Paper-1

JEE Advanced, 2015

Part III –Mathematics

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.

2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.

3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.

4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.

5. Blank spaces are provided within this booklet for rough work.

6. Write your name and roll number in the space provided on the back cover of this booklet.

7. After breaking the seal of the booklet. Verify that the booklet contains 32 pages and that all the 60 questions along with the options are legible.

Question paper format and marking scheme:

8. The question paper has three parts: Physics, Chemistry and Mathematics. Each part has three sections.

9. Carefully read the instructions given at the beginning of each section.

10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).

    Marking scheme: +4 correct answer and 0 in all other cases.

11. Section 2 contains 10 multiple choice questions with one or more than one correct option.

    Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

12. Section 3 contains 2 " match the following" type questions and you will have to match entries in Column I with the entries in Column II.

Marking scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and -1 in all other cases.

OPTICAL RESPONSE SHEET :

13. The ORS consists of an original (top sheet) and its carbon-less copy. (bottom sheet).

14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.

15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.

16. You will be allowed to take away the carbon-less copy at the end of the examination.

17. Do not tamper with or mutilate the ORS.

18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Q.41 Let $F(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x \in R$ and $f: \left[0,\frac{1}{2}\right] \rightarrow [0,\infty)$ be a continuous functions. For $\alpha \in \left[0,\frac{1}{2}\right]$, if $F'(\alpha)+2$ is the area of the region bounded by $x = 0$, $y=0$, $f(x)$ and $x = \alpha$, then $f(0)$ is

Ans.41 (3)

Q.42 A cylindrical container is to be made from certain solid material with the following constrains: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the value of $\frac{V}{250\pi}$ is

Ans.42 (4)

Q.43 Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Ans.43 (5)

Q.44 The minimum number of times a fair coin need to be tossed, so that the probability of getting at least two heads is at least 0.96 is

Ans.44 (8)

Q45. If the normal the parabola $y^2 = 4x$ drawn at the end points of its latus return are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is

Ans.45 (2)

Q.46 Let f : R → R be a function defined by $f(x)=\begin{cases}[x], x \le 2\\ 0, x > 2\end{cases}$

Where [x] is the greatest integer less then or equal to x. If $l = \int_1^2 \frac{xf(x^2)}{2+f(x+1)}dx$, then the value of (4l - 1) is

Ans.46 (0)

Q.47 The number of distinct solution of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

Ans.47 (8)

Q.48 Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$. Then the distance between A and B is

Ans.48 (4)

SECTION 2 (Maximum Marks: 40)

- This section contains **TEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubbles(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:
  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
  0 If none of the bubbles is darkened
  -2 In all other cases

Q.49 Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

(A) $(4, 2\sqrt{2})$ (B) $(9, 3\sqrt{2})$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $(1, \sqrt{2})$

Ans.49 (A,D)

Q.50 Let y(x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If y (0) = 2, then which of the following statements is (are) true?

(A) y (-4) = 0

(B) y (-2) = 0

(C) y (x) hs a critical point in the interval (-1,0)

(D) y(x) has no critical point in the interval (-1,0)

Ans.50 (A,C)

Q.51 Consider the family of all circle whose centres lie on the straight line y = x. if this family of circle is represented by the differential equation Py" + Qy' + 1 = 0, where P,Q are functions of x, y and y' (here $y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}$) , then which of the following statements is (are) true?

(A) P = y+x (B) P = y – x

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

Ans.51 (B,C)

Q.52 Let g: R →R be a differentiable function with g (0)= 0, g' (0)= 0 and g'(1)≠0, Let

$$f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and $h(x) = e^{|x|}$ for all $x \in R$. Let $(f \cdot h)(x)$ denote $f(h(x))$ and $(h \cdot f)(x)$ denote $h(f(x))$.

Then which of the following is (are)true?

(A) f is differentiable at x = 0

(B) h is differentiable at x = 0

(C) $f \cdot h$ is differentiable at x = 0

(D) $h \cdot f$ is differentiable at x = 0

Ans.52 (A,D)

Q.53 Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{\pi}{2}\sin x$ for all $x \in R$. Let $(f \cdot g)(x)$ denote $f(g(x))$ and $(g \cdot f)(x)$denote $g(f(x))$. Then which of the following is (are)true?

(A) Range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) Range of f·g is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

(D) There is an $x \in R$ such that $(g \cdot f)(x) = 1$

Ans.53 (A,B,C)

Q.54 Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are)true?

(A) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

(B) $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

(C) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

(D) $\vec{a}.\vec{b} = -72$

Ans.54 (A,C,D)

Q.55 Let X and Y be two arbitrary, 3× 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4 - Z^4Y^3$

(B) $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$

(D) $X^{23} + Y^{23}$

Ans.55 (C,D)

Q.56 Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648a\ ?$$

(A) -4 (B)9 (C) -9 (D)4

Ans.56 (B,C)

Q.57 In R^3, consider the planes P_1 :y = 0 and P_2:x+z = 1. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α,β,γ)from P_3 is 2, then which of the following relations is (are)true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$

(B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$

(D) $2\alpha - \beta + 2\gamma - 8 = 0$

Ans.57 (B,C)

Q.58 In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes P_1:x+2y – z + 1= 0 and P_2 :2x – y + z – 1 = 0. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$

(B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

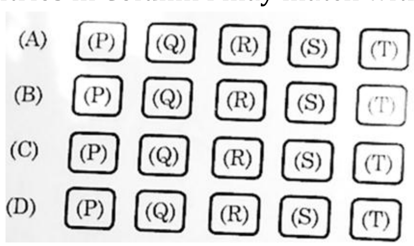
(C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

(D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Ans.58 (A), (B), (C), (D)

SECTION 3 (Maximum Marks: 16)

- This section contains TWO questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has four entries (A), (B), (C) and (D)
- **Column II** has five entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **column II**
- One or more entries in Column I may match with one or more entries in **Column II**



- For each entry in **Column I,** darken the bubbles of all the matching entries For example, if entry (A) in
  Column I matches with entries (Q), (R) and (T) then darken these three bubbles in the ORS. Similarly, for entries, (B), (C) and (D).
- Marking scheme:
  <u>For each entry in **Column I.**</u>
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened
  0 If none of the bubbles is darkened
  -1 In all other cases

Q.59 **Column I** **Column II**

(A) In R^2, if the magnitude of the projection Vector of the vector $\alpha\hat{\imath} + \beta\hat{\jmath}$ on $\sqrt{3}\hat{\imath} + \hat{\jmath}$ is $\sqrt{3}$ and If $\alpha = 2 + \sqrt{3}\beta$, then possible value(s) of $|\alpha|$ is (are) — (P) 1

(B)Let a and b be real numbers such that the function — (Q) 2

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ bx + a^2, & x \geq 1 \end{cases}$$

is differentiable for all x ∈ R. then possible value(s) of a is (are)

(C) Let ω ≠1 be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+$ (R) 3

$(2+3\omega-3\omega^2)^{4n+3} + (-3+2\omega+3\omega^2)^{4n+3} = 0$, then possible

Value(s)of n is (are)

(D) Let the harmonic mean of two positive real numbers (S) 4

a and b be 4. If q is a positive real number such that a,5,q,b

is an arithmetic progression, then value(s) of |q −a | is (are)

(T) 5

Ans.59 (A→ P.Q), (B →P, Q), (C→ P,Q,S,T), (D → Q,T)

Q.60 **Column I** **Column II**

(A) in a triangle ΔXYZ, let a, b and c be the (P) 1

length of the sides opposite to the angles X, Y

and Z, respectively, If $2(a^2-b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$,

then possible values of n for which $\cos(n\pi\lambda) = 0$

is (are)

(B) In a triangle ΔXYZ, let a,b and c be the length (Q) 2

of the sides opposite to the angles X,Y and Z, respectively.

If 1 + cos2X – 2cos2Y = 2sinX sin Y, then possible value(s)

of $\frac{a}{b}$ is (are)

(C) In R^2,let $\sqrt{3}\hat{\imath} + \hat{\jmath}$, $\hat{\imath} + \sqrt{3}\,\hat{\jmath}$ and $\beta\hat{\imath} + (1 - \beta)\hat{\jmath}$ (R) 3

be the position vectors of X,Y and Z with respect to the

origin O, respectively, If the distance of Z from the bisector

of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $|\beta|$ is (are)

(D) Suppose that $F(\alpha)$ denotes that area of the region bounded   (S) 5

By $x = 0$, $x = 2$, $y^2 = 4x$ and $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x$, where

$\alpha \in \{0,1\}$ .then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$,

when $\alpha = 0$ and $\alpha = 1$, is (are)

(T) 6

Ans.60 (A → P, R, S), (B → P), (C → P,Q), (D →S,T)

Answer Keys and Explanations

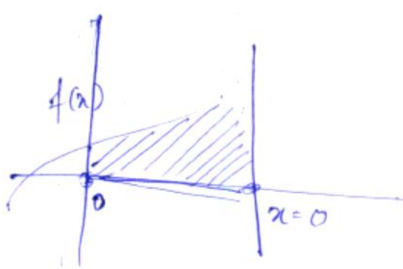
Sol.41 (3)

$$F(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$$

$$F`(x) = 2\cos^2\left(x^2+\frac{\pi}{6}\right)(2x) - 2\cos^2 x$$

$$\left[\int_{h(x)}^{g(x)} f(t)\,dt = \left\{f\left(g(x)\right)g'(x) - f\left(h(x)\right)h'(x)\right\}\right]$$

$$F`(\alpha) = 4\alpha\cos^2\left(a^2+\frac{\pi}{6}\right) - 2\cos^2 a$$



Area bounded = $\int_0^{\alpha} f(x)\,dx$

Acc to question

$$\int_0^{\alpha} f(x)\,dx = F'(\alpha) + 2$$

$$\int_0^{\alpha} f(x)\,dx = 2\cos^2\left[\alpha^2+\pi/6\right]\times 2\alpha - 2\cos^2\alpha\times 1 + 2$$

Differentiating both the sides

$$f(x) = 4x\cos^2\left[x^2+\pi/6\right] + 4x\times 2\cos\left[x^2+\pi/6\right]\times -\sin$$

$$\left[x^2+\pi/6\right] + 2\times 2\cos x\times\sin x$$

$$f(0) = 4\times\frac{3}{4} + 0 = 3$$

Sol.42 (4)

Fixed volume = V Let height be h

Let inner radius be r $V = \pi r^2 h$

Outer radius = r + 2 $h = \frac{V}{\pi r^2}$

(Volume of material = Outer Volume – Inner Volume around curved surface area)

$= \pi(r+2)^2 h - V$

Total volume of material (say V_1) $= \pi(r+2)^2 h - V + \pi(r+2)^2 \times 2$

$= \pi \frac{(r+2)^2 V}{\pi r^2} - V + 2\pi\,(r+2)^2$

$V_1 = (1+\frac{2}{r})^2 V - V + 2\pi(r+2)^2$

$V_1' = 2V\left(1+\frac{2}{r}\right)\left(\frac{-2}{r^2}\right) + 4\pi(r+2)$

Now V_1 is minimum at r = 10

$V_1' = 0 \; at \; r = 10$

On solving $\frac{V}{250\pi} = 4$

Sol.43 (5)

When five girls are standing consecutively let us consider them as 1 unit. So now 1 unit of girls and 5 boys can be arrange in 6! Ways Girls themselves can be arranged in 5! Ways

Total ways n = 6! × 5!

When 4 girls are standing consecutively

First select 4 girls out of 5 in $^5C_4$ ways and then consider them as 1 unit

4 girls and other girl will be arranged in gaps between 5 Boys.

$m = {}^5C_4 5! \, {}^6C_2 \, 2! \, 4!$

So, $\frac{m}{n} = 5$

Sol.44 (8)

Let n be the number of times coin is tossed

Using binomial distribution Probability At least two heads = Total – 1 head – No head

$$P(x \geq 2) = 1 - {}^{n}C_{1}\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} - {}^{n}C_{o}\left(\frac{1}{2}\right)^{n}$$

$$= 1 - n\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{n}$$

Now $P(x \geq 2) \geq 0.96$

$1 - \frac{(n+1)}{2^n} \geq 0.96$

$0.04 \geq \frac{(n+1)}{2^n}$

$(0.04)2^n \geq (n+1)$

By hit and trial.

For n = 8 (min.)

Satisfies above equation

$n = 8$

Sol.45 (2)

$y^2 = 4x$

So, a = 1, so coordinates of LR: (1, 2) and (1, -2).

So, $m_T = \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y} = \frac{2}{2} = 1$

$m_N = -1$

So, eqn. of normal is,

$y - 1 = -1(x - 2)$

$y + x = 3$

Now, solve this line with circle and put D = 0 as this time is tangent to circle so it will touch at one point only.

$(-y)^2 + (y + 2)^2 = r^2$

$2y^2 + 4y + (4 - r^2) = 0$

$D = 16 - 8(4 - r^2) = 0$

$2 = 4 - r^2$

Hence, $r^2 = 2$

Sol.46 (0)

$$f(x) = \begin{cases} [x] & x \le 2 \\ 0 & x > 2 \end{cases}$$

$$I = \int_1^2 \frac{xf(x^2)}{2+f(x+1)}\, dx$$

$$f(x^2) = \begin{cases} [x^2] & x^2 \le 2 \\ 0 & x^2 > 2 \end{cases}$$

$$= \begin{cases} [x^2] & -\sqrt{2} < x \le \sqrt{2} \\ 0 & x > \sqrt{2} \quad x < -\sqrt{2} \end{cases}$$

$-1 < x < 2$

$f(x)^2 = [x^2]\ -1 \le x \le \sqrt{2}$

$0 \qquad x > \sqrt{2}$

$$f(x+1) = \begin{cases} [x+1] & x+1 \le 2 \\ 0 & x+1 > 2 \end{cases}$$

$$f(x+1) = \begin{cases} [x+1] & x \le 1 \\ 0 & x > 1 \end{cases}$$

In (-1, 2)

$$f(x+1) = \begin{cases} [x+1] & -1 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

	(-1, 0)	(0, 1)	(1, √2)	(√2, √3)	(√3, 2)
$f(x^2)$	0	0	1	0	0
$f(x+1)$	0	1	0	0	0

$$I = \int_1^2 \frac{xf(x^2)y1}{2+f(x+1)} = \int_1^{\sqrt{2}} \frac{x(1)dx}{2+0}$$

$= \frac{x^2}{4} \Big|_1^{\sqrt{2}} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$4I - 1 = 4 \times \frac{1}{4} - 1 = 0$

Sol.47 (8)

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x)^2 + (\sin^2 x)^2 + (\cos^2 x)^3 + (\sin^2 x)^3 = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x + (\cos^2 x + \sin^2 x)^3 - 3\sin^2 x\cos^2 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - 2\cos^2 x \sin^2 x + 1 - 3\cos^2 x \sin^2 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x = 5\cos^2 x \sin^2 x$

$\Rightarrow \cos^2 2x - \sin^2 2x = 0$

$\cos 4x = 0$

x lies in $[0, 2\pi]$

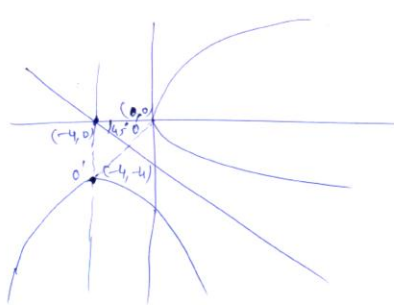
4x lies in $[0, 8\pi]$

In 0, 2π $\cos t = 0$

At $t = \frac{\pi}{2}, \frac{3\pi}{2}$

In 0, 8π there will be 8 solutions.

Sol.48 (4)



So equation of image parabola is $(x+4)^2 = -4(y+4)$

Now solve with $y = -5$

$(x+4)^2 = -4(-5+4)$

$(x+4)^2 = 4$

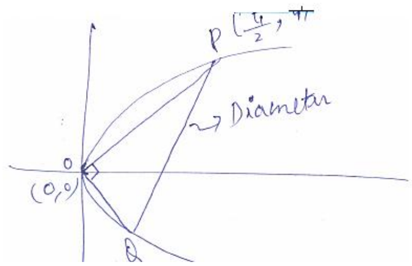
$x + 4 = \pm 2$

$x = -4 \pm 2$

$x = -6, -2$

So distance b/w then 4.

Sol.49 (A,D)



$Y^2 = 2x$

$\therefore a = \frac{1}{2}$

Now, $m_{op}\, m_{od} = -1$

$\frac{t_1}{\left(\frac{t_1^2}{2}\right)} \times \frac{t_2}{\left(\frac{t_2^2}{2}\right)} = -1$

$t_1 t_2 = -4$ ..... (1)

Area of triangle$(\Delta) = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{t_1^2}{2} & t_1 & 1 \\ \frac{t_2^2}{2} & t_2 & 1 \end{vmatrix} = 3\sqrt{2}$ ..... (2)

Solving (1) and (2), we get,

$t_1 = 2\sqrt{2}$ and $t_2 = \sqrt{2}$

Sol.50 (A,C)

$(1 + e^x)\, y' + e^x y = 1$

$\frac{d}{dx}[(1 + e^x)y] = 1$

Integrating both sides

$(1+e^x)Y = x + c$

$Y = \frac{x+c}{1+e^x}$

$Y(0) = 2$

$\frac{c}{1+1} = 2 \Rightarrow C = 4$

$Y = \frac{x+4}{1+e^x}$

$Y(-4) = 0$

$Y' = \frac{(1+e^x)-(x+4)e^x}{(1+e^x)^2}$

$Y' = \frac{1^{-e^x}[x+3]}{(1+e^x)^2}$

$Y'(0) = \frac{1-3}{2^2} = \frac{-2}{2^2} = \frac{-1}{2}$

$Y'(-1) = \frac{1-2e^{-1}}{(1+e^{-1})^2} > 0$

So it will have critical point in (-1,0)

Sol.51 (B,C)

$(x-a)^2 + (y-a)^2 = r^2$

$2(x-a) + 2(y-a)\frac{dy}{dx} = 0$

$x - a + (y - a)\frac{dy}{dx} = 0 \Rightarrow x - a + yy' - ay' = 0$

$a = \frac{x+yy'}{1+y'}$

again differentiating

$(y - a)y'' + y'y' + 1 = 0$

$1 + (y - \frac{x+yy'}{1+y'})y'' + y'y' = 0$

$1 + y' + (y - x)y'' + y'y' + y'y'y' = 0$

$Y' + y'[1 + y' - y' \cdot y'] + 1 = 0$

$P = Y - X$

P + Q = 1 + - X + Y + Y' + (Y')2

Sol.52 (A,D)

$$f(x) = \begin{cases} g(x) & x > 0 \\ -g(x) & x < 0 \\ 0 & x = 0 \end{cases}$$

$$h(x) = \begin{cases} e^{x} & x > 0 \\ e^{-x} & x < 0 \end{cases}$$

$$f(h(x)) = \begin{cases} g(e^{x}) & e^{x} > 0, \quad x \geq, 0 \\ -g(e^{x}) & e^{x} < 0, \quad x \geq, 0 \quad \text{its wrong} \\ 0 & e^{x} > 0, \quad x \geq 0x \quad \text{its wrong} \end{cases}$$

$$= \begin{cases} g(e^{-x}) & e^{-x} > 0, \; x < 0 \quad \text{its correct} \\ -g(e^{-x}) & e^{-x} < 0, \; x < 0 \quad \text{its wrong} \\ 0 & e^{-x} > 0, \; x < 0 \quad \text{its wrong} \end{cases}$$

$$f(h(x)) = \begin{cases} g(e^{x}) & x \geq 0 \\ g(e^{-x}) & x < 0 \end{cases}$$

$$(foh)' = \begin{cases} g'\left(e^{x}\right) \times e^{x} & x \geq 0 \\ -g'\left(e^{-x}\right) \times e^{-x} & x < 0 \end{cases}$$

$$h(f(x)) = \begin{cases} e^{g(x)} & g(x) > 0, \; x > 0 \\ e^{-g(x)} & -g(x) > 0, \; x < 0 \; g(x) < 0 \\ e^{0} & g(x) > 0, \; x < 0 \end{cases}$$

$$h(f(x)) = \begin{cases} e^{-g(x)} & g\left(x\right) < 0, & x > 0 & g\left(x\right) < 0 \\ e^{g(x)} & -g\left(x\right) < 0, & x < 0 & g\left(x\right) > 0, x < 0 \\ e^{-0} & 0 > 0, & x = 0 & \end{cases}$$

$$h(f(x)) = \begin{cases} e^{g(x)} & g(x) > 0 \\ e^{-g(x)} & g(x) < 0 \\ 1 & \end{cases}$$

$h(f(x)) = e^{g(x)} \times g'x$

$h(f(x)) = e^{-g(x)} \times g'(x) \times -1$

Sol.53 (A,B,C)

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$g\ (x) = \frac{\pi}{2}\sin x$

Now, $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$\theta = \frac{\pi}{2}\sin x\ \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{\pi}{6}\sin\theta\right) \in \left[\frac{-1}{2}, \frac{1}{2}\right]\ \therefore$ (A)

$f(g(x)) = \sin\left[\frac{\pi}{6}\sin\frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\sin x\right)\right)\right]$

$\frac{\pi}{2}\sin x = \theta$ for $x \in R$

$\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$f\ (g(x) = \sin\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\theta\right)$

$\frac{\pi}{2}\sin\theta = \propto$

$\propto \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$f(g(x)) = \sin\left(\frac{\pi}{6}\sin\propto\right)$

For $\propto \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\ \frac{\pi}{6}\sin\propto \in \left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$

$\Rightarrow \sin\left(\frac{\pi}{6}\sin\propto\right) \in \frac{-1}{2}, \frac{1}{2}\ \therefore$ (B)

Now, $\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{6}sin\left(\frac{\pi}{2}sin\ x\right)\right)}{\frac{\pi}{2}\sin x}$

$= \lim_{x\to 0} \frac{2}{\pi} \frac{\sin\left(\frac{\pi}{6} sin\left(\frac{\pi}{2} sinx\right)\right)}{\frac{\pi}{6}\sin(\frac{\pi}{2} sinx)} \times \frac{\frac{\pi}{6}\sin(\frac{\pi}{2}\sin x)}{\frac{sinx}{x}xx}$

$= \lim_{x\to 0} \frac{2}{\pi} \times \frac{\pi}{6} \times \frac{\sin\left(\frac{\pi}{2} sinx\right)}{\frac{\pi}{2} sinx} \times \frac{\frac{\pi}{2} sinx}{x}$

$\frac{1}{3} \times \frac{\pi}{2} = \frac{\pi}{6}$ $\quad\quad \therefore$ (C)

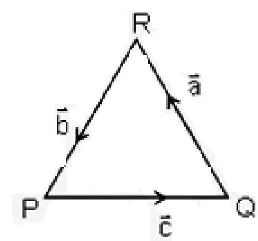
$\Rightarrow \frac{\pi}{2}\sin\left(\frac{\pi}{2}\left(\frac{\pi}{2} sinx\right)\right) = 1$

$\Rightarrow \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2} sinx\right)\right) = \frac{2}{\lambda} \cong \frac{2}{3.14} > \frac{1}{2}$ $\quad \therefore$ (D)

Sol.54 (A,C,D)

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$



$\Rightarrow 48 + \vec{c}^2 + 48 = 144$

$\Rightarrow \vec{c}^2 = 48$

$\Rightarrow \frac{|\vec{c}|^2}{2} - |\overrightarrow{a}| = 24 - 12 = 12$ $\quad$ Ans $\quad$ (A)

Further

$\vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow 144 + 48 + 2\vec{a}\vec{b} = 48$

$\Rightarrow \vec{a}\vec{b} = -72$ $\quad$ Ans $\quad$ (D)

$\because \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2\,|\vec{a} \times \vec{b}| = 2.\sqrt{144.48 - (72)^2} = 48\sqrt{3}$ $\quad$ Ans. $\quad$ (C)

Sol.55 (C,D)

(A) $(Y^3Z^4 - Z^4 Y^3)^T = -Y^3Z^4 + Z^4Y^3$

$\Rightarrow Y^3Z^4 - Z^4Y^3$ is Skew-symmetric

(C) $(X^4Z^3 - Z^3X^4)^T = (X^4Z^3)^T(Z^3X^4)^T$

$= Z^3X^4 - X^4Z^3$

$= -(X^4Z^3 - Z^3X^4)$

(D) $(X^{23} + Y^{23})^T = -X^{23} - Y^{23}$ $\Rightarrow X^{23} + Y^{23}$ is Skew – symmetric

Sol.56 (B,C)

$R_3 \to R_3 - R_2$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+a)^2 & (1+2a)^2 & (1+3a)^2 \\ 3+2a & 3+4a & 3+6a \\ 5+2a & 5+4a & 5+6a \end{vmatrix} = -648a$$

$R_3 \to R_3 \to R_2$

$$\begin{vmatrix} (1+a)^2 & (1+2a)^2 & (1+3a)^2 \\ 3+2a & 3+4a & 3+6a \\ 2 & 2 & 2 \end{vmatrix} = -648\,a$$

$C_3 \to C_3 - C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+a)^2 & a(2+3a) & a(2+5a) \\ 3+2a & 2a & 2a \\ 2 & 0 & 0 \end{vmatrix} = 648a$$

$\Rightarrow 2a^2(2+3a) - 2a^2(2+5a) = -324a$

$\Rightarrow -4a^3 = -324a$ $\Rightarrow a = 0, \pm 9$

Sol.57 (B,C)

$y + \lambda\,[x + z - 1] = 0$

$$\left|\frac{1-\lambda}{\sqrt{\lambda^2+\lambda^2+1}}\right| = 1$$

$1 + \lambda^2 - 2\lambda = 2\lambda^2 + 1$

$\lambda^2 + 2\lambda = 0$

$\lambda = 0 \,/\, \lambda = -2$

$2x + 2z - y - 2 = 0$

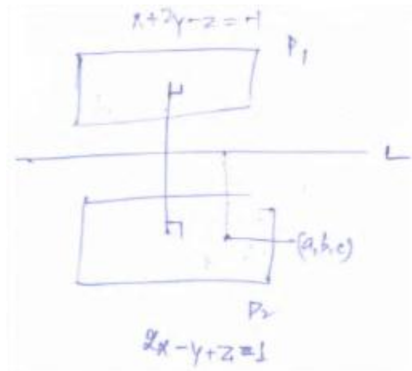
$$\left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{2^2+2^2+1}}\right| = 2$$

$2\alpha + 2\gamma - \beta - 2 = \pm 6$

$2\,\alpha - ß + 2\,\gamma = 8$

$2\,\alpha - ß + 2\,\gamma = -4$

Sol.58 (A), (B), (C), (D)



$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$$

a + 2b – c = 0

2a – b + c = 0

3a + b = 0 $\qquad a = \frac{-b}{3}$

b = -3a

2a + 4b – 2c = 0

-2a + b – c = 0

5b – 3c = 0 $\qquad c = \frac{5b}{3}$

$\therefore \frac{x}{\frac{-b}{3}} = \frac{y}{b} = \frac{z}{\frac{5b}{3}} = k$

$\frac{x}{\frac{-1}{3}} - \frac{y}{1} = \frac{z}{\frac{5}{3}} = k'$

$\frac{x}{-1} = \frac{y}{3} = \frac{z}{5} = k$

2a – b + c = -1

Sol.59 (A→ P.Q)

(A) $\left|(\alpha\hat{\imath} + \beta\hat{\jmath}) \cdot \left(\frac{\sqrt{3}\hat{\imath} + \hat{\jmath}}{2}\right)\right| = \sqrt{3}$ $\qquad \Rightarrow \sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$

$\sqrt{3}\,\alpha + \left(\frac{\alpha - 2}{\sqrt{3}}\right) = \pm 2\sqrt{3}$

$\Rightarrow 3\alpha + \alpha - 2 = \pm 6 \qquad \Rightarrow 4\alpha = 8, -4 \qquad \Rightarrow \alpha = 2, -1$

(B →P, Q)

Continuous ⇒ $-3a - 2 = b + a^2$

Differentiable ⇒ $-6a = b$ ⇒ $\qquad 6a = a^2 + 3a + 2$

(C→ P,Q,S,T)

Let $a = 3 - 3\omega + 2\omega^2$

$a\omega = 3\omega - 3\omega^2 + 2$

Now, $a^{4n+3}(1 + \omega^{4n+3} + (\omega^2)^{4n+3}) = 0$

$\Rightarrow$ n should not be a multiple of 3 Hence P,Q,S,T

(D → Q,T)

$\frac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b$ . . . . . (i)

$q = 10 - a$ and $2q = 5 + b$

$\Rightarrow 20 - 2a = 5 + b \Rightarrow 15 = 2a + b$ . . . . . (II)

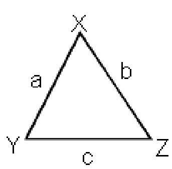
From (I) and (II) $a(15-2a) = 2a + 2(15-2a)$

$\Rightarrow 15a - 2a^2 = -2a + 30 \Rightarrow 2a^2 - 17a + 30 = 0 \Rightarrow a = 6, \frac{5}{2}$

$\Rightarrow q = 4, \frac{15}{2} \Rightarrow |q - a| = 2, 5$

Sol.60 (A → P,R,S)

(A)



Given $2(a^2 - b^2) = c^2$

$\Rightarrow 2(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow 2\sin(x + y)\sin(x - y) = \sin^2 z$

$\Rightarrow 2\sin(\pi - z)\sin(x - y) = \sin^2 z$

Z $\Rightarrow \sin(x - y) = \frac{\sin z}{2}$ . . . . . (i)

Also given,

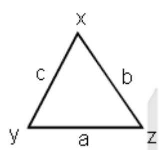
$\lambda = \frac{\sin(x-y)}{sin} = \frac{1}{2}$

Now, $\cos(n\pi\lambda) = 0$

$\Rightarrow \cos\left(\frac{n\pi}{2}\right) = 0$

$\because n = 1,3,5$ $\therefore (A \to P,R,S)$

$(B \to P)$



$1 + \cos2x - 2\cos2y = 2\sin x \sin y$

$2\cos^2x - 2\cos2y = 2\sin X \sin y$

$1 - \sin^2x - 1 + 2\sin^2y = \sin x \sin y$

$\sin^2x + \sin x \sin y = 2\sin^2y$

$\sin x(\sin x + \sin y) = 2\sin^2y$ $\sin x = ak, \sin y = bk$

$a^2 + ab - 2b^2 = 0$

$\left(\frac{a}{b}\right)^2 + \frac{a}{b} - 2 = 0$

$\frac{a}{b} = -2, 1$

$\frac{a}{b} = 1$ $(B \to P)$

$(C \to P,Q)$

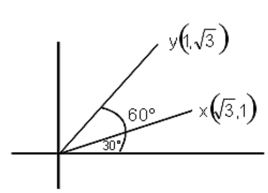
Hence equation of acute angle bisector of OX and OY is $y = x$

Hence $x - y = 0$

Now, distance of $\beta\hat{\imath} + (1 - \beta)\hat{\jmath} \equiv z\ (\beta, 1 - \beta)$ from $x - y$ is $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$|2\beta - 1| = 3$

$2\beta - 1 = \pm 3$



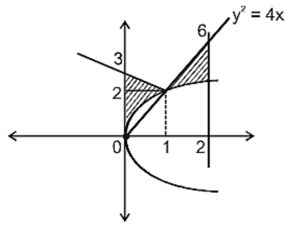
$2\beta = 4, -2$

$|\beta| = 2, 1$ Ans. (P,Q)

For α 1

$$Y = |x-1| + |x-2| + x = \begin{cases} 3-x \ ; \ x<1 \\ 1+x \ ; \ 1 \le x < 2 \\ 3x-3 \ ; \ x \ge 2 \end{cases}$$

$A = \frac{1}{2}$

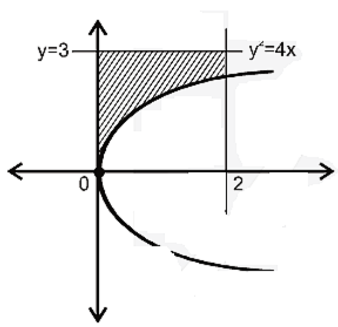


$(2+3) \times 1 + \frac{1}{2}(2+3) \times 1 - \int_0^2 2\sqrt{x}\, dx$

$A = 5 - \frac{8}{3}\sqrt{2}$

$\therefore F(1) + \frac{8}{3}\sqrt{2} = 5$

For $\alpha = 0$, $y = |-1| + |-2| = 3$



$A = 6 - \int_0^2 2\sqrt{x}\, dx$ $\Rightarrow A = 6 - \frac{8}{3}\sqrt{2}$

$\therefore F(0) + \frac{8}{3}\sqrt{2} = 6$

$\therefore (D \rightarrow S, T)$