Paper-2

JEE Advanced, 2016

Part III: Mathematics

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 2:00 pm, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon – less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with of mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.

16. Darken the bubble completely.

17. The correct way of darkening a bubble is as:

18. The ORS is machine – gradable. Ensure that the bubbles are darkened in the correct way.

19. Darken the bubbles only if you are sure of the answer. There is no way to erase or "un–darken" a darkened bubble.

PART - III : MATHEMATICS

SECTION–1 : (Maximum Marks : 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases.

37. Let $P=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$ and I be the identity matrix of order 3. If Q = $[q_{ij}]$ is a matrix such that $P^{50}-Q=I$, then $\dfrac{q_{31}+q_{32}}{q_{21}}$ equals

(A) 52 (B) 103 (C) 201 (D) 205

Ans. (B)

Sol. $P=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix} \Rightarrow P^2=\begin{bmatrix}1 & 0 & 0\\ 8 & 1 & 0\\ 16+32 & 8 & 1\end{bmatrix}$

so, $P^3=\begin{bmatrix}1 & 0 & 0\\ 12 & 1 & 0\\ 16+32+48 & 12 & 1\end{bmatrix}$ (from the symmetry)

$$P^{50}=\begin{bmatrix}1 & 0 & 0\\ 200 & 1 & 0\\ \dfrac{16.50.51}{2} & 200 & 1\end{bmatrix}$$

As, $P^{50}-Q=I \Rightarrow q_{31}=\dfrac{16.50.51}{2}$

$q_{32}=200$ and $q_{21}=200$

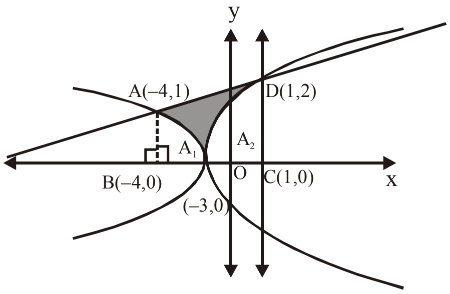
$\therefore \dfrac{q_{31}+q_{32}}{q_{21}}=\dfrac{16.50.51}{2.200}+1=102+1=103$

38. Area of the region $\left\{(x,y)\in\mathbb{R}^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$ is equal to -

(A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Ans. (C)

Sol.



Clearly required area = area (trapezium ABCD) − (A_1 + A_2) .....(i)

area (trapezium ABCD) $=\frac{1}{2}(1+2)(5)=\frac{15}{2}$

$$A_1 = \int_{-4}^{-3}\sqrt{-(x+3)}dx$$

$$=\frac{2}{3}$$

and $A_2 = \int_{-3}^{1}(x+3)^{1/2}dx = \frac{16}{3}$

$\therefore$ From equation (1), we get required area $=\frac{15}{2}-\left(\frac{2}{3}+\frac{16}{3}\right)=\frac{3}{2}$

39. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to

(A) $3-\sqrt{3}$ (B) $2(3-\sqrt{3})$ (C) $2(\sqrt{3}-1)$ (D) $2(2+\sqrt{3})$

Ans. (C)

Sol. We have,

$$=2.\sum_{k=1}^{13}\frac{\sin\left(\left(\frac{k\pi}{6}+\frac{\pi}{4}\right)-\left((k-1)\frac{\pi}{6}+\frac{\pi}{4}\right)\right)}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right).\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}=2\sum_{k=1}^{13}\left(\cot\left((k-1)\frac{\pi}{6}+\frac{\pi}{4}\right)-\cot\left(\frac{k\pi}{6}+\frac{\pi}{4}\right)\right)$$

$$=2\left[\cot\frac{\pi}{4}-\cot\left(\frac{13\pi}{6}+\frac{\pi}{4}\right)\right]=2\left(1-\cot\left(\frac{5\pi}{12}\right)\right)=2\left(1-\left(2-\sqrt{3}\right)\right)=2\left(\sqrt{3}-1\right)$$

40. Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$ then

(A) $s > t$ and $a_{101} > b_{101}$

(B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$

(D) $s < t$ and $a_{101} < b_{101}$

Ans. (B)

Sol. If $\log_e b_1, \log_e b_2 \ldots\ldots \log_e b_{101} \rightarrow$ AP ; $D = \log_e 2$

$\Rightarrow b_1\ b_2\ b_3 \ldots b_{101} \rightarrow$ GP ; $r = 2$

$\therefore\ b_1, 2b_1, 2^2 b_1, \ldots\ldots\ldots, 2^{100} b_1 \ldots\ldots$ GP

$a_1\ a_2\ a_3 \ldots\ldots\ldots a_{101} \ldots\ldots$ AP

Given, $a_1 = b_1$ & $a_{51} = b_{51}$

$\Rightarrow\ a_1 + 50D = 2^{50} b_1$

$\therefore\ a_1 + 50D = 2^{50} a_1$ (As $b_1 = a_1$)

Now, $t = b_1(2^{51} - 1)$; $s = \frac{51}{2}(2a_1 + 50D)$

$t = a_1.2^{51} - a_1 \Rightarrow t < a_1.2^{51}$ ....(i) ; $s = \frac{51}{2}(a_1 + a_1 + 50D)$

$$s = \frac{51}{2}(a_1 + 2^{50} a_1)$$

$$s = \frac{51a_1}{2} + \frac{51}{2}.2^{50} a_1$$

$\Rightarrow s > a_1.2^{51}$ ....(ii)

clearly $s > t$ (from equation (i) and (ii))

Also $a_{101} = a_1 + 100D$; $b_{101} = b_1.2^{100}$

$\therefore\ a_{101} = a_1 + 100\left(\frac{2^{50} a_1 - a_1}{50}\right)$; $b_{101} = 2^{100} a_1$ ....(iii)

$a_{101} = a_1 + 2^{51} a_1 - 2a_1 \Rightarrow\ a_{101} = 2^{51} a_1 - a_1\ \Rightarrow a_{101} < 2^{51} a_1$ ....(iv)

clearly $b_{101} > a_{101}$ (from equation (iii) and (iv))

41. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$ is equal to

(A) $\dfrac{\pi^2}{4}-2$ (B) $\dfrac{\pi^2}{4}+2$ (C) $\pi^2 - e^{\frac{\pi}{2}}$ (D) $\pi^2 + e^{\frac{\pi}{2}}$

Ans. (A)

Sol. Let I = $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx = \int_0^{\pi/2} \left(\frac{1}{1+e^x} + \frac{1}{1+e^{-x}}\right) x^2 \cos x \, dx$

$$= \int_0^{\pi/2} \underset{\text{(I)}}{x^2}\ \underset{\text{(II)}}{\cos x}\, dx = \left(x^2 \sin x\right)_0^{\pi/2} - 2\int_0^{\pi/2} \underset{\text{(I)}}{x}\,.\underset{\text{(II)}}{\sin x}\, dx$$

$$= \frac{\pi^2}{4} - 2\left[-\left(x.\cos x\right)_0^{\pi/2} + \int_0^{\pi/2} 1.\cos x\right] = \frac{\pi^2}{4} - 2[0 + 1] = \left(\frac{\pi^2}{4} - 2\right)$$

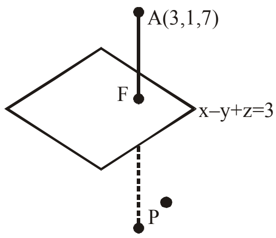
42. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{1}$ is

(A) $x + y - 3z = 0$ (B) $3x + z = 0$

(C) $x - 4y + 7z = 0$ (D) $2x - y = 0$

Ans. (C)

Sol. Line AP : $\dfrac{x-3}{1} = \dfrac{y-1}{-1} = \dfrac{z-7}{1} = \lambda$



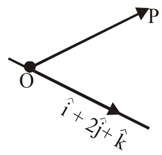
$\Rightarrow$ F$(3 + \lambda, 1 - \lambda, \lambda + 7)$ lies in the plane

$\therefore\ 3 + \lambda - (1 - \lambda) + \lambda + 7 = 3$

$3\lambda = -6 \Rightarrow \lambda = -2$

$\Rightarrow$ F(1,3,5)

$\Rightarrow$ P(–1,5,3)



so required plane is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = 0$

$\therefore\quad x - 4y + 7z = 0$

SECTION–2 : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

43. Let a, b $\in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is -

(A) differentiable at x = 0 if a = 0 and b = 1

(B) differentiable at x = 1 if a = 1 and b = 0

(C) **NOT** differentiable at x = 0 if a = 1 and b = 0

(D) **NOT** differentiable at x = 1 if a = 1 and b = 1

Ans. (A,B)

Sol. If $x^3 - x \geq 0 \Rightarrow \cos|x^3 - x| = \cos(x^3 - x)$

$x^3 - x < 0 \Rightarrow \cos|x^3 - x| = \cos(x^3 - x)$

Similarly $b|x|\sin|x^3 + x| = bx\sin(x^3 + x)$ for all $x \in R$

$\therefore \quad f(x) = a\cos(x^3 - x) + bx\sin(x^3 + x)$

which is composition and sum of differentiable functions

therefore always continuous and differentiable.

44. Let $f(x) = \lim\limits_{n\to\infty}\left(\dfrac{n^n(x+n)\left(x+\frac{n}{2}\right).....\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right).....\left(x^2+\frac{n^2}{n^2}\right)}\right)^{x/n}$, for all x > 0. Then

(A) $f\left(\frac{1}{2}\right) \geq f(1)$ (B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$ (C) $f'(2) \leq 0$ (D) $\dfrac{f'(3)}{f(3)} \geq \dfrac{f'(2)}{f(2)}$

Ans. (B,C)

Sol. $\ell n f(x)=\lim\limits_{n\to\infty}\frac{x}{n}\ell n\left[\frac{\prod\limits_{r=1}^{n}\left(x+\frac{1}{r/n}\right)}{\prod\limits_{r=1}^{n}\left(x^2+\frac{1}{(r/n)^2}\right)}\frac{1}{\prod\limits_{r=1}^{n}(r/n)}\right]$

$$=x\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ell n\left(\frac{x\frac{r}{n}+1}{\left(x\frac{r}{n}\right)^2+1}\right)$$

$$=x\int_0^1\ell n\left(\frac{1+tx}{1+t^2x^2}\right)dt \qquad \text{put } tx=z$$

$$\ell n f(x)=\int_0^x\ell n\left(\frac{1+z}{1+z^2}\right)dz$$

$$\Rightarrow\quad \frac{f'(x)}{f(x)}=\ell n\left(\frac{1+x}{1+x^2}\right)$$

sign scheme of $f'(x)$ $\quad \overset{+}{\text{————}}\underset{1}{|}\overset{-}{\text{————}}\quad$ also $f'(1)=0$

$$\Rightarrow\quad f\left(\frac{1}{2}\right)<f(1),\ f\left(\frac{1}{3}\right)<f\left(\frac{2}{3}\right),\ f'(2)<0$$

Also $\frac{f'(3)}{f(3)}-\frac{f'(2)}{f(2)}=\ell n\left(\frac{4}{10}\right)-\ell n\left(\frac{3}{5}\right)$

$$=\ell n\left(\frac{4}{6}\right)<0\Rightarrow\frac{f'(3)}{f(3)}<\frac{f'(2)}{f(2)}$$

45. Let $f:\mathbb{R}\to(0,\infty)$ and $g:\mathbb{R}\to\mathbb{R}$ be twice differentiable function such that f'' and g'' ar continuous functions on $\mathbb{R}$. Suppose $f'(2)=g(2)=0$, $f''(2)\neq 0$ and $g'(2)\neq 0$. If $\lim\limits_{x\to 2}\frac{f(x)g(x)}{f'(x)g'(x)}=1$, then

(A) f has a local minimum at $x = 2$ (B) f has a local maximum at $x = 2$

(C) $f''(2) > f(2)$ (D) $f(x) - f''(x) = 0$ for at least one $x \in \mathbb{R}$

Ans. (A,D)

Sol. Using L'Hôpital's Rule

$$\lim_{x\to 2}\frac{f'(x)g(x)+f(x)g'(x)}{f''(x)g'(x)+f'(x)g''(x)}=1$$

$$\Rightarrow\quad \frac{f(2)g'(2)}{f''(2)g'(2)}=1\quad\Rightarrow\qquad f'(2)=f(2)>0$$

option (D) is right and option (C) is wrong

also $f'(2) = 0$ and $f''(2) > 0$ $\quad\therefore\quad$ $x = 2$ is local minima.

46. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^2$ and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u}\times\vec{v}| = 1$ and $\hat{w}.(\hat{u}\times\vec{v}) = 1$. Which of the following statement(s) is(are) correct?

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choice for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

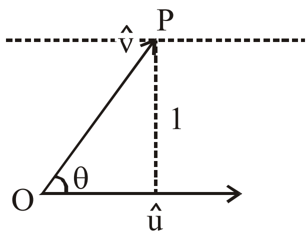
(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

Ans. **(B,C)**

Sol. $|\hat{w}||\hat{u}\times\hat{v}|\cos\phi = 1 \Rightarrow \phi = 0$

$\Rightarrow \quad \hat{u}\times\vec{v} = \hat{w}$ also $|\vec{v}|\sin\theta = 1$

$\Rightarrow$ there may be infinite vectors $\vec{v} = \overrightarrow{OP}$ such that P is always 1 unit dist. from $\hat{u}$



For option (C) : $\hat{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & 0 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\hat{w} = (u_2v_3)\hat{i} - (u_1v_3)\hat{j} + (u_1v_2 - u_2v_1)\hat{k}$

$u_2v_3 = \frac{1}{\sqrt{6}}, -u_1v_3 = \frac{1}{\sqrt{6}} \quad \Rightarrow \quad |u_1| = |u_2|$

for option (D) : $\hat{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & 0 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\hat{w} = (-v_2u_3)\hat{i} - (u_1v_3 - u_3v_1)\hat{j} + (u_1v_2)\hat{k}$

$-v_2u_3 = \frac{1}{\sqrt{6}}, u_1v_2 = \frac{2}{\sqrt{6}}$

$\Rightarrow \quad 2|u_3| = |u_1|$ So (D) is wrong

47. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then-

(A) $SP = 2\sqrt{5}$

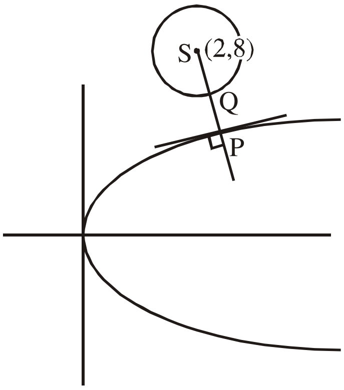
(B) $SQ : QP = (\sqrt{5}+1) : 2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

Ans. **(A,C,D)**

Sol..



$y^2 = 4x$

point P lies on normal to parabola passing through centre of circle

$y + tx = 2t + t^3$ ....(i)

$8 + 2t = 2t + t^3$

$t = 2$

$P(4, 4)$

$SP = \sqrt{(4-2)^2 + (4-8)^2}$

$SP = 2\sqrt{5}$

$SQ = 2$

$\Rightarrow \quad PQ = 2\sqrt{5} - 2$

$\frac{SQ}{QP} = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4}$

To find x intercept

put $y = 0$ in (i)

$\Rightarrow \quad x = 2 + t^2$

$x = 6$

$\because \quad$ Slope of common normal $= -t = -2$

$\therefore \quad$ Slope of tangent $= \frac{1}{2}$

48. Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in \mathbb{C} : z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0\right\}$, where $i = \sqrt{-1}$.

If $z = x + iy$ and $z \in S$, then (x,y) lies on

(A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

(B) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

(C) the x-axis for $a \neq 0, b = 0$

(D) the y-axis for $a = 0, b \neq 0$

Ans. (A,C,D)

Sol.. $x + iy = \dfrac{1}{a + ibt}$

$x + iy = \dfrac{a - ibt}{a^2 + b^2t^2}$

Let $a \neq 0$ & $b \neq 0$

$x = \dfrac{a}{a^2 + b^2t^2}$ ....(1)

$y = \dfrac{-bt}{a^2 + b^2t^2}$ ....(2)

$\dfrac{y}{x} = \dfrac{-bt}{a} \Rightarrow t = -\dfrac{ay}{bx}$

put in (1)

$x\left\{a^2 + b^2.\dfrac{a^2y^2}{b^2x^2}\right\} = a$

$a^2(x^2 + y^2) = ax$

$x^2 + y^2 - \dfrac{1}{a}x = 0$

$\left(x - \dfrac{1}{2a}\right)^2 + y^2 = \dfrac{1}{4a^2}$

$\Rightarrow$ option (A) is correct

for $a \neq 0$, $b = 0$

$x + iy = \dfrac{1}{a}$

$x = \dfrac{1}{a}, y = 0$ $\Rightarrow$ z lies on x-axis $\Rightarrow$ option (C) is correct

for $a = 0$, $b \neq 0$

$x + iy = \dfrac{1}{ibt}$

$y = -\dfrac{1}{bt}i, x = 0$

$\Rightarrow$ z lies on y-axis. $\Rightarrow$ option (D) is correct

49. Let $a,\lambda,m \in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct ?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

Ans. (B,C,D)

Sol. $ax + 2y = \lambda$
$3x - 2y = \mu$
for $a = -3$ above lies will be parallel or coincident
parallel for $\lambda + \mu \neq 0$ and coincident if $\lambda + \mu = 0$
and if $a \neq -3$ lies are intersecting $\Rightarrow$ unique solution.

50. Let $f:\left[-\frac{1}{2},2\right]\to\mathbb{R}$ and $g:\left[-\frac{1}{2},2\right]\to\mathbb{R}$ be function defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|\,f(x) + |4x - 7|\,f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(A) f is discontinuous exactly at three points in $\left[-\frac{1}{2},2\right]$

(B) f is discontinuous exactly at four points in $\left[-\frac{1}{2},2\right]$

(C) g is NOT differentiable exactly at four points in $\left(-\frac{1}{2},2\right)$

(D) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2},2\right)$

Ans. **(B,C)**

Sol. $f(x) = [x^2] - 3$
$g(x) = (|x| + |4x - 7|)([x^2] - 3)$

$\because$ f is discontinuous at $x = 1, \sqrt{2}, \sqrt{3}, 2$ in $\left[-\frac{1}{2},2\right]$

and $|x| + |4x - 7| \neq 0$ at $x = 1, \sqrt{2}, \sqrt{3}, 2$

$\Rightarrow$ g(x) is discontinuous at $x = 1, \sqrt{2}, \sqrt{3}$ in $\left(-\frac{1}{2},2\right)$

In $(0 - \delta, 0 + \delta)$
$g(x) = (|x| + |4x - 7|).(-3)$
$\Rightarrow$ 'g' is non derivable at $x = 0$.

In $\left(\frac{7}{4}-\delta, \frac{7}{4}+\delta\right)$

$g(x) = 0$ as $f(x) = 0$

$\Rightarrow$ Derivable at $x = \frac{7}{4}$

$\therefore$ 'g' is non-derivable at $0, 1, \sqrt{2}, \frac{7}{4}$

SECTION–3 : (Maximum Marks : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

PARAGRAPH 1

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

51. P(X > Y) is-

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

Ans. (B)

Sol.. $P(X > Y) = P(T_1 \text{ win})\, P(T_1 \text{ win}) + P(T_1 \text{ win})\, P(\text{match draw}) + P(\text{match draw}).P(T_1 \text{ win})$

$$= \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{2} = \frac{5}{12}$$

52. P(X = Y) is-

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

Ans. (C)

Sol. $P(X = Y) = P(\text{match draw})\, P(\text{match Draw}) + P(T_1 \text{ win})\, P(T_2 \text{ win}) + P(T_2 \text{ win})\, P(T_1 \text{ win})$

$$= \frac{1}{6}\times\frac{1}{6} + \frac{1}{2}\times\frac{1}{3} + \frac{1}{3}\times\frac{1}{2} = \frac{13}{36}$$

PARAGRAPH 2

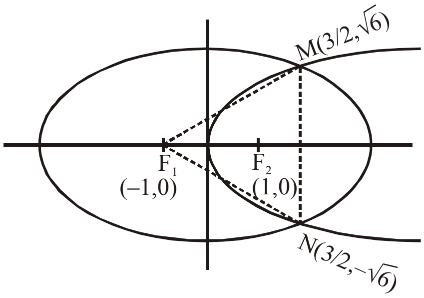
Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

53. The orthocentre of the triangle F_1MN is-

(A) $\left(-\frac{9}{10},0\right)$ (B) $\left(\frac{2}{3},0\right)$ (C) $\left(\frac{9}{10},0\right)$ (D) $\left(\frac{2}{3},\sqrt{6}\right)$

Ans. (A)

Sol.



Orthocentre lies on x-axis

Equation of altitude through M : $y-\sqrt{6}=\frac{5}{2\sqrt{6}}\left(x-\frac{3}{2}\right)$

Equation of altitude through F_1 : $y = 0$

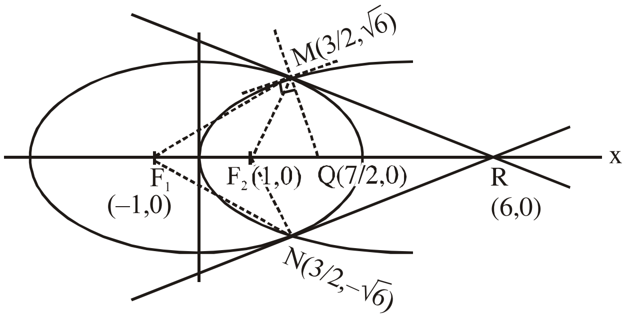
solving, we get orthocentre $\left(-\frac{9}{10},0\right)$

54. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is-

(A) 3 : 4 (B) 4 : 5 (C) 5 : 8 (D) 2 : 3

Ans. (C)

Sol.



Normal to parabola at M : $y-\sqrt{6}=-\frac{\sqrt{6}}{2.1}\left(x-\frac{3}{2}\right)$

Solving it with y = 0, we get $Q \equiv \left(\frac{7}{2}, 0\right)$

Tangent to ellipse at M : $\frac{x.\frac{3}{2}}{9} + \frac{y(\sqrt{6})}{8} = 1$

Solving it with y = 0, we get R ≡ (6, 0)

$\therefore$ Area of triangle MQR $= \frac{1}{2}.\left(6 - \frac{7}{2}\right).\sqrt{6} = \frac{5\sqrt{6}}{4}$

Area of quadrilateral $MF_1NF_2 = 2.\frac{1}{2}.\left(1-(-1)\right).\sqrt{6} = 2\sqrt{6}$

Required ratio = 5 : 8