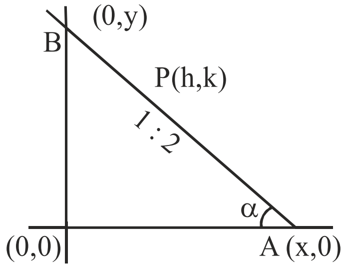
(2) A circle of radius $\frac{\ell}{2}$

(3) An ellipse of eccentricity $\frac{1}{2}$

(4) A circle of radius $\frac{\sqrt{3}}{2}\ell$

Ans. (1)

Sol.



Let any time one end is A (x, 0) & other and B(0, y) so

$\ell^2 = x^2 + y^2$ ...(1)

Let P is (h, k) using section formula

$(h, k) = \left(\frac{x}{3}, \frac{2y}{3}\right)$

$x = 3h$ & $y = \frac{3k}{2}$

use in (1)

$9h^2 + \frac{9k^2}{4} = \ell^2$

Locus of Pt p is ellipse

which equation is $\left(9x^2 + \frac{9y^2}{4} = \ell^2\right)$

$$\frac{x^2}{\left(\frac{\ell^2}{9}\right)} + \frac{9^2}{\left(\frac{4\ell^2}{9}\right)} = 1$$

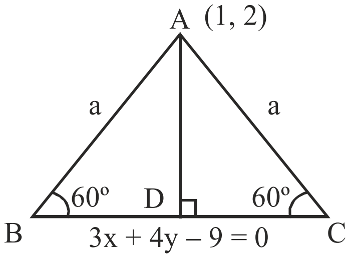
$$e = \sqrt{1 - \frac{\ell^2}{9 \times \frac{4\ell^2}{2}}} = \frac{\sqrt{3}}{2}$$

30. Ihe base of an equilateral triangle is along the line given by 3x + 4y = 9. If a vertex of the triangle is (1, 2), then the length of a side of the triangle is :

(1) $\frac{4\sqrt{3}}{15}$ (2) $\frac{4\sqrt{3}}{5}$ (3) $\frac{2\sqrt{3}}{15}$ (4) $\frac{2\sqrt{3}}{5}$

Ans. (1)

Sol. Let BC is base of equilater triangle ABC with side a and A (1, 2)



AD = a sin 60°

AD is perpendicular distance of PtA from line 3x + 4y – 9 = 0

$$AD = \left|\frac{3 \times 1 + 4 \times 2 - 9}{\sqrt{3^2 + 4^2}}\right|$$

$$a \sin 60^\circ = \frac{2}{5}$$

$$a = \frac{4}{5\sqrt{3}} = \frac{4\sqrt{3}}{15}$$