JEE ADVANCED (Paper - 1)

PHYSICS

SECTION – 1 (One or More Than One Options Correct Type)

This section contains **10 multiple choice type questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE** or **MORE THAN ONE** are correct.

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*1. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m , the gas in the tube is (**Useful information:** $\sqrt{167RT} = 640\ J^{1/2}mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

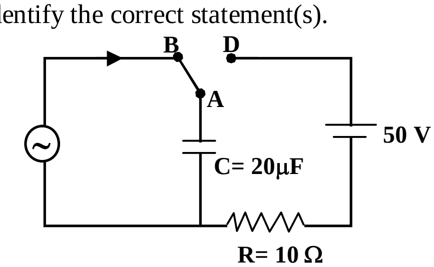
(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

2. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with I_0 = 1A and ω = 500 rad/s starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement(s).



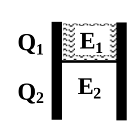
(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) $Q = 2 \times 10^{-3}$C.

3. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$

(B) $\frac{E_1}{E_2} = \frac{1}{K}$

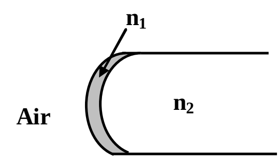
(C) $\frac{Q_1}{Q_2} = \frac{3}{K}$

(D) $\frac{C}{C_1} = \frac{2+K}{K}$

*4. One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

(A) $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$
(B) $y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$
(C) $y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$
(D) $y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$

5. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then

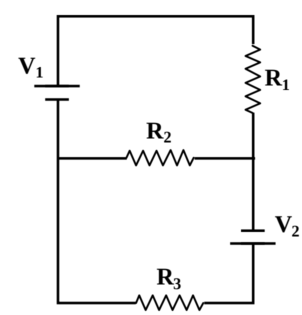


(A) $|f_1| = 3R$
(B) $|f_1| = 2.8 R$
(C) $|f_2| = 2R$
(D) $|f_2| = 1.4 R$

6. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?

(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in series
(D) 0.5 if wires are in parallel.

7. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

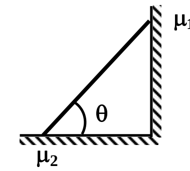
8. Let $E_1(r), E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$
(B) $r_0 = \frac{\lambda}{2\pi\sigma}$
(C) $E_1(r_0/2) = 2E_2(r_0/2)$
(D) $E_2(r_0/2) = 4E_3(r_0/2)$

9. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

(A) $\beta_2 > \beta_1$
(B) $m_1 > m_2$
(C) From the central maximum, 3^{rd} maximum of λ_2 overlaps with 5^{th} minimum of λ_1
(D) The angular separation of fringes for λ_1 is greater than λ_2

*10. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0 \;\; \mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0 \;\; \mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0 \;\; \mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0 \;\; \mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

SECTION – 2: (Only Integer Value Correct Type)

This section contains **10 questions.** Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

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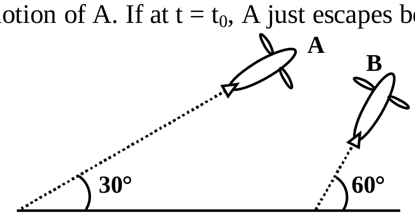
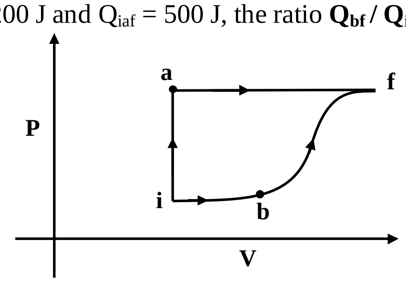
11. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

*12. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



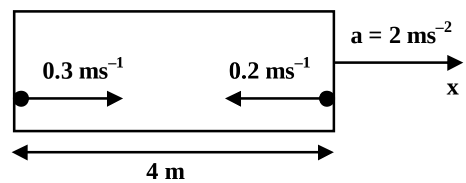
*13. A thermodynamic system is taken from an initial state *i* with internal energy U_i = 100 J to the final state *f* along two different paths *iaf* and *ibf*, as schematically shown in the figure. The work done by the system along the paths *af*, *ib* and *bf* are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio $\mathbf{Q_{bf} / Q_{ib}}$ is



14. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is

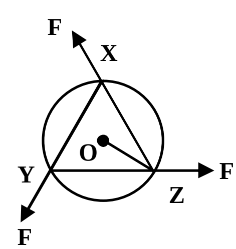
15. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

*16. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is

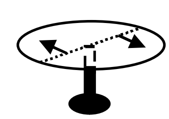


17. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 – 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 – 1.5 A. The value of n is

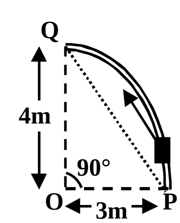
*18. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



*19. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is



*20. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n×10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



JEE(ADVANCED)-2014
PAPER 1 CODE 5
ANSWERS

PHYSICS

1.	**D**	2.	**C, D**	3.	**A, D**	4.	**A, C, D**
5.	**A, C**	6.	**B, D**	7.	**A, B, D**	8.	**C**
9.	**A, B, C**	10.	**C, D**	11.	**4**	12.	**5**
13.	**2**	14.	**3**	15.	**3**	16.	**2**
17.	**5**	18.	**2**	19.	**4**	20.	**5**

HINTS AND SOLUTIONS

PHYSICS

1. $\ell = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}$

Calculations for $\frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}$ for gases mentioned in options A, B, C and D, work out to be 0.459 m, 0.363 m 0.340 m & 0.348 m respectively. As ℓ = (0.350 ± 0.005)m ; Hence correct option is D.

2. As current leads voltage by $\pi/2$ in the given circuit initially, then ac voltage can be represent as
$V = V_0 \sin \omega t$
$\therefore$ $q = CV_0 \sin \omega t = Q \sin \omega t$
where, $Q = 2 \times 10^{-3}$ C

- At $t = 7\pi/6\omega$; $I = -\frac{\sqrt{3}}{2}I_0$ and hence current is anticlockwise.
- Current 'i' immediately after $t = \frac{7\pi}{6\omega}$ is

$i = \frac{V_c + 50}{R} = 10$ A

- Charge flow = $Q_{final} - Q_{(7\pi/6\omega)} = 2 \times 10^{-6}$C

Hence C & D are correct options.

3. As E = V/d
$E_1/E_2 = 1$ (both parts have common potential difference)
Assume C_0 be the capacitance without dielectric for whole capacitor.

$$k\frac{C_0}{3} + \frac{2C_0}{3} = C$$

$$\frac{C}{C_1} = \frac{2+k}{k}$$

$$\frac{Q_1}{Q_2} = \frac{k}{2}.$$

4. Taking y(t) = A f(x) g(t) & Applying the conditions:
1; here x = 3m is antinode & x = 0 is node
2; possible frequencies are odd multiple of fundamental frequency.
where, $\nu_{fundamental} = \frac{v}{4\ell} = \frac{25}{3}$Hz
The correct options are A, C, D.

5. For air to glass

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

$\therefore$ $f_1 = 3R$
For glass to air.

$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$\therefore$ $f_2 = 2R$

6. $$H = \frac{V^2}{R}4 = \frac{V^2}{R/2}t_1 = \frac{V^2}{R/8}t_2$$

$t_1 = 2$ min.

$t_2 = 0.5$ min.

7. $$V_1 = \frac{R_1(V_1+V_2)}{R_1+R_3} \Rightarrow V_1R_3 = V_2R_1$$

$$V_2 = \frac{R_3(V_1+V_2)}{R_1+R_3} \Rightarrow V_2R_1 = V_2R_3$$

8. $$\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

$$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{\pi\varepsilon_0 r_0^2}\ ,\ E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{\pi\varepsilon_0 r_0}\ ,\ E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\varepsilon_0}$$

$$\therefore \quad E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$$

9. $$\beta = \frac{D\lambda}{d}$$

$\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$

Also $m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$

Also $3\left(\frac{D}{d}\right)(600\ \text{nm}) = (2\times5-1)\left(\frac{D}{2d}\right)400\,\text{nm}$

Angular width $\theta = \frac{\lambda}{d}$

10. Condition of translational equilibrium

$N_1 = \mu_2N_2$

$N_2 + \mu_1N_1 = Mg$

Solving $N_2 = \frac{mg}{1+\mu_1\mu_2}$

$$N_1 = \frac{\mu_2 mg}{1+\mu_1\mu_2}$$

Applying torque equation about corner (left) point on the floor

$$mg\frac{\ell}{2}\cos\theta = N_1\ell\sin\theta + \mu_1N_1\ell\cos\theta$$

Solving $\tan\theta = \frac{1-\mu_1\mu_2}{2\mu_2}$

11. $Y = \frac{FL}{\ell A}$ since the experiment measures only change in the length of wire

$$\therefore \quad \frac{\Delta Y}{Y}\times100 = \frac{\Delta\ell}{\ell}\times100$$

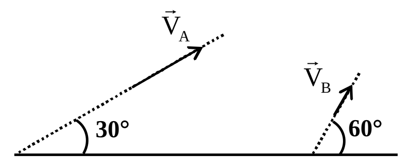
From the observation ℓ_1 = MSR + 20 (LC)

ℓ_2 = MSR + 45 (LC)

$\Rightarrow$ change in lengths = 25(LC)
and the maximum permissible error in elongation is one LC

$$\therefore \frac{\Delta Y}{Y} \times 100 = \frac{(LC)}{25(LC)} \times 100 = 4$$

12. The relative velocity of B with respect to A is perpendicular to line of motion of A.



$\therefore \quad V_B \cos 30° = V_A$

$\Rightarrow \quad V_B = 200$ m/s

And time t_0 = (Relative distance) / (Relative velocity)

$$= \frac{500}{V_B \sin 30°} = 5 \sec$$

13. $U_b = 200$ J, $U_i = 100$ J

Process iaf

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ia		0	
af		200	
Net	300	200	500

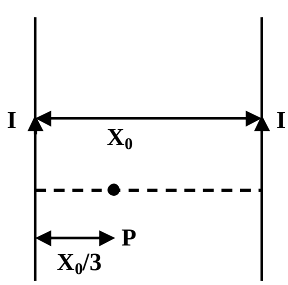
$\Rightarrow \quad U_f = 400$ Joule

Process ibf

Process	W(in Joule)	ΔU(in Joule)	Q(in Joule)
ib	100	50	150
bf	200	100	300
Net	300	150	450

$$\Rightarrow \quad \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

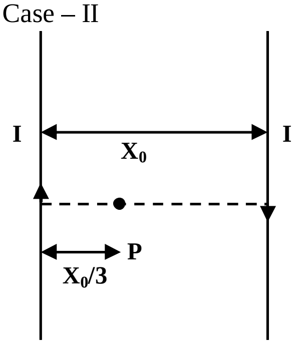
14. Case – I



$$B_1 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\right)\left(\frac{3I}{x_0}\right)$$

$$R_1 = \frac{mv}{qB_1}$$

Case – II



$$R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{1/3}{1/9} = 3$$

15. $d \propto \rho^x S^y F^z$

$\Rightarrow \quad [L] \equiv [ML^{-3}]^x [MT^{-3}]^y \, [T^{-1}]^z$

$\Rightarrow \quad x + y = 0, \; -3x = 1, \; -3y - z = 0$

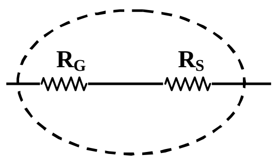
$\Rightarrow \quad x = \frac{-1}{3},\ y = \frac{1}{3},\ z = -1$

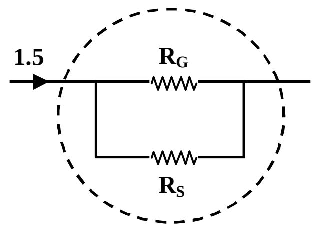
$\Rightarrow \quad y = \frac{1}{n}$

$\Rightarrow \quad n = 3$

16. Maximum displacement of the left ball from the left wall of the chamber is 2.25 cm, so the right ball has to travel almost the whole length of the chamber (4m) to hit the left ball. So the time taken by the right ball is 1.9 sec (approximately 2 sec)

17.





$i = \frac{V}{R}$

$0.006 = \frac{30}{4990 + R}$

R = 10

$i_{RG} = 0.006$

$i_{RS} = 1.494$

Since R_G and R_S are in parallel, $i_G R_G = i_S R_S$

$0.006\,R = 1.494\left(\frac{2n}{249}\right)$

$\therefore\ n = 5$

18. $\tau = I\alpha$

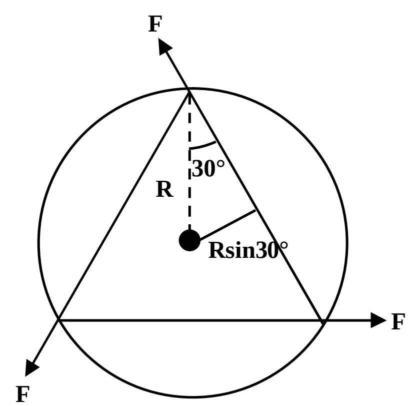
$3\ FR\sin 30° = I\alpha$

$I = \frac{MR^2}{2}$

$\alpha = 2$

$\omega = \omega_0 + \alpha t$

$\omega = 2$ rad/s



19. Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$L_i = L_f$

$0 = I\omega + 2mv\ (r/2)$; comparing magnitude

$\therefore\ \left(\frac{0.45 \times 0.5 \times 0.5}{2}\right)\omega = 0.05 \times 9 \times \frac{0.5}{2} \times 2$

$\therefore\ \omega = 4$

20. Using work energy theorem

$W_{mg} + W_F = \Delta KE$

$-\,mgh + Fd = \Delta KE$

$-1 \times 10 \times 4 + 18(5) = \Delta KE$

$\Delta KE = 50$

$\therefore\ n = 5$