JEE ADVANCED (Paper - 2)

PHYSICS Code - 4

Section 1 (Maximum Marks: 32)

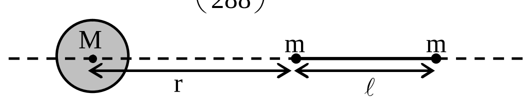
- This section contains **EIGHT** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, **both** inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- Marking scheme:

  +4 If the bubble corresponding to the answer is darkened.

  0 In all other cases.

1. An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

*2. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



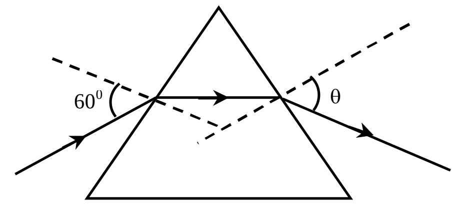
3. The energy of a system as a function of time t is given as $E(t) = A^2\exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25 %. If the error in the measurement of time is 1.50 %, the percentage error in the value of E(t) at t = 5 s is

*4. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

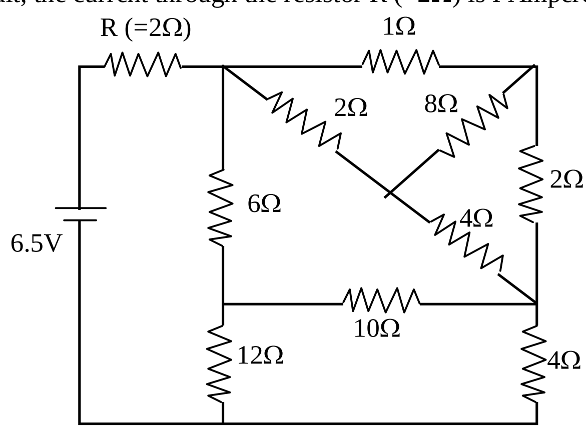
*5. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

6. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q(mean life 2τ) have the same activity at t = 0. Their rates of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

7. A monochromatic beam of light is incident at 60^0 on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60^0 and $\frac{d\theta}{dn} = m$. The value of m is



8. In the following circuit, the current through the resistor R (=2Ω) is I Amperes. The value of I is



Section 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
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  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

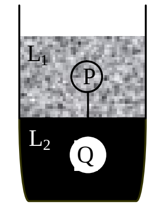
  0 If none of the bubbles is darkened

  –2 In all other cases

9. A fission reaction is given by ${}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2MeV) and K_y(2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV respectively. Considering different conservation laws, the correct option(s) is(are)

(A) x = n, y = n, K_{Sr} = 129MeV, K_{Xe} = 86 MeV
(B) x = p, y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV
(C) x = p, y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV
(D) x = n, y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

*10. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$

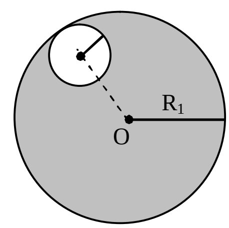
(C) $\vec{V}_P . \vec{V}_Q > 0$ (D) $\vec{V}_P . \vec{V}_Q < 0$

11. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are)

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$

(C) $I = \varepsilon_0 c V$ (D) $\mu_0 c I = \varepsilon_0 V$

12. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = a = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are)



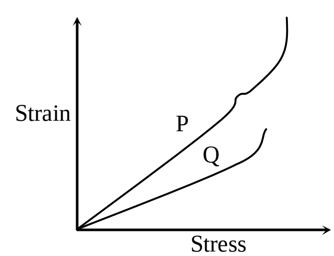
(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of *a* but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

*13. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



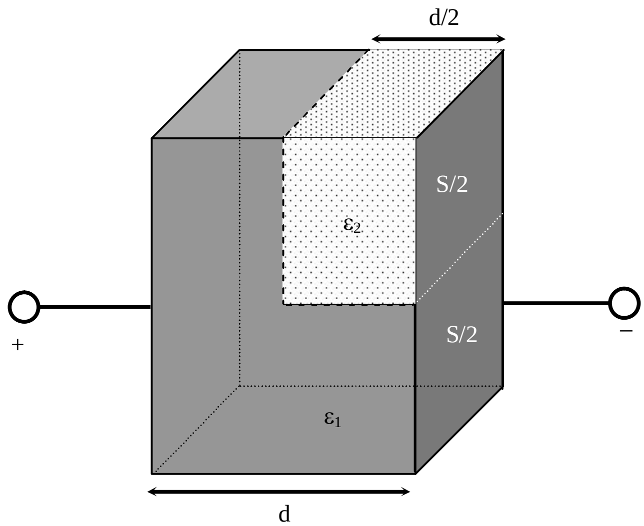
(A) P has more tensile strength than Q
(B) P is more ductile than Q
(C) P is more brittle than Q
(D) The Young's modulus of P is more than that of Q

*14. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are)

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

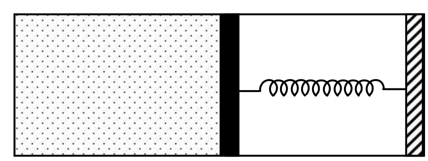
(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$ (D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

15. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is



(A) 6/5 (B) 5/3
(C) 7/5 (D) 7/3

*16. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are)



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
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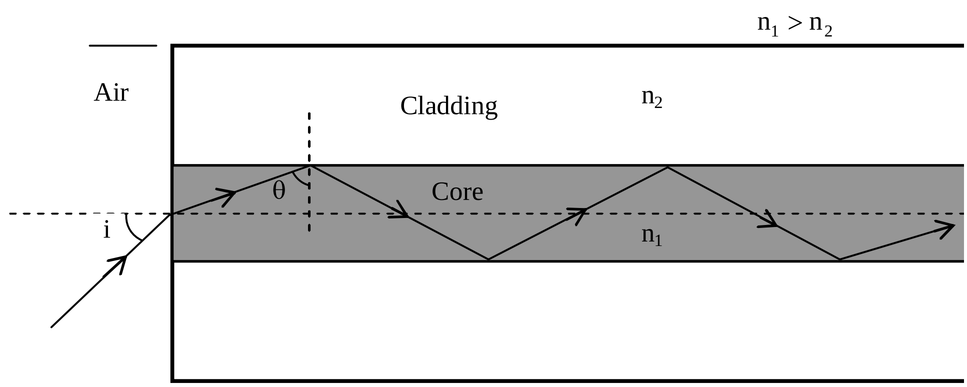
PARAGRAPH 1

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



17. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with n_1 = 8/5 and n_2 = 7/5 and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are)

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.

(D) NA of S_1 placed in air is the same as that of S_2 placed in water

18. If two structures of same cross-sectional area, but different numerical apertures NA_1 and $NA_2 (NA_2 < NA_1)$ are joined longitudinally, the numerical aperture of the combined structure is

(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$

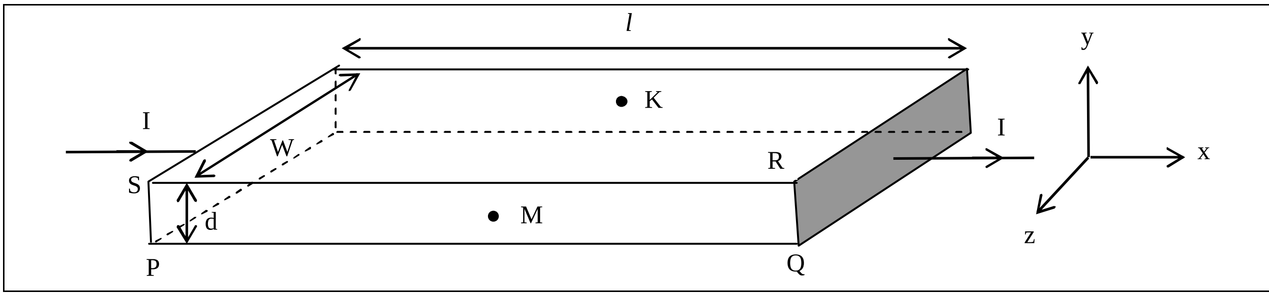
(B) $NA_1 + NA_2$

(C) NA_1

(D) NA_2

PARAGRAPH 2

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are ℓ, w and d, respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



19. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$
(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$
(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

20. Consider two different metallic strips (1 and 2) of same dimensions (lengths ℓ, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$
(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

PAPER-2 [Code – 4]
JEE (ADVANCED) 2015
ANSWERS

PHYSICS

1.	**2**	2.	**7**	3.	**4**	4.	**6**
5.	**3**	6.	**2**	7.	**2**	8.	**1**
9.	**A**	10.	**A, D**	11.	**A, C**	12.	**D**
13.	**A, B**	14.	**B, C**	15.	**D**	16.	**B or A, B, C**
17.	**A, C**	18.	**D**	19.	**A, D**	20.	**A, C**

SOLUTIONS

PHYSICS

1. $mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi}$

de-Broglie Wavelength $\lambda = \frac{h}{mv} = \frac{2\pi r}{3} = \frac{2\pi}{3}\frac{a_0(3)^2}{z_{Li}} = 2\pi a_0$

2. For m closer to M

$\frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2} = ma$ ...(i)

and for the other m :

$\frac{Gm^2}{\ell^2} + \frac{GMm}{16\ell^2} = ma$ ...(ii)

From both the equations,

k = 7

3. $E(t) = A^2 e^{-\alpha t}$

$\Rightarrow dE = -\alpha A^2 e^{-\alpha t} dt + 2AdAe^{-\alpha t}$

Putting the values for maximum error,

$\Rightarrow \frac{dE}{E} = \frac{4}{100} \Rightarrow$ % error = 4

4. $I = \int \frac{2}{3}\rho 4\pi r^2 r^2 dr$

$I_A \propto \int (r)(r^2)(r^2)\, dr$

$I_B \propto \int (r^5)(r^2)(r^2) dr$

$\therefore \frac{I_B}{I_A} = \frac{6}{10}$

5. First and fourth wave interfere destructively. So from the interference of 2nd and 3rd wave only,

$\Rightarrow I_{net} = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) = 3I_0$

$\Rightarrow n = 3$

6. $\lambda_P = \frac{1}{\tau}$; $\lambda_Q = \frac{1}{2\tau}$

$\frac{R_P}{R_Q} = \frac{(A_0\lambda_P)e^{-\lambda_P t}}{A_0\lambda_Q e^{-\lambda_Q t}}$

At $t = 2\tau$; $\frac{R_P}{R_Q} = \frac{2}{e}$

7. Snell's Law on 1st surface : $\frac{\sqrt{3}}{2} = n\sin r_1$

$\sin r_1 = \frac{\sqrt{3}}{2n}$ ...(i)

$\Rightarrow \cos r_1 = \sqrt{1-\frac{3}{4n^2}} = \frac{\sqrt{4n^2-3}}{2n}$

$r_1 + r_2 = 60°$ ...(ii)

Snell's Law on 2nd surface :

$n\sin r_2 = \sin\theta$

Using equation (i) and (ii)

$n\sin(60° - r_1) = \sin\theta$

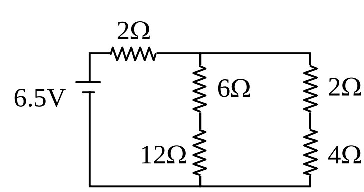
$n\left[\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1\right] = \sin\theta$

$\frac{d}{dn}\left[\frac{\sqrt{3}}{4}\left(\sqrt{4n^2-3}-1\right)\right] = \cos\theta\frac{d\theta}{dn}$

for $\theta = 60°$ and $n = \sqrt{3}$

$\Rightarrow \frac{d\theta}{dn} = 2$

8. Equivalent circuit :



$R_{eq} = \frac{13}{2}\Omega$

So, current supplied by cell = 1 A

9. Q value of reaction = (140 + 94) × 8.5 – 236 × 7.5 = 219 Mev

So, total kinetic energy of Xe and Sr = 219 – 2 – 2 = 215 Mev

So, by conservation of momentum, energy, mass and charge, only option (A) is correct

10. From the given conditions, $\rho_1 < \sigma_1 < \sigma_2 < \rho_2$

From equilibrium, $\sigma_1 + \sigma_2 = \rho_1 + \rho_2$

$V_P = \frac{2}{9}\left(\frac{\rho_1 - \sigma_2}{\eta_2}\right)g$ and $V_Q = \frac{2}{9}\left(\frac{\rho_2 - \sigma_1}{\eta_1}\right)g$

So, $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ and $\vec{V}_P \cdot \vec{V}_Q < 0$

11. $BI\ell c \equiv VI \Rightarrow \mu_0 I^2 c \equiv VI \Rightarrow \mu_0 Ic = V$

$\Rightarrow \mu_0^2 I^2 c^2 = V^2$

$\Rightarrow \mu_0 I^2 = \varepsilon_0 V^2 \Rightarrow \varepsilon_0 cV = I$

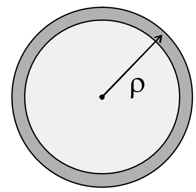
12. $\vec{E} = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1C_2}$

$C_1 \Rightarrow$ centre of sphere and $C_2 \Rightarrow$ centre of cavity.

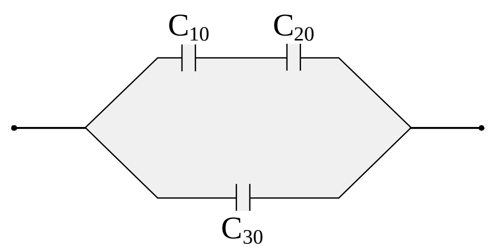
13. $Y = \dfrac{\text{stress}}{\text{strain}}$

$\Rightarrow \dfrac{1}{Y} = \dfrac{\text{strain}}{\text{stress}} \Rightarrow \dfrac{1}{Y_P} > \dfrac{1}{Y_\theta} \Rightarrow Y_P < Y_Q$

14. P(r) = $K\left(1 - \dfrac{r^2}{R^2}\right)$



15. $C_{10} = \dfrac{4\varepsilon_0 \frac{S}{2}}{d/2} = \dfrac{4\varepsilon_0 S}{d}$



$C_{20} = \dfrac{2\varepsilon_0 S}{d}$, $C_{30} = \dfrac{\varepsilon_0 S}{d}$

$\dfrac{1}{C'_{10}} = \dfrac{1}{C_{10}} + \dfrac{1}{C_{10}} = \dfrac{d}{2\varepsilon_0 S}\left[1 + \dfrac{1}{2}\right]$

$\Rightarrow C'_{10} = \dfrac{4\varepsilon_0 S}{3d}$

$C_2 = C_{30} + C'_{10} = \dfrac{7\varepsilon_0 S}{3d}$

$\dfrac{C_2}{C_1} = \dfrac{7}{3}$

16. P (pressure of gas) = $P_1 + \dfrac{kx}{A}$

$W = \int PdV = P_1(V_2 - V_1) + \dfrac{kx^2}{2} = P_1(V_2 - V_1) + \dfrac{(P_2 - P_1)(V_2 - V_1)}{2}$

$\Delta U = nC_V\Delta T = \dfrac{3}{2}(P_2V_2 - P_1V_1)$

$Q = W + \Delta U$

Case I: $\Delta U = 3P_1V_1$, $W = \dfrac{5P_1V_1}{4}$, $Q = \dfrac{17P_1V_1}{4}$, $U_{spring} = \dfrac{P_1V_1}{4}$

Case II: $\Delta U = \dfrac{9P_1V_1}{2}$, $W = \dfrac{7P_1V_1}{3}$, $Q = \dfrac{41P_1V_1}{6}$, $U_{spring} = \dfrac{P_1V_1}{3}$

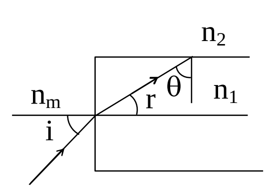
Note: A and C will be true after assuming pressure to the right of piston has constant value P_1.

17. $\theta \geq c$

$\Rightarrow 90° - r \geq c$

$\Rightarrow \sin(90° - r) \geq c$

$\Rightarrow \cos r \geq \sin c$



using $\dfrac{\sin i}{\sin r} = \dfrac{n_1}{n_m}$ and $\sin c = \dfrac{n_2}{n_1}$

we get, $\sin^2 i_m = \dfrac{n_1^2 - n_2^2}{n_m^2}$

Putting values, we get, correct options as A & C

18. For total internal reflection to take place in both structures, the numerical aperture should be the least one for the combined structure & hence, correct option is D.

19. $I_1 = I_2$

$\Rightarrow neA_1v_1 = neA_2v_2$

$\Rightarrow d_1w_1v_1 = d_2w_2v_2$

Now, potential difference developed across MK

$V = Bvw$

$$\Rightarrow \frac{V_1}{V_2} = \frac{v_1w_1}{v_2w_2} = \frac{d_2}{d_1}$$

& hence correct choice is A & D

20. As $I_1 = I_2$

$n_1w_1d_1v_1 = n_2w_2d_2v_2$

Now, $$\frac{V_2}{V_1} = \frac{B_2v_2w_2}{B_2v_1w_1} = \left(\frac{B_2w_2}{B_1w_1}\right)\left(\frac{n_1w_1d_1}{n_2w_2d_2}\right) = \frac{B_2n_1}{B_1n_2}$$

$\therefore$ Correct options are A & C