JEE (ADVANCE) - 2016
MATHEMATICS

SECTION 1 (Maximum Marks: 18)

- This section contains **SIX** questions
- Each question has FOUR option (A), (B), (C) and (D). **ONLY ONE** of these four option is correct.
- For each question, darken the bubble corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –1 In all other cases.

37. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

(A) $x + y - 3z = 0$ (B) $3x + z = 0$ (C) $x - 4y + 7z = 0$ (D) $2x - y = 0$

Key (C)

Sol: Image of point (3, 1, 7) in $x - y + z = 3$

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = -2\frac{(3-1+7-3)}{3}$$

$x = -1, y = 5, z = 3$

So Image is P(–1, 5, 3)

Now line is $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$

It passes through O(0, 0, 0)

d.r. of OP = –1, 5, 3

Let d.r. of normal to plane be a, b, c

$a + 2b + c = 0$

$-a + 5b + 3c = 0$

$$\frac{a}{6-5} = \frac{b}{-1-3} = \frac{c}{5+2}$$

So, equation of plane will be

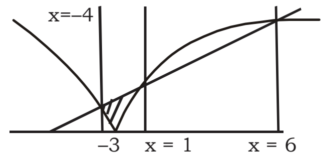
$x - 4y + 7z = 0$

38. Area of the region $\left\{(x, y) \in ¡^{2} : y \ge \sqrt{|x+3|}, 5y \le x + 9 \le 15\right\}$ is equal to

(A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Key (C)

Sol: $\sqrt{|x+3|} = \frac{x+9}{5} \Rightarrow x = -4, 1, 6$



So required area will be

$$\int_{-4}^{-3}\left(\frac{x+9}{5}-\sqrt{-x-3}\right)dx+\int_{-3}^{1}\left(\frac{x+9}{5}-\sqrt{x+3}\right)dx$$

$$=\left[\frac{x^2}{10}+\frac{9x}{5}+\frac{2}{3}(-x-3)^{3/2}\right]_{-4}^{-3}+\left[\frac{x^2}{10}+\frac{9x}{5}-\frac{2}{3}(x+3)^{\frac{3}{2}}\right]_{-3}^{-1}=\frac{3}{2}$$

39. Let $b_i > 1$ for I = 1, 2, …, 101. Suppose $\log_e b_1$, $\log_e b_2$, …., $\log_e b_{101}$ are in Arithmetic Progression (A.P) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$, then

(A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

Key (B)

Sol: $\log_e b_1, \log_e b_2, \ldots\ldots \log_e b_{101}$ are in A.P. with common difference $\log_e 2 = \log_e 2$

$\log_e b_2 - \log_e b_1 = \log_e b_3 - \log_e b_2$

$\frac{b_2}{b_1} = 2, \frac{b_3}{b_2} = 2$

$b_2 = 2b_1, b_3 = 2b_2, \ldots. b_{101} = 2b_{100}$

$b_1, b_2, b_3 \ldots. b_{101}$ are in G.P. ⇒ first term = b common ratio = r

$a_1, a_2 \ldots.. a_{101}$ are in A.P. ⇒ first term = a common difference = d

$a_1 = b_1$ and $a + 50d = a.r^{50}$ …(1)

If given t = $b_1 + b_2 + \ldots.. b_{51}$

$= \frac{b(r^{50}-1)}{r-1} = \frac{a(r^{50}-1)}{r-1}$

$S = \frac{51}{2}(2a+50d)$

Now, $S - t = \frac{51}{2}(2a+50d) - \left(\frac{a.r^{50}-a}{r-1}\right)$

$= \frac{51}{2}(a+a_{51}) - (a_{51}-a)$

$= \frac{51(a+a_{51}) - a_{51} + a}{2} = \frac{52a + 50a_{51}}{2} > 0$

Now, $a_{101} - b_{101}$

$a + 100d - a.r^{100}$

$a + 50d + 50d - ar^{100}$

$ar^{50} + 50d - ar^{100}$

$ar^{50} - ar^{100} + ar^{50} - a$

$a\left(2^{51} - 2^{100} - 1\right) < 0$

40. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

(A) $3 - \sqrt{3}$ (B) $2\left(3 - \sqrt{3}\right)$ (C) $2\left(\sqrt{3} - 1\right)$ (D) $2\left(2 + \sqrt{3}\right)$

Key (C)

Sol: $T_k = \frac{1}{\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right).\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$

let $A = \frac{\pi}{4} + \frac{k\pi}{6}$ and $B = \frac{\pi}{4} + (k-1)\frac{\pi}{6}$

Now $T_k = 2(\cot B - \cot A)$

$= 2\left(\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right) - \cos\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)$

$T_1 = 2\left[\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right]$

$T_2 = 2\left[\cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)\right]$

$T_{13} = 2\left[\cot\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right]$

$S_{13} = 2\left[\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right]$

$S_{13} = 2\left(1 - \frac{\sqrt{3} - 1}{\sqrt{3} + 1}\right)$

$= 2\left(\sqrt{3} - 1\right)$

41. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If Q = [q_{ij}] is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals

(A) 52 (B) 103 (C) 201 (D) 205

Key (B)

Sol: $P\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 0 \end{bmatrix}$$

$$\text{So } P^n = \begin{bmatrix} 1 & 0 & 0 \\ 4n & 1 & 0 \\ 8n(n+1) & 4n & 1 \end{bmatrix}$$

$$\Rightarrow P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix}$$

$$\text{Given } Q = P^{50} - I = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 20400 & 200 & 0 \end{bmatrix}$$

Now $q_{31} = 20400$

$q_{32} = 200$

$q_{21} = 200$

$$\frac{q_{31} + q_{32}}{q_{21}} = \frac{20600}{200} = 103$$

42. The value of $\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx$ is equal to

(A) $\frac{\pi^2}{4} - 2$ (B) $\frac{\pi^2}{4} + 2$ (C) $\pi^2 - e^{\pi/2}$ (D) $\pi^2 + e^{\pi/2}$

Key (A)

Sol: (A)

$$I = \int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx \qquad \text{... (i)}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{(-x)^2 \cos(-x)}{1+e^{-x}} dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^x x^2 \cos x}{1+e^x} dx \quad \text{... (ii)}$$

On adding (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx$$

On solving, we will get

$$I = \frac{\pi^2}{4} - 2$$

SECTION 2 (Maximum Marks: 18)

- This section contains **EIGHT** questions
- Each question has Four options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +4 if only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

  Zero Marks : 0 if none of the bubbles is darkened.

  Negative Marks : –2 in all other cases
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

43. Let $f : R \to (0, \infty)$ and $g : R \to R$ be twice differentiable functions such that f'' and g'' are continuous functions on R. Suppose $f'(2) = g(2) = 0$, $f''(2) \neq 0$ and $g'(2) \neq 0$. If $\lim_{x\to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(A) f has a local minimum at x = 2

(B) f has a local maximum at x = 2

(C) $f''(2) > f(2)$

(D) $f(x) - f''(x) = 0$ for at least one $x \in R$

Key (A, D)

Sol. (A, D)

$$\lim_{x\to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1 \qquad \left(\frac{0}{0}\right) \text{ form}$$

$$\lim_{x\to 2} \frac{f(x)g'(x) + g(x)f'(x)}{f'(x)g''(x) + f''(x)g'(x)} = 1$$

$$\Rightarrow \frac{f(2)}{f''(2)} = 1 \Rightarrow f(2) = f''(2)$$

$$\Rightarrow f''(2) > 0$$

Hence f(x) has local minimum at x = 2.

44. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the centre S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

(A) SP = $2\sqrt{5}$

(B) SQ : QP = $(\sqrt{5}+1):2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

Key (A, C, D)

Sol. (A, C, D)

Since P is at the shortest distance from S, hence SP is common normal of circle and parabola.

Equation of normal to the parabola $y^2 = 4x$ is $y = mx - 2m - m^3$

This passes through S(2, 8)

$\Rightarrow 8 = 2m - 2m - m^3 \Rightarrow m = -2$

Hence, equation of normal is $2x + y = 12$

P is given by (4, 4) and SP = $2\sqrt{5}$

and, SQ : QP = $1:\sqrt{5}-1$ or $\sqrt{5}+1:4$

45. Let $a, b \in R$ and $f : R \to R$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is

(A) differentiable at x = 0 if a = 0 and b = 1

(B) differentiable at x = 1 if a = 1 and b = 0

(C) **NOT** differentiable at x = 0 if a = 1 and b = 0

(D) **NOT** differentiable at x = 1 if a = 1 and b = 1

Key (A, B)

Sol. (A, B)

$f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$

f(x) can be written as

$f(x) = a\cos(x^3 - x) + bx\sin(x^3 + x)$

f(x) is differentiable everywhere

Hence correct options are ‘A’ and ‘B’

46. Let $f:\left[-\frac{1}{2},2\right] \to R$ and $g:\left[-\frac{1}{2},2\right] \to R$ be function defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where [y] denotes the greatest integer less than or equal to y for $y \in R$. Then

(A) f is discontinuous exactly at three points in $\left[-\frac{1}{2},2\right]$

(B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

(C) g is **NOT** differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$

(D) g is **NOT** differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

Key (B, C)

Sol. $f(x) = \begin{cases} -3 & \frac{-1}{2} \le x < 1 \\ -2 & 1 \le x < \sqrt{2} \\ -1 & \sqrt{2} \le x < \sqrt{3} \\ 0 & \sqrt{3} \le x < 2 \\ 1 & x = 2 \end{cases}$

Clearly function f is discontinuous exactly at four points is $\left[-\frac{1}{2}, 2\right]$

$$g(x) = \begin{cases} 15x - 21 & -\frac{1}{2} \le x < 0 \\ 9x - 21 & 0 \le x < 1 \\ 6x - 14 & 1 \le x < \sqrt{2} \\ 3x - 7 & \sqrt{2} \le x < \sqrt{3} \\ 0 & \sqrt{3} \le x < \frac{7}{4} \\ 0 & \frac{7}{4} \le x < 2 \\ 3 & x = 2 \end{cases}$$

Function g is non-differentiable at $x = 0, 1, \sqrt{2}, \sqrt{3}$ i.e at 4 points in $\left(-\frac{1}{2}, 2\right)$

47. Let $f(x) = \lim_{n\to\infty} \left(\dfrac{n^n (x+n)\left(x+\frac{n}{2}\right)........\left(x+\frac{n}{n}\right)}{n!\left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right).....\left(x^2+\frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$, for all $x > 0$ Then

(A) $f\left(\frac{1}{2}\right) \ge f(1)$ (B) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$ (C) $f'(2) \le 0$ (D) $\frac{f'(3)}{f(3)} \ge \frac{f'(2)}{f(2)}$

Key (A, B, C, D)

Sol. $f(x)=\lim_{n\to\infty}\left(\frac{n^n.n^n\left(\frac{x}{n}+1\right)\left(\frac{x}{n}+\frac{1}{2}\right)\left(\frac{x}{n}+\frac{1}{3}\right).....\left(\frac{x}{n}+\frac{1}{n}\right)}{n!\left(n^2\right)^n\left(\frac{x^2}{n^2}+1\right)\left(\frac{x^2}{n^2}+\frac{1}{4}\right).....\left(\frac{x^2}{n^2}+\frac{1}{n^2}\right)}\right)^{\frac{x}{n}}$

$f(x)=1$

So A,B,C,D all are correct

48. Let $\alpha,\lambda,\mu\in R.$ Consider the system of linear equations

$$\alpha x+2y=\lambda$$
$$3x-2y=\mu$$

Which of the following statement(s) is (are) correct?

(A) If $\alpha=-3,$ then the system has infinitely many solutions for all values of λ and μ

(B) If $\alpha\neq-3,$ then the system has a unique solution for all values of λ and μ

(C) If $\lambda+\mu=0$, then the system has infinitely many solutions for $\alpha=-3$

(D) If $\lambda+\mu\neq0,$ then the system has no solution for $\alpha=-3$

Key (B,C,D)

Sol. In the Equation.

$a_1x+b_1y+c_1=0$
$a_2x+b_2y+c_2=0$

If $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\Rightarrow$ unique solution

So 'B' is correct

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}\Rightarrow$ no solution

So 'D' is correct

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow$ infinitely many solution

So 'C' is correct

49. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^3 and $\hat{\omega}=\frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right).$ given that there exists a vector $\vec{v}$ in R^3 such that $|\hat{u}\times\vec{v}|=1$ and $\hat{\omega}.\left(\hat{u}\times\vec{v}\right)=1.$ Which of the following statement(s) is (are) correct?

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choices for such $\vec{v}$

(C) If $\hat{u}$ lies in the *xy*-plane then $|u_1|=|u_2|$

(D) If $\hat{u}$ lies in the *xz*-plane then $2|u_1|=|u_3|$

Key (B, C)

Sol. $|\hat{u}\times\vec{v}|=1$ and $\hat{\omega}.(\hat{u}\times\vec{v})=1$

$\Rightarrow |\hat{\omega}|\ |\hat{u}\times\vec{v}|\cos\theta=1$

$\Rightarrow |\hat{\omega}|\ \cos\theta=1$

$\Rightarrow \cos\theta=1$

$\Rightarrow \hat{\omega}$ is parallel to $\hat{u}\times\vec{v}$

Hence $\hat{\omega}$ is perpendicular to $\hat{u}$ and $\hat{\omega}$ is perpendicular to $\vec{v}$.

Let $\vec{v}=x_1\hat{i}+x_2\hat{j}+x_3\hat{k}$ then $\hat{\omega}.\vec{v}=0$

$\Rightarrow \quad x_1+x_2+2x_3=0 \qquad (i)$

u_1,u_2,u_3 are constants.

$|\hat{u}\times\vec{v}|=1$

$\Rightarrow |\hat{u}||\vec{v}|\sin\alpha=1$

$\Rightarrow |\vec{v}|\sin\alpha=\frac{1}{|\hat{u}|}=1$

$\Rightarrow \sqrt{x_1^2+x_2^2+x_3^3}\sin\alpha=1$

So by changing angle α and values of x_1,x_2,x_3 we have. In finitely choices for $\vec{v}$

Now if $\hat{u}$ lies in the xy plane.

Then. $u_3=0$ and $\hat{\omega}.\hat{u}=0 \Rightarrow u_1+u_2=0$

$\Rightarrow u_1=-u_2$

Hence $|u_1|=|u_2|$

Hence option C is correct

Now if $\hat{u}$ lies in the $xz-$plane, $u_2=0$

$\hat{\omega}.\hat{u}=0$

$\Rightarrow u_1+2u_3=0$

$\Rightarrow u_1=-2u_3$

$\Rightarrow |u_1|=|-2u_3|=2|u_3|$

Hence option D is incorrect

50. Let $a,b\in R$ and $a^2+b^2\neq 0$. Suppose $S=\left\{z\in C: z=\frac{1}{a+ibt}, t\in R, t\neq 0\right\}$, where $i=\ -1$. If $z=x+iy$ and $z\in S$, then (x,y) lies on

(A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a},0\right)$ for $a>0, b\neq 0$

(B) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a},0\right)$ for $a<0, b\neq 0$

(C) the x-axis for $a\neq 0, b=0$

(D) the y-axis for $a=0, b\neq 0$

Key (A, C, D)

Sol. $x+iy=\frac{1}{a+ibt}$

$\Rightarrow (x+iy)(a+ibt)=1$

$\Rightarrow ax-byt-1+i(btx+ay)=0$

$\Rightarrow ax-1-byt=0$

And $btx+ay=0$

$\Rightarrow t=\frac{-ay}{bx}$

$\Rightarrow ax-by\left(\frac{-ay}{bx}\right)-1=0$

$\Rightarrow ax^2+ay^2-x=0$

$\Rightarrow x^2+y^2-\frac{x}{a}=0$

$\Rightarrow x^2-2.\frac{1}{2a}x+\left(\frac{1}{2a}\right)^2-\left(\frac{1}{2a}\right)^2+y^2=0$

$\Rightarrow \left(x-\frac{1}{2a}\right)^2+y^2=\left(\frac{1}{2a}\right)^2$

Hence locus is circle with centre $\left(\frac{1}{2a},0\right)$ and radius $\frac{1}{2a}$ if $a>0$, $b\neq 0$

Hence option A is correct

If $a<0.$ then as radius remains positive radius = $\frac{-1}{2a}$ but centre will be remain $\left(\frac{1}{2a},0\right)$

Hence option B is incorrect.

Now if , $a\neq 0, b=0$

$x+iy=\frac{1}{a}$

$ax+ayi-1=0$

$\Rightarrow ax-1=0$ and $ay=0$

$\Rightarrow x=\frac{1}{a}$ and $y=0$

Hence option C is correct

If $a = 0, \quad b \neq 0$

$$x + iy = \frac{1}{ibt}$$

$\Rightarrow xbti - byt = 1$

$\Rightarrow xbti = 1 + byt.$

$\Rightarrow xbt = 0 \,\&\, byt = -1$

$$t = \frac{-1}{by}$$

$$\Rightarrow \frac{-x}{y} = 0$$

$\Rightarrow x = 0$

Hence locus is y-axis

Hence option D is correct.

Section 3 (Maximum Marks : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question darken the bubble corresponding to the correct option in the ORS.
- For each question marks will e awarded in one of the following categories

Full marks : +3 If only the bubble corresponding to the correct option is darkend.

Zero marks: 0 In all other cases

Paragraph 1

Football teams T_1 *and* T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ *and* $\frac{1}{3}$, respectively. Each team gets 3 point for a win, 1 point for a draw and 0 point for a loss in game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

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51. $P(X > Y)$ is

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$

(C) $\frac{1}{2}$ (D) $\frac{7}{12}$

Key. B

Sol. For Team T_1

Probability of T_1 wining $=\frac{1}{2}$

Probability of T_1 drawing $=\frac{1}{6}$

Probability of T_1 Losing $=\frac{1}{3}$

Number of possibilities in which $P(X>Y)$

$=P\{WW,WD,DW\}$

$=P(WW)+P(WD)+P(DW)$

$=P(W)P(W)+P(W)P(D)+P(D)P(W)$

$=\frac{1}{4}+\frac{1}{6}\times\frac{1}{2}+\frac{1}{6}\times\frac{1}{2}=\frac{1}{4}+\frac{1}{12}+\frac{1}{12}=\frac{5}{12}$

52. $P(X=Y)$ is

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$

(C) $\frac{13}{36}$ (D) $\frac{1}{2}$

Key (C)

Sol. Number of possible cases are WL, DD, LW

$P(X=Y)=P(WL)+P(DD)+P(LW)$

$=P(W)P(L)+P(D)P(D)+P(L)P(W)$

$=\frac{1}{2}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}=\frac{13}{36}$

Paragraph 2

Let $F_1(x_1,0)$ and $F_2(x_2,0)$, for $x_1<0$ *and* $x_2>0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

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53. The orthocentre of the triangle F_1MN is

(A) $\left(-\frac{9}{10},0\right)$ (B) $\left(\frac{2}{3},0\right)$

(C) $\left(\frac{9}{10},0\right)$ (D) $\left(\frac{2}{3},\sqrt{6}\right)$

Key (A)

Sol. $\frac{x^2}{9}+\frac{y^2}{8}=1$ $\qquad F_2=(1,0)$

$F_1=(-1,0)$

$\frac{x^2}{3^2}+\frac{y^2}{\left(2\sqrt{2}\right)^2}=1$

$e=\sqrt{1-\frac{8}{9}},\ e=\frac{1}{3},\ a=3$

Equation of parabola $y^2=4x$

By solving parabola & ellipse $\frac{x^2}{9}+\frac{4x}{8}=1$

$2x^2+9x-18=0$

$2x^2+12x-3x-18=0$

$2x(x+6)-3(x+6)=0$

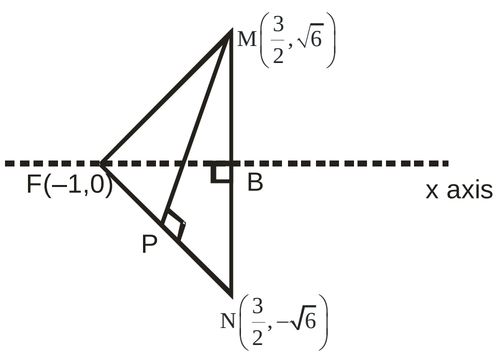
$x=\frac{3}{2}$

$x=-6$

We take $x=\frac{3}{2}$ only as $x>0$

So $y^2=4\times\frac{3}{2}$

$y=\pm\sqrt{6}$



Solving MP and F_1B

We get orthocentre $\equiv\left(-\frac{9}{10},0\right)$

54. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the $x-axis$ at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(A) 3 : 4 (B) 4 : 5

(C) 5 : 8 (D) 2 : 3

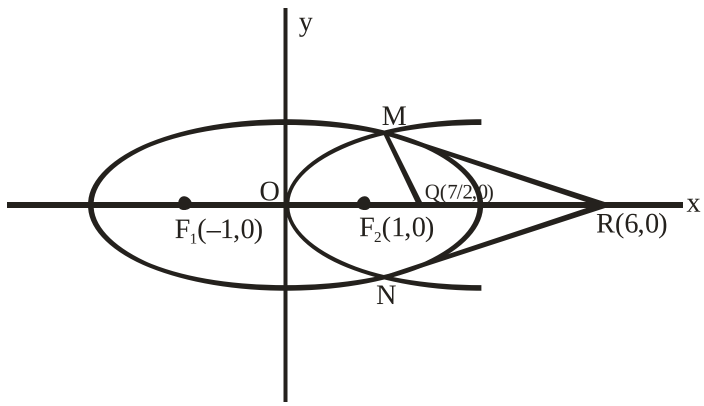
Key (C)

Sol. Equation of tangent $\frac{x}{9}\times\frac{3}{2}+\frac{y}{8}\sqrt{6}-1=0$

$R\equiv(6,0)$

Equation of Normal at M to the parabola $y^2=4x$ is $y-\sqrt{6}=-\frac{\sqrt{6}}{2}\left(x-\frac{3}{2}\right)$

$Q\equiv\left(\frac{7}{2},0\right)$



Area of ΔQMR $\Delta QMR=\frac{5\sqrt{6}}{4}$

Area of $MF_1NF_2=2\sqrt{6}$

Required Ratio $=\frac{5}{8}$