

# PART C – MATHEMATICS



1. In a set of  $2n$  distinct observations, each of the observation below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations :
- (1) increases by 1      (2) decreases by 2  
 (3) increases by 2      (4) decreases by 1

**Ans.** (1)

**Sol.**  $\frac{t_1 + t_2 + t_3 + \dots + t_n + t_{n+1} + \dots + t_{2n}}{2n} = M$

$$\frac{t_1 + 5 + t_2 + 5 + \dots + t_n + 5 + t_{n+1} - 3 + t_{n+2} - 3 + \dots + t_{2n} - 3}{2n}$$

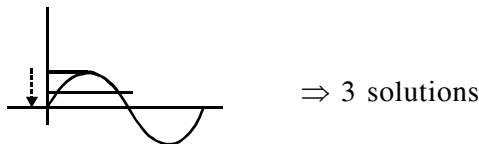
$$\frac{t_1 + t_2 + \dots + t_{n-1} + 5(n) + t_n + t_{n+1} + \dots + t_{2n} - 3(n)}{2n}$$

$$\frac{t_1 + t_2 + t_3 + \dots + t_{2n}}{2n} + \frac{5n - 3n}{2n} = M + 1$$

2. The number of values of  $\alpha$  in  $[0, 2\pi]$  for which  $2 \sin^3 \alpha - 7 \sin^2 \alpha + 7 \sin \alpha = 2$ , is:
- (1) 6      (2) 1      (3) 4      (4) 3

**Ans.** (4)

**Sol.**  $2 \sin^3 \alpha - 2 = 7 \sin^2 \alpha - 7 \sin \alpha$   
 $2 (\sin \alpha - 1) (\sin^2 \alpha + 1 + \sin \alpha) = 7 \sin \alpha (\sin \alpha - 1)$   
 $\Rightarrow \sin \alpha = 1$  or  
 $2 \sin^2 \alpha + 2 + 2 \sin \alpha = 7 \sin \alpha (\sin \alpha - 1)$   
 $\Rightarrow \sin \alpha = 1$  or  $\sin \alpha = \frac{1}{2} \quad \because \sin \alpha \neq -2$



3. Let  $P$  be the relation defined on the set of all real numbers such that  $P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$ . Then  $P$  is :
- (1) reflexive and transitive but not symmetric.  
 (2) reflexive and symmetric but not transitive  
 (3) symmetric and transitive but not reflexive  
 (4) an equivalence relation

**Ans.** (4)

**Sol.** for reflexive :  $\sec^2 a - \tan^2 a = 1$  an identity for all  $x \in \mathbb{R} \Rightarrow$  reflexive

for symmetric :  $\sec^2 a - \tan^2 b = 1 \dots(i)$  to prove  $\sec^2 b - \tan^2 a = 1$

$$\sec^2 b - \tan^2 a = 1 + \tan^2 b - (\sec^2 a - 1) = 1 + \tan^2 b + 1 - \sec^2 a = \sec^2 a - \tan^2 b = 1 \Rightarrow \text{symmetric}$$

[ $\because$  from (1)]

for transitive :

$$\sec^2 a - \tan^2 b = 1 \dots\dots (ii)$$

$$\sec^2 b - \tan^2 c = 1 \dots\dots (iii)$$

to prove :  $\sec^2 a - \tan^2 c = 1$

proof L.H.S.

$$1 + \tan^2 b + 1 - \sec^2 b \text{ from (ii) \& (iii)}$$

$$= \sec^2 b - \tan^2 b \text{ identity}$$

$$= 1$$

$\Rightarrow P$  is reflexive, symmetric and transitive.

4.  $\int \frac{\sin^8 x - \cos^8 x}{(1 - 2 \sin^2 x \cos^2 x)} dx$  is equal to:

(1)  $-\frac{1}{2} \sin 2x + c$       (2)  $-\sin^2 x + c$

(3)  $\frac{1}{2} \sin 2x + c$       (4)  $-\frac{1}{2} \sin x + c$

**Ans.** (1)

**Sol.**  $I = \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{\{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x\}}$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^2 x - \cos^2 x)}{(\sin^4 x + \cos^4 x)}$$

$$= \int -\cos 2x$$

$$= -\frac{\sin 2x}{2} + 2$$

5. The integral  $\int_0^{\frac{1}{2}} \frac{\ell n(1+2x)}{1+4x^2} dx$ , equals:

(1)  $\frac{\pi}{32} \ell n 2$       (2)  $\frac{\pi}{8} \ell n 2$

(3)  $\frac{\pi}{16} \ell n 2$       (4)  $\frac{\pi}{4} \ell n 2$

**Ans.** (3)

**Sol.**  $\int_0^{1/2} \frac{\ln(1+2x)}{1+(2x)^2} dx$  Put  $2x = \tan \theta$

$$dx = \frac{1}{2} \sec^2 \theta d\theta$$

at  $x = 0, \theta = 0$ , at  $x = \frac{1}{2}, \theta = \frac{\pi}{4}$

$$I = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \cdot \frac{1}{2} \sec^2 \theta d\theta$$

$$I = \frac{1}{2} \int_0^{\pi/4} \log(1+\tan\theta) d\theta, \quad \frac{1}{2} I_1$$

$$I_1 = \int_0^{\pi/4} \log[1+\tan(\frac{\pi}{4}-\theta)] \text{ using property}$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan\theta}\right] = \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1+\tan\theta) d\theta$$

$$I_1 = \frac{\pi}{4} \log 2 - I_1$$

$$I_1 = \frac{\pi}{8} \ln 2$$

$$\Rightarrow I = \frac{\pi}{16} \ln 2$$

6. If  $f(x)$  is continuous and  $f(9/2) = 2/9$ , then

$$\lim_{x \rightarrow 0} f\left(\frac{1-\cos 3x}{x^2}\right) \text{ is equal to:}$$

- (1)  $9/2$  (2)  $0$  (3)  $2/9$  (4)  $8/9$

**Ans.** (3)

**Sol.**  $f\left(\frac{2\sin^2 \frac{3x}{2}}{\frac{4}{9} \cdot \frac{3x}{2} \cdot \frac{3x}{2}}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}$

7. The number of terms in the expansion of  $(1+x)^{101} (1+x^2-x)^{100}$  in powers of  $x$  is:

- (1) 202 (2) 302 (3) 301 (4) 101

**Ans.** (1)

**Sol.**  $(1+x)(1+x)^{100} (1+x^2-x)^{100} = (1+x)(1+x^3)^{100}$

$$= \underbrace{1(1+x^3)^{100}}_{101 \text{ terms}} + \underbrace{x(1+x^3)^{100}}_{101 \text{ terms}}$$

and no term is of same exponent of  $x$

$$\Rightarrow 202 \text{ terms}$$

8. Equation of the plane which passes through the point of intersection of lines

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} \text{ and } \frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

and has the largest distance from the origin is:

- (1)  $5x + 4y + 3z = 57$  (2)  $7x + 2y + 4z = 54$   
 (3)  $4x + 3y + 5z = 50$  (4)  $3x + 4y + 5z = 49$

**Ans.** (3)

**Sol.**  $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = \alpha$

$$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \beta$$

Solve the above equation to find the point of intersection i.e.  $(4, 3, 5)$

e.g. of plane with dr's  $l, m, n$  as distance from origin is  $d$  is  $lx + my + nz = d$

dr's of  $(4, 3, 5)$  joined with origin

$$\left(\frac{4}{\sqrt{50}}, \frac{3}{\sqrt{50}}, \frac{5}{\sqrt{50}}\right)$$

$\therefore$  eq of plane

$$\frac{4}{\sqrt{50}}x + \frac{3}{\sqrt{50}}y + \frac{5}{\sqrt{50}}z = \sqrt{50}$$

$$4x + 3y + 5z = 50$$

9. A line in the 3-dimensional space makes an angle  $\theta$  ( $0 < \theta \leq \frac{\pi}{2}$ ) with both the  $x$  and  $y$  axes.

Then the set of all values of  $\theta$  is the interval :

(1)  $\left[0, \frac{\pi}{4}\right]$  (2)  $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

(3)  $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$  (4)  $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

**Ans.** (3)

**Sol.** for min, if the line lies on  $x$   $y$  plane it makes angle of  $45^\circ$

for max. If line at  $z$ -axis it makes an angle of  $90^\circ$

$$\Rightarrow \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

10. If  $\frac{1}{\sqrt{\alpha}}$  and  $\frac{1}{\sqrt{\beta}}$  are the roots of the equation,  $ax^2 + bx + 1 = 0$  ( $a \neq 0, a, b \in \mathbb{R}$ ), then the equation,  $x(x + b^3) + (a^3 - 3abx) = 0$  has roots:

- (1)  $\alpha^{\frac{3}{2}}$  and  $\beta^{\frac{3}{2}}$       (2)  $\alpha^{\frac{3}{2}}$  and  $\beta^{\frac{3}{2}}$   
 (3)  $\alpha\beta^{\frac{1}{2}}$  and  $\alpha^{\frac{1}{2}}\beta$       (4)  $\sqrt{\alpha\beta}$  and  $\alpha\beta$

Ans. (2)

Sol.  $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = -\frac{b}{a}$  also  $\frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a} \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = -b$   
 now  $x(x + b^3) + a^3 - 3abx$   
 $= x^2 + (b^3 - 3ab)x + a^3 = x^2 + b(b^2 - 3a)x + a^3$   
 $= x^2 - (\sqrt{\alpha} + \sqrt{\beta})\{\alpha + \beta + 2\sqrt{\alpha\beta} - 3\sqrt{\alpha\beta}\}x + \alpha\beta\sqrt{\alpha\beta}$   
 $= x^2 - (\alpha\sqrt{\alpha} + \beta\sqrt{\beta}) + \alpha\beta\sqrt{\alpha\beta}$   
 $\Rightarrow$  roots are  $\alpha\sqrt{\alpha}$  and  $\beta\sqrt{\beta}$

11. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4<sup>th</sup> term is :

- (1) 20      (2) 16      (3) 8      (4) 24

Ans. (1)

Sol.  $(12 - d) + 12 + (12 + d) + (12 + 2d) + \dots + 12 + 7d$   
 $= 12 \times 9 + 27d = 108 + 27d$   
 now according to question  
 $200 < 108 + 27d < 220$   
 $92 < 27d < 112$   
 $\frac{92}{27} < d < \frac{112}{27} \Rightarrow d = 4$  only integer  
 $\Rightarrow$  4th term =  $12 + 2d = 12 + 8 = 20$

12. Let  $w$  ( $\text{Im } w \neq 0$ ) be a complex number. Then the set of all complex numbers  $z$  satisfying the equation  $w - \bar{w}z = k(1 - z)$ , for some real number  $k$ , is :

- (1)  $\{z : z = \bar{z}\}$       (2)  $\{z : |z| = 1, z \neq 1\}$   
 (3)  $\{z : |z| = 1\}$       (4)  $\{z : z \neq 1\}$

Ans. (2)

Sol.  $w - \bar{w}z = k - kz$

$$kz - \bar{w}z = k - w$$

$$z = \frac{k - w}{k - \bar{w}} \dots\dots(i)$$

$$\bar{z} = \frac{k - \bar{w}}{k - w} \dots\dots(ii)$$

$$(i) \times (ii)$$

$$z\bar{z} = 1$$

$$|z| = 1$$

$$\text{but } z \neq 1$$

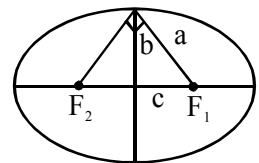
13. If  $OB$  is the semi-minor axis of an ellipse,  $F_1$  and  $F_2$  are its foci and the angle between  $F_1B$  and  $F_2B$  is a right angle, then the square of the eccentricity of the ellipse is :

- (1)  $\frac{1}{\sqrt{2}}$       (2)  $\frac{1}{2}$   
 (3)  $\frac{1}{4}$       (4)  $\frac{1}{2\sqrt{2}}$

Ans. (2)

Sol.  $a^2 = b^2 + c^2$   
 .....(i)

given  $a^2 + a^2 = (2c)^2$   
 $2a^2 = 4c^2$   
 $a^2 = 2c^2$   
 .....(ii)



$$c = ae$$

$$\frac{c}{a} = e$$

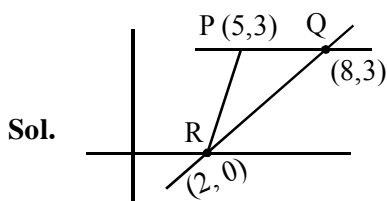
$$\frac{c^2}{a^2} = e^2$$

$$\frac{c^2}{2c^2} = e^2 \Rightarrow e^2 = \frac{1}{2}$$

14. Given three points  $P, Q, R$  with  $P(5, 3)$  and  $R$  lies on the  $x$ -axis. If equation of  $RQ$  is  $x - 2y = 2$  and  $PQ$  is parallel to the  $x$ -axis, then the centroid of  $\Delta PQR$  lies on the line:

- (1)  $x - 2y + 1 = 0$       (2)  $5x - 2y = 0$   
 (3)  $2x + y - 9 = 0$       (4)  $2x - 5y = 0$

Ans. (4)



equation of RQ  $\equiv x - 2y = 2$

$\Rightarrow R(2, 0)$

equation of PQ  $= y = 3$

point of intersection of PQ and RQ

$x - 2(3) = 2$

$x = 8$

$\Rightarrow R(8, 3)$

Centroid  $\left(\frac{2+8+5}{3}, \frac{0+3+3}{3}\right)$

$\equiv (5, 2)$  as is simplified by  $2x - 5y = 0$

15. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the number 3, 4, 5 and 6, without repetition, is:

- (1) 432    (2) 36    (3) 18    (4) 108

Ans. (4)

Sol.  $(6 + 5 + 4 + 3) \cdot 3$

$= 18 \times 6$

$= 108$

16. If  $\operatorname{cosec} \theta = \frac{p+q}{p-q}$  ( $p \neq q \neq 0$ ), then

$\left| \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \right|$  is equal to:

- (1)  $pq$     (2)  $\sqrt{pq}$     (3)  $\sqrt{\frac{q}{p}}$     (4)  $\sqrt{\frac{p}{q}}$

Ans. (3)

Sol.  $\left| \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \right| = \left| \frac{1 - \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2}} \right|$

$= \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}} \times \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}$

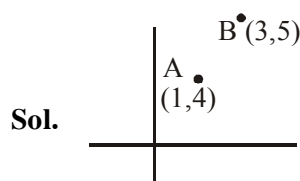
$= \frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} - 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta}$

$= \frac{1 - \frac{p-q}{p+q}}{\sqrt{1 - \left(\frac{p-q}{p+q}\right)^2}} = \frac{\sqrt{q}}{\sqrt{p}}$

17. If the point (1, 4) lies inside the circle  $x^2 + y^2 - 6x - 10y + p = 0$  and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval:

- (1) (25, 29)    (2) (25, 39)  
(3) (9, 25)    (4) (0, 25)

Ans. (1)



$AB = \sqrt{2^2 + 1} = \sqrt{5}$

according to question

$\sqrt{5} < \sqrt{3^2 + 5^2 - p} < q$

$5 < 34 - p < q$

$-29 < -p < -25$

$29 > p > 25$

18. If  $y = e^{nx}$ , then  $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$  is equal to:

- (1) 1    (2)  $-ne^{-nx}$     (3)  $ne^{-nx}$     (4)  $n e^{nx}$

Ans. (2)

**Sol.**  $y = e^{nx}$   $\left\{ \begin{array}{l} \frac{1}{n} \log y = x \\ \frac{1}{n} \left( \frac{1}{y} \right) = \frac{dx}{dy} \\ -\frac{1}{ny^2} = \frac{d^2x}{dy^2} \dots\dots(ii) \end{array} \right.$

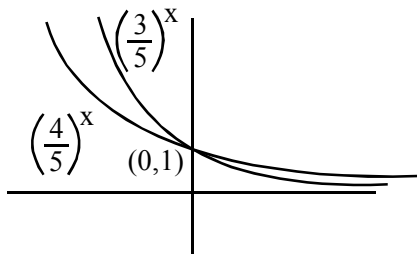
$\frac{dy}{dx} = ne^{nx}$

$\frac{d^2y}{dx^2} = n^2 e^{nx} \dots(i)$

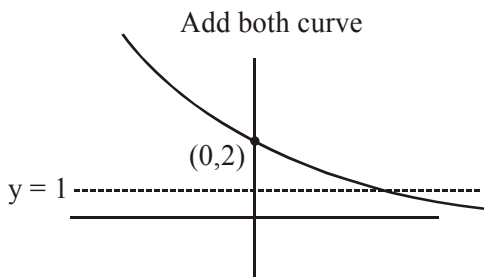
$(i) \times (ii) = n^2 e^{nx} \cdot \frac{1}{ny^2} = \frac{n^2 y}{ny^2} = \frac{n}{y} = \frac{n}{e^{nx}}$

- 19.** If  $f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$ ,  $x \in \mathbb{R}$ , then the equation  $f(x) = 0$  has:
- (1) One solution
  - (2) no solution
  - (3) more than two solutions
  - (4) two solutions

**Ans.** (1)

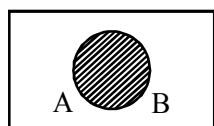


**Sol.**



- 20.** If A and B are two events such that  $P(A \cup B) = P(A \cap B)$ , then the **incorrect** statement amongst the following statements is:
- (1)  $P(A) + P(B) = 1$
  - (2)  $P(A' \cap B) = 0$
  - (3)  $P(A \cap B') = 0$
  - (4) A and B are equally likely

**Ans.** (4)



**Sol.**

$\Rightarrow A = B$

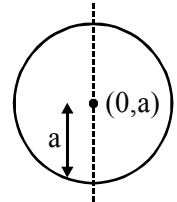
- 21.** If the differential equation representing the family of all circles touching x-axis at the origin

is  $(x^2 - y^2) \frac{dy}{dx} = g(x) y$ , then  $g(x)$  equals:

- (1)  $2x^2$
- (2)  $2x$
- (3)  $\frac{1}{2}x^2$
- (4)  $\frac{1}{2}x$

**Ans.** (2)

**Sol.**  $x^2 + (y - a)^2 = a^2$   
 $x^2 + y^2 - 2ay = 0 \dots(i)$   
 diff. w.r.t. x



$2x + 2y \frac{dy}{dx} - 2a \frac{dy}{dx} = 0$

$a = \frac{x + y \cdot y'}{y'} \dots(ii)$

put (ii) in (i)

$x^2 + y^2 - 2y \left( \frac{x + y \cdot y'}{y'} \right) = 0$   
 $(x^2 - y^2)y' = 2xy \dots(iii)$

compare (iii) with  $(x^2 - y^2) \frac{dy}{dx} = g(x) \cdot y$

gives  $g(x) = 2x$

- 22.** The contrapositive of the statement "I go to school if it does not rain" is:

- (1) If it rains, I go to school.
- (2) If it rains, I do not go to school.
- (3) If I go to school, it rains.
- (4) If I do not go to school, it rains.

**Ans.** (4)

**Sol.** Contrapositive of  $p \rightarrow q$  is  $\sim q \rightarrow \sim p$

- 23.** Let a and b be any two numbers satisfying

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$ . Then, the foot of perpendicular

from the origin on the variable line,  $\frac{x}{a} + \frac{y}{b} = 1$ ,

lies on:

- (1) a circle of radius = 2
- (2) a circle of radius =  $\sqrt{2}$
- (3) a hyperbola with each semi-axis =  $\sqrt{2}$
- (4) a hyperbola with each semi-axis = 2

**Ans.** (1)

**Sol.** Equation of  $\perp$

$$\frac{h-0}{\frac{1}{a}} = \frac{k-0}{\frac{1}{b}} = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}}$$

$$\Rightarrow a = \frac{4}{h} \dots\dots(i) \text{ and } b = \frac{4}{k} \dots\dots(ii)$$

to find locus of (h, k) put

$$(i) \text{ and } (ii) \text{ in } \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$$

$$\text{i.e. } \frac{h^2}{16} + \frac{k^2}{16} = \frac{1}{4}$$

$$\Rightarrow \text{locus is } \frac{x^2}{16} + \frac{y^2}{16} = \frac{1}{4}$$

$$\Rightarrow x^2 + y^2 = 4$$

**24.** If  $|\vec{a}|=2$ ,  $|\vec{b}|=3$  and  $|2\vec{a}-\vec{b}|=5$ , then

$|2\vec{a}+\vec{b}|$  equals:

- (1) 1 (2) 17  
(3) 5 (4) 7

**Ans.** (3)

**Sol.**  $|2\vec{a}-\vec{b}|^2=25$

$$4|\vec{a}|^2+|\vec{b}|^2-4\vec{a}\cdot\vec{b}=25$$

$$16+9-4\vec{a}\cdot\vec{b}=25$$

$$4\times\vec{a}\cdot\vec{b}=0 \dots\dots(i)$$

now

$$|2\vec{a}+\vec{b}|=k$$

$$(2\vec{a}+\vec{b})(2\vec{a}+\vec{b})=k^2$$

$$4|\vec{a}|^2+|\vec{b}|^2+4\vec{a}\cdot\vec{b}=k^2$$

$$\sqrt{16+9+0}=k$$

$$5=k$$

**25.** If the sum

$$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \dots + \text{upto 20 terms}$$

is equal to  $\frac{k}{21}$ , then k is equal to:

- (1) 240 (2) 120 (3) 180 (4) 60

**Ans.** (2)

$$\text{Sol. } t_n = \frac{2n+1}{n(n+1)(2n+1)} = \frac{6}{n(n+1)} = 6\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

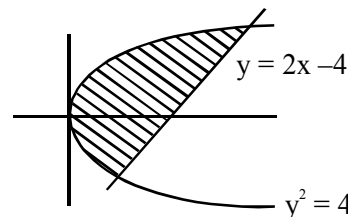
$$S_n = 6\left\{\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \dots - \frac{1}{21}\right\} = 6\left\{\frac{1}{1} - \frac{1}{21}\right\}$$

$$= 6\left(\frac{20}{21}\right) = \frac{120}{21} \Rightarrow k = 120$$

**26.** Let  $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$ . The area (in square units) of the region A is :

- (1) 11 (2) 9 (3) 8 (4) 10

**Ans.** (2)



**Sol.**

solve for y;  $y^2 = 4x$  as  $y - 2x = -4$   
gives  $y = -2, 4$

$$\Rightarrow \text{Area} = \int_{-2}^4 \left(\frac{y+4}{2} - \frac{y^2}{4}\right) dy = \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^4 = 9$$

**27.** If equations  $ax^2 + bx + c = 0$ , ( $a, b, c \in \mathbb{R}$ ,  $a \neq 0$ ) and  $2x^2 + 3x + 4 = 0$  have a common root, then  $a : b : c$  equals :

- (1) 2 : 3 : 4 (2) 3 : 2 : 1  
(3) 1 : 2 : 3 (4) 4 : 3 : 2

**Ans.** (1)

**Sol.**  $2x^2 + 3x + 4 = 0$  as  $D \leq 0$

$\Rightarrow$  both roots are imaginary  $\Rightarrow$  both roots are common

$$\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$$

**28.** If a, b, c are non-zero real numbers and if the system of equations

$$(a-1)x = y+z,$$

$$(b-1)y = z+x,$$

$$(c-1)z = x+y,$$

has a non-trivial solution, then  $ab + bc + ca$  equals :

- (1) 1 (2)  $a+b+c$  (3)  $abc$  (4)  $-1$

**Ans.** (3)

**Sol.** for non-trivial solution  $D = 0$

$$\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_3 \quad \begin{vmatrix} -a & 0 & c \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix}$$

$$\Rightarrow a \{(1-b)(1-c)-1\} + c\{1-(1-b)\}=0$$

$$\Rightarrow ab + ac + bc - abc = 0$$

$$\Rightarrow ab + ac + bc = abc$$

**29.** If the Rolle's theorem holds for the function  $f(x) = 2x^3 + ax^2 + bx$  in the interval  $[-1, 1]$  for

the point  $c = \frac{1}{2}$ , then the value of  $2a + b$  is:

- (1) 2      (2) -2      (3) -1      (4) 1

**Ans.** (3)

**Sol.**  $f(-1) = -2 + a - b$ ,  $f(1) = 2 + a + b$

$$f(-1) = f(1) \Rightarrow -2 + a - b = 2 + a + b$$

$$-2 = b$$

$$f'(x) = 6x^2 + 2ax + b$$

$$f'\left(\frac{1}{2}\right) = 6 \cdot \frac{1}{4} + 2 \cdot a \cdot \frac{1}{2} + b = 0$$

$$\Rightarrow \frac{3}{2} + a + b = 0 \quad (\because b = -2)$$

$$\Rightarrow a = \frac{1}{2} \quad \because 2a + b = -1$$

**30.** If  $B$  is a  $3 \times 3$  matrix such that  $B^2 = 0$ , then  $\det. [(I + B)^{50} - 50B]$  is equal to:

- (1) 1                                      (2) 2  
(3) 3                                      (4) 50

**Ans.** (1)

**Sol.**  $[(1 + B)^{50} - 50B] = 1 + 50B + \frac{50 \cdot 49}{2} B^2 + \dots - 50B$

$$= 1 + B^2 \{ \dots \} = 1 + 0 \{ \dots \} = 1$$