PART C – MATHEMATICS

12th

1. Two tangents are drawn from a point (–2,–1) to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan \alpha|$ is equal to :-

(1) $\sqrt{3}$ (2) 3

(3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

Ans. (2)

Sol. Let equation of tangent from (– 2, –1) be

$y + 1 = m(x + 2)$

$\Rightarrow y = mx + (2m - 1)$

Condition of tangency, $C = \frac{a}{m}$

i.e., $2m - 1 = \frac{1}{m}$

$\Rightarrow 2m^2 - m - 1 = 0$

$(2m + 1)(m - 1) = 0$

$m = -\frac{1}{2}, 1$

Now, $|\tan\alpha| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right| = 3$

2. The general solution of the diffferential equation, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is :-

(1) $y\sqrt{\tan x} = \cot x + c$

(2) $y\sqrt{\cot x} = x + c$

(3) $y\sqrt{\tan x} = x + c$

(4) $y\sqrt{\cot x} = \tan x + c$

Ans. (2)

Sol. $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

$\frac{dy}{dx} - \frac{y}{\sin 2x} = \sqrt{\tan x}$

I.F. $= e^{-\int \operatorname{cosec} 2x\,dx} = e^{-\frac{1}{2}\ln \tan x} = \frac{1}{\sqrt{\tan x}}$

$\Rightarrow$ General solution

$y.\frac{1}{\sqrt{\tan x}} = \int \sqrt{\tan x}.\frac{1}{\sqrt{\tan x}} + c$

$y\sqrt{\cot x} = x + c$

3. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a :-

(1) circle (2) straight line

(3) parabola (4) hyperbola

Ans. (2)

Sol. $x + a(2y + 1) = 0$

$x + b(3y + 1) = 0$

$x + a(4y + 1) = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 2a & 0 \end{vmatrix} = 0$$

$\Rightarrow 2a(b - a) = 0$

$2a = 0$ or $b = a$

Locus of (a, b) $\Rightarrow x = 0$ or $y = x$

4. If

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$ $\lambda \neq 0$, then k is equal to :-

(1) $4\lambda abc$ (2) $-4\lambda^2$

(3) $4\lambda^2$ (4) $-4\lambda abc$

Ans. (3)

Sol. $R_2 \to R_2 - R_1,\ R_1 \to R_1 - R_3$

$$\begin{vmatrix} \lambda(2a-\lambda) & \lambda(2b-\lambda) & \lambda(2c-\lambda) \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$= R_3 \to R_3 + R_1,\ R_1 \to R_1 - \frac{1}{2}R_2$

$$= \begin{vmatrix} -\lambda^2 & -\lambda^2 & -\lambda^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= -4\lambda^3 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= +4\lambda^3 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\therefore\ K = 4\lambda^2$

5. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :-

(1) -2 (2) $\sqrt{2}$

(3) $-\sqrt{2}$ (4) 2

Ans. (2)

Sol. $x^2 + |2x - 3| - 4 = 0$

Case-1 : $x \geq \frac{3}{2}$

$\Rightarrow x^2 + 2x - 3 - 4 = 0$

$\Rightarrow x = -1 \pm 2\sqrt{2}$

$\Rightarrow x = -1 + 2\sqrt{2}$

Case-2 : $x < \frac{3}{2}$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = 1 \pm \sqrt{2}$

$\Rightarrow x = 1 - \sqrt{2}$

$\Rightarrow$ Sum $= \sqrt{2}$

6. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is :-

(1) any non-zero real number other than 1.

(2) a purely imaginary number.

(3) 0

(4) any non- zero real number

Ans. (4)

Sol. Let $Z = x + iy$

$\frac{z-i}{z+i}$ is a purely imaginary number

$\Rightarrow \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$ is a purely imaginary

$\Rightarrow \frac{(x^2+y^2-1)-i(2x)}{x^2+(y+1)^2}$ is purely imaginary

$\Rightarrow x^2 + y^2 - 1 = 0 \Rightarrow x^2 + y^2 = 1$ ...(i)

$z + \frac{1}{z} = x + iy + \frac{1}{x+iy}$

$= (x+iy) + \frac{1}{(x+iy)} \times \frac{(x-iy)}{(x-iy)}$

$= (x+iy) + \frac{(x-iy)}{x^2+y^2} = 2x$

$x \neq 1$

($\because$ if $x = 1$ then $y = 0$, from (i) & z won't be complex number)

If $x = 1 \Rightarrow y = 0$

$\Rightarrow z = 1$

$\Rightarrow \frac{z-i}{z+i} = \frac{1-i}{1+i}$ cannot be purely imaginary.

7. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then :-

(1) $y = -2x$ (2) $y = 2x$

(3) $y = x$ (4) $y = -x$

Ans. (2)

Sol. $AB = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$y + 3x = 6$

$3y - x = 6$

$y + 3x = 3y - x$

$4x = 4y$

$\boxed{x = y}$

8. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observation x_i, i = 1,2,...., n. If each of the observation is increased by 5, then the new mean and the mean deviation about the new mean respectively , are :-

(1) $\overline{X}$ + 5, M.D. (2) $\overline{X}$ + 5, M.D. + 5

(3) $\overline{X}$, M.D. (4) $\overline{X}$, M.D. + 5

Ans. (1)

Sol. If all the observations are increased by K then mean is increased by K but M.D. remains same.

9. If for a continuous function f(x), $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$, for all $t \geq -\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to :-

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$ (4) π

Ans. (4)

Sol. $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx + \int_{-\pi}^{t} xdx = \pi^2 - t^2$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \frac{3}{2}(\pi^2 - t^2)$

$\Rightarrow \int_{-\pi}^{t} f(x)dx = \int_{-\pi}^{t} -3xdx \Rightarrow f(x) = -3x$

$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi$

10. A number x is chosen at random from the set {1, 2, 3, 4, ..., 100}. Define the event : A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

Then P (A) is :-

(1) 0.20 (2) 0.51

(3) 0.71 (4) 0.70

Ans. (3)

Sol. S = {1, 2, 3...........100}

A : Chosen no. x satisfies

$\frac{(x-10)(x-50)}{(x-30)} \geq 0$

$\therefore\ x \in \{10, 11, 12......29\} \cup \{50, 51.....100\}$

$P(A) = \frac{71}{100} = 0.71$

11. Let f and g be two diffrentiable functions on **R** such that f'(x) > 0 and g' (x) < 0, for all x ∈ **R**. Then for all x :-

(1) g(f(x)) > g (f(x –1))

(2) f(g(x)) > f (g(x –1))

(3) g(f(x)) < g (f(x + 1))

(4) f(g(x)) > f (g (x – 1))

Ans. (4)

Sol. $f'(x) > 0 \Rightarrow f(x)$ is increasing function

$g'(x) < 0 \Rightarrow g(x)$ is decreasing function

Now,

(i) $x > x - 1$

$f(x) > f(x - 1)$

$g(f(x)) < g(f(x - 1))$

and

(ii) $x + 1 > x$

$f(x + 1) > f(x)$

$g(f(x + 1)) < g(f(x))$

(iii) $x > x - 1$

$g(x) < g(x - 1)$

$f(g(x)) < f(g(x - 1))$

(iv) $x + 1 > x$

$g(x + 1) < g(x)$

$f(g(x + 1)) < f(g(x))$

12. If $f(x) = x^2 - x + 5, x > \frac{1}{2}$, and g(x) is its inverse function, then g'(7) equals :-

(1) $\frac{1}{3}$ (2) $-\frac{1}{3}$

(3) $-\frac{1}{13}$ (4) $\frac{1}{13}$

Ans. (1)

Sol. $f(x) = x^2 - x + 5$

$g(f(x)) = x$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow g'(f(2)) = \frac{1}{f'(2)}$$

$$\Rightarrow g'(7) = \frac{1}{3}$$

13. **Statement I :** The equation $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$ has a solution for all $a \geq \frac{1}{32}$.

Statement II : For any $x \in R$,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and

$$0 \leq \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$$

(1) Both statement I and II are true

(2) Statement I is true and statement II is false

(3) Statement I is false and statement II is true

(4) Both statements I and II are false

Ans. (3)

Sol. **Statement-I**

$(\sin^{-1}x)^3 + (\cos^{-1}x)^3 - a\pi^3 = 0$

$\Rightarrow (\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3$

$\Rightarrow (\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x) = a\pi^3$

$$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi}{2}\left(\frac{\pi}{2} - \cos^{-1} x\right)\cos^{-1} x = a\pi^3$$

$$\Rightarrow \frac{\pi^3}{8} - \frac{3\pi^2}{4}\cos^{-1} x + \frac{3\pi}{2}(\cos^{-1} x)^2 = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[(\cos^{-1} x)^2 - \frac{\pi}{2}\cos^{-1} x\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\Rightarrow \frac{3\pi}{2}\left[\left(\cos^{-1} x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\left(\cos^{-1} x - \frac{\pi}{4}\right)^2 = \left(\frac{2a}{3} - \frac{1}{48}\right)\pi^2 \qquad ...(1)$$

$\because\ 0 \leq \cos^{-1}x \leq \pi$

$$-\frac{\pi}{4} \leq \left(\cos^{-1} x - \frac{\pi}{4}\right) \leq \frac{3\pi}{4}$$

$$0 \leq \left(\cos^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16}$$

From equation (1)

$$\frac{1}{32} \leq a \leq \frac{7}{8}$$

$\therefore$ Statement-I is false

Statement-II

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\ \forall\ x \in [-1, 1]$

not for any $x \in R$ so Statement-II is false

14. Let f, g: R → R be two functions defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), x \neq 0 \\ 0 \qquad , x = 0 \end{cases}$$, and g(x) = xf (x) :-

Statement I : f is a continuous function at x = 0.

Statement II : g is a differentiable function at x = 0.

(1) Statement I is false and statement II is true

(2) Statement I is true and statement II is false

(3) Both statement I and II are true

(4) Both statements I and II are false

Ans. (3)

Sol. **Statement-I**

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$$

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)=0=f(0)$$

Hence f(x) is continuous function

$g(0) = 0 = \lim_{x\to 0} g(x)$

LHD :

$$\lim_{x\to 0}\frac{g(o-h)-g(0)}{-h}$$

$$\lim_{x\to 0}\frac{h^2\sin\left(-\frac{1}{h}\right)-0}{-h}$$

LHD = 0

RHD :

$$\lim_{h\to 0}\frac{g(0+h)-g(0)}{h}$$

$$\lim_{h\to 0}\frac{h^2\sin\left(\frac{1}{n}\right)-0}{h}$$

RHD = 0

LHD = RHD

Hence g(x) at x = 0 is diff. function.

15. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$, there is / are:-

(1) no common tangent

(2) two pairs of common tangents

(3) one pair of common tangents

(4) three common tangents

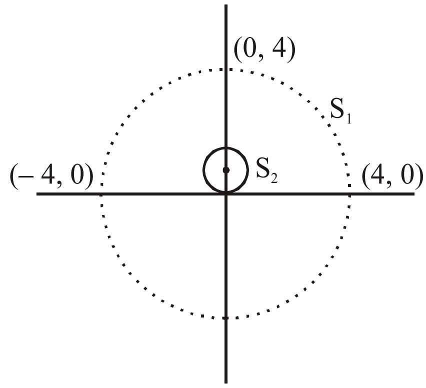
Ans. (1)

Sol. S-I : $x^2 + y^2 = 4^2$

S-II : $x^2 + (y - 1)^2 = 1$

Aliter : Distance between their centres is 1 units and sum of their radii is 3 so one of them lie completely inside the other, hence no common tangent.

Also, from figure we can say no common tangent :



16. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are :-

(1) 28^{th} and 29^{th} (2) 8^{th} and 9^{th}

(3) 7^{th} and 8^{th} (4) 27^{th} and 28^{th}

Ans. (2)

Sol. $\left(2+\frac{x}{3}\right)^{55}$

General term

$$^{55}C_r\times 2^{55-r}\times\left(\frac{x}{3}\right)^r$$

Let T_{r+1} and T_{r+2} are having some co-efficients

$\Rightarrow$ Coff. of T_{r+1} = Coff. of T_{r+2}

$$^{55}C_r\times 2^{55-r}\times\left(\frac{1}{3}\right)^r = {}^{55}C_{r+1}\times(2)^{54-r}\times\left(\frac{1}{3}\right)^{r+1}$$

$\Rightarrow$ r = 6

$\Rightarrow$ Coff. of T_7 = Coff. of T_8

17. If the distance between planes, $4x - 2y - 4z + 1 = 0$ and $4x - 2y - 4z + d = 0$ is 7, then d is :-

(1) 41 or – 42 (2) – 42 or 44

(3) 42 or – 43 (4) – 41 or 43

Ans. (4)

Sol. $\left|\frac{d-1}{\sqrt{4^2+2^2+4^2}}\right|=7 \Rightarrow \left|\frac{d-1}{6}\right|=7$

d– 1 = ± 42

d = + 43, – 41

18. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81}=1$ and the co-ordinate axes is :-

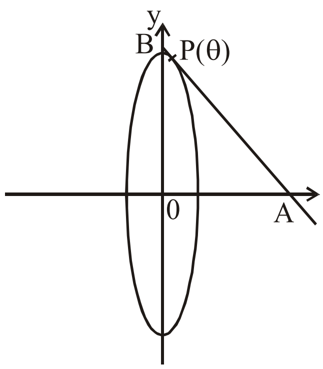
(1) 26 (2) 18

(3) 36 (4) 12

Ans. (3)

Sol. Let P(4cosθ, 9sinθ) be a point on ellipse equation of tangent $\frac{x}{4}\cos\theta+\frac{y}{9}\sin\theta=1$

Let A & B are point of intersection of tangent at P with co-ordinate axes.



$A\left(\frac{4}{\cos\theta},0\right)$ $B\left(0,\frac{9}{\sin\theta}\right)$

Area of ΔOAB = $\frac{1}{2}\left(\frac{4}{\cos\theta}\right)\left(\frac{9}{\sin\theta}\right)=\frac{36}{\sin 2\theta}$

$(\text{Area})_{\min}=36$ as $\sin 2\theta=1$

19. If $\hat{x},\hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2$ is :-

(1) $\frac{3}{2}$ (2) $3\sqrt{3}$ (3) 3 (4) 6

Ans. (3)

Sol. $|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2=K$ let

$K=2\hat{x}^2+2\hat{y}^2+2\hat{z}^2+2\hat{x}.\hat{y}+2\hat{y}.\hat{z}+2\hat{z}.\hat{x}$

$K = 2 + 2 + 2 + 2[\cos\theta_1+\cos\theta_2+\cos\theta_3]$

When $\theta_1=\theta_2=\theta_3=\frac{2\pi}{3}$

Then $K_{\min}=6+2\left(-\frac{3}{2}\right)=6-3=3$

20. Let G be the geometric mean of two positive numbers a and b, and M be the arthemetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5 , then a : b can be :-

(1) 2 : 3 (2) 1 : 4 (3) 1 : 2 (4) 3 : 4

Ans. (2)

21. The least positive integer n such that $1-\frac{2}{3}-\frac{2}{3^2}-\ldots\ldots-\frac{2}{3^{n-1}}<\frac{1}{100}$, is :-

(1) 7 (2) 4 (3) 5 (4) 6

Ans. (4)

Sol. $1-2\left(\frac{1}{3^1}+\frac{1}{3^2}\ldots\ldots\frac{1}{3^{n-1}}\right)<\frac{1}{100}$

$$1-2\left(\frac{1}{3}\right)\frac{\left[1-\frac{1}{3^{n-1}}\right]}{\left(\frac{2}{3}\right)}<\frac{1}{100}$$

$\Rightarrow 1-1+\frac{1}{3^{n-1}}<\frac{1}{100}$

$100 < 3^{n-1}$

$n-1=5$

$n=6$

22. If a line intercepted between the coordinate axes is trisected at a point A (4, 3), which is nearer to x-axis, then its equation is :-

(1) 3x + 8y = 36

(2) 4x – 3y = 7

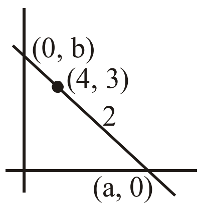
(3) x + 3y = 13

(4) 3x + 2y = 18

Ans. (4)

Sol. $4=\frac{a}{3}$ a = 12

$3=\frac{2b}{3}$ $b=\frac{9}{2}$



$\frac{x}{12}+\frac{y}{9}(2)=1$

$\boxed{3x+8y=36}$

23. If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and

A and B are respectively the maximum and the minimum values of f (θ), then (A, B) is equal to :-

(1) $(4, 2-\sqrt{2})$

(2) $(2+\sqrt{2}, 2-\sqrt{2})$

(3) $(3, -1)$

(4) $(2+\sqrt{2}, -1)$

Ans. (2)

Sol. Expanding the determinant, we get

$f(\theta) = 1(1 + \sin\theta\cos\theta) + \cos\theta(\sin\theta + \cos\theta) + 1(1-\sin^2\theta)$

$= 1 + 2\sin\theta\cos\theta + 2\cos^2\theta$

$= 1 + \sin 2\theta + (1 + \cos 2\theta)$

$= 2 + \sin 2\theta + \cos 2\theta$

Now,

$\sin 2\theta + \cos 2\theta$ lies between $-\sqrt{2}$ to $\sqrt{2}$

$$\left[\sqrt{2}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right) \to \pm\sqrt{2}\right]$$

$\therefore$ A = $2+\sqrt{2}$; $\quad B = 2-\sqrt{2}$

24. The integral $\int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$ is equal to :-

(1) $-\frac{\cos^3 x}{3(1+\sin^3 x)}+c$ (2) $\frac{1}{(1+\cot^3 x)}+c$

(3) $-\frac{1}{3(1+\tan^3 x)}+c$ (4) $\frac{\sin^3 x}{(1+\cos^3 x)}+c$

Ans. (3)

Sol. $\int \frac{\sin^2 x\cos^2 x\,dx}{\left(\sin^3 x+\cos^3 x\right)^2} = \int \frac{\tan^2 x\sec^2 x\,dx}{\left(1+\tan^3 x\right)^2}$

Put $\tan^3 x = t$

$3\tan^2 x.\sec^2 x\,dx = dt$

$$\int \frac{dt}{3(1+t)^2} = \frac{-1}{3(1+t)}+C$$

$$= \frac{-1}{3(1+\tan^3 x)}+C$$

25. A symmetrical form of the line of intersection of the planes x = ay + b and z = cy + d is :-

(1) $\frac{x-a}{b} = \frac{y-0}{1} = \frac{z-c}{d}$

(2) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$

(3) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

(4) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$

Ans. (3)

Sol. $\frac{x-b}{a} = y = \frac{z-d}{c}$

$$\Rightarrow \frac{x-b}{a}-1 = y-1 = \frac{z-d}{c}-1$$

$$\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$$

26. 8-digit numbers are formed using the digits 1,1,2,2,2,3,4,4. The number of such number in which the odd digits do not occupy odd places, is :-

(1) 60 (2) 48 (3) 160 (4) 120

Ans. (4)

Sol. No. of ways of selecting 3 odd places out of 4 odd places.

$${}^4C_3 \times \frac{3!}{2!} \times \frac{5!}{3!2!}$$

$= 4 \times 3 \times 5 \times 2$

$= 120$

27. If $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i (1 + x)^i$, for all x in **R**, then a_2 is :-

(1) – 8 (2) 6 (3) 10 (4) – 4

Ans. (4)

Sol. $1 + x^4 + x^5 = a_0 + a_1(1 + x) + a_2(1 + x)^2 + a_3(1 + x)^3 + a_4(1 + x)^4 + a_5(1 + x)^5$

$= a_0 + a_1(1 + x) + a_2(1 + 2x + x^2) + a_3(1 + 3x + 3x^2 + x^3) + a_4(1 + 4x + 6x^2 + 4x^3 + x^4) + a_5(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

So, Coeff. of x^i in LHS = Coeff. of x^i on RHS

$i = 5 \Rightarrow 1 = a_5$ ...(i)

$i = 4 \Rightarrow 1 = a_4 + 5a_5 = a_4 + 5$

$\Rightarrow a_4 = -4$ ...(ii)

$i = 3 \Rightarrow 0 = a_3 + 4a_4 + 10a_5$

$\Rightarrow a_3 - 16 + 10 = 0$

$\Rightarrow a_3 = 6$ ...(iii)

$i = 2 \Rightarrow 0 = a_2 + 3a_3 + 6a_4 + 10a_5$

$\Rightarrow a_2 + 18 - 24 + 10 = 0$

$\Rightarrow a_2 = -4$

Put x = – 1

$1 = a_0$

Now differentiate w.r.t. x.

$4x^3 + 5x^4 = a_1 + 2a_2(1 + x) + 3a_3(1 + x)^2 + ...$

Put x = – 1

$\Rightarrow 1 = a_1$

Again differentiate w.r.t. x

$12x^2 + 20x^3 = 2xa_2 + 6a_3 (1 + x)$

Put x = –1

$12 - 20 = 2a_2 \Rightarrow a_2 = -4$

28. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is :-

(1) $(p \vee q) \Rightarrow r$

(2) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$

(3) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

(4) $(p \Rightarrow q) \vee (p \Rightarrow r)$

Ans. (4)

Sol. $p \rightarrow (q \vee r)$

$\sim p \vee (q \vee r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$(p \Rightarrow q) \vee (p \Rightarrow r)$

29. If [ ] denote the greatest integer function, then the integral $\int_0^{\pi} [\cos x]\, dx$ is equal to :-

(1) 0 (2) $\frac{\pi}{2}$ (3) $-\frac{\pi}{2}$ (4) –1

Ans. (3)

Sol. $\int_0^{\pi} [\cos x] dx$

$= \int_0^{\pi/2} 0.\, dx + \int_{\frac{\pi}{2}}^{\pi} -1 dx$

$= [-x]_{\frac{\pi}{2}}^{\pi} = -\frac{\pi}{2}$

30. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is : :-

(1) 32 (2) 64 (3) 16 (4) 8

Ans. (3)

Sol. A = {– 2, – 1, 0, 1, 2}

R = {(– 2, 2) (0, 0) (1, 1), (1, 2)}

$n(P(R)) = 2^4 = 16$