JEE Main – 2018 (CBT) Exam

Test Date: 15/04/2018
Test Time: 9:30 AM – 12:30 PM
Subject: JEE Main 2018 CBT EH

Mathematics

Q1:

The set of all $\alpha \in R$, for which $w = \dfrac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number , for all $z \in C$ Satifying $|z| = 1$ and $\text{Re}\, z \neq 1$, is:

Option

1. An empty set
2. $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$
3. equal to R
4. $\{0\}$

Q2:

Let $y = y(x)$be the solution of the differential equation $\dfrac{dy}{dx} + 2y = f(x)$, where $f(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & \text{Otherwise} \end{cases}$ If $y(0)$, then $y\left(\dfrac{3}{2}\right)$ is:

Options

1. $\frac{e^2-1}{2e^3}$
2. $\frac{e^2+1}{2e^4}$
3. $\frac{e^2-1}{e^3}$
4. $\frac{1}{2e}$

Q3:

An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane, is:

Options

1. 750
2. 720
3. 1440
4. 1500

Q4:

The value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log\left(\frac{2 + \sin x}{2 - \sin x}\right)\right) dx \text{ is:}$$

Options

1. $\frac{3}{4}$

2. 0

3. $\frac{3}{8}\pi$

4. $\frac{3}{16}\pi$

Q5:

A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is:

Options

1. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{11}{6}$

2. $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$

3. $x + y + z = 6$

4. $\frac{x}{3} + \frac{y}{2} + \frac{z}{1} = 6$

Q6:

An angle between the plane, x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z – 1 = 0 and 5x + 8y + 2z + 14 = 0, is:

Options

1. $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$
2. $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$
3. $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$
4. $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$

Q7:

If n is the degree of the polynomial,

$$\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right] \text{ and}$$

m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to :

Options

1. $(24, (10)^8)$
2. $(12, (20)^4)$
3. $(8, 5(10)^4)$
4. $(12, 8(10)^4)$

Q8:

If β is one of the angles between the normals to the ellipse, $x^2+3y^2=9$ at the points $\left(3\cos\theta, \sqrt{3}\sin\theta\right)$ and $\left(-3\sin\ \theta\sqrt{3}\ \cos\theta\right)'$; $\theta\epsilon\left(0,\frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to:

Options

1. $\frac{2}{\sqrt{3}}$
2. $\frac{1}{\sqrt{3}}$
3. $\frac{\sqrt{3}}{4}$
4. $\sqrt{2}$

Q9:

In a triangle ABC, coordinates of A are (1, 2) and the equations of the medians through B and C are respectively, x + y = 5 and x = 4. Then area of ΔABC (in sq. units) is :

Options

1. 4
2. 5
3. 9
4. 12

Q10:
The mean of a set of 30 observations is 75. If each observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to :
Options
1. $\frac{10}{3}$

2. $\frac{2}{3}$

3. $\frac{4}{3}$

4. $\frac{1}{3}$

Q11:
A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, y — 4x + 3 = 0, then its radius is equal to :
Options
1. $\sqrt{5}$
2. 2
3. $\sqrt{2}$
4. 1

Q12:
Let S be the set of all real values of k for which the system of linear equations
x+ y + z = 2
2x + y - z = 3
3x + 2y + kz = 4
has a unique solution.
Then S is :
Options
1. 1 equal to {0}
2. equal to R- {0}
3. an empty set
4. equal to R

Q13:
If tanA and tanB are the roots of the quadratic equation, $3x^2$ —10x — 25 = 0, then the value of 3 $\sin^2$(A + B) —10 sin(A +B).cos(A+B) —25 $\cos^2$(A + B)
Options
1. -25
2. 10
3. -10
4. 25

Q14:

If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point $(-2, 0)$ is:

Options

1. -34
2. -2
3. 4
4. -32

Q15:

If $x_1, x_2 \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}, \ldots, \frac{1}{h_n}$ are two A. P. s such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals:

Options

1. 2650
2. 2560
3. 3200
4. 1600

Q16:

If $\vec{a}, \vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = 0$, then $|\vec{a} \times \vec{c}|$ is equal to:

Options

1. $\frac{\sqrt{15}}{4}$
2. $\frac{\sqrt{15}}{16}$
3. $\frac{1}{4}$
4. $\frac{15}{4}$

Q17:

If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid-point of AB is:

Options

1. $4x^2 - y^2 + 16x^2y^2 = 0$
2. $X^2 - 4y^2 - 16x^2y^2 = 0$
3. $4x^2 - y^2 + 16x^2y^2 = 0$
4. $4x^2 - y^2 - 16x^2y^2 = 0$

Q18:

The area (in sq. units) of the region

$\{x \in R : x \geq 0, y \geq x - 2 \text{ and } y \geq \sqrt{x}\}$ is :

Options

1. $\frac{8}{3}$

2. $\frac{10}{3}$

3. $\frac{5}{3}$

4. $\frac{13}{3}$

Q19:

Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ isa scalar matrix and $|3A| = 108$. Then A^2 equals:

Options

1. $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

2. $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

Q20:

If $f\left(\frac{x-4}{x+2}\right) = 2x + 1, (x \in R - \{1, -2\})$, then $\int f(x)dx$ is equal to:

(where C is a constant of integration)

Options

1. $12 \log_e |1 - x| - 3x + C$
2. $-12 \log_e |1 - x| + 3x + C$
3. $-12 \log_e |1 - x| - 3x + C$
4. $12 \log_e |1 - x| + 3x + C$

Q21:
If a right circular cone, having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is:
Options
1. $8\sqrt{2}\,\pi$
2. $6\sqrt{2}\,\pi$
3. $6\sqrt{3}\,\pi$
4. $8\sqrt{3}\,\pi$

Q22:
If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval:
Options
1. (0, 10)
2. [10, ∞]
3. (- 10, 0)
4. (-∞, -10]

Q23:
Consider the following two binary relations on the set A = {a, b, c}:
R_1 = {(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)} and R_2 = {(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c)}.
Then:
Options
1. both R_1 and R_2 are transitive.
2. both R_1 and R_2 are not symmetric.
3 R_1 is not symmetric but it is transitive.
4 R_2 is symmetric but it is not transitive.

Q24:
$\text{Let } S = \{\lambda, \mu) \epsilon\, R \times R :=\ f(t) = (|\lambda| e^{|t|} - \mu).\ \sin(2(2|t|), t \in R, \text{is a differentiable function}\}.$
$\text{Then S is a subset of :}$
Options
1. R × [0, ∞)
2. R × (-∞, 0)
3. (-∞, 0) × R
4. [-∞, 0) × R

Q25:

A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is:

Options

1. $\frac{9}{16}$
2. $\frac{9}{32}$
3. $\frac{7}{8}$
4. $\frac{7}{16}$

Q26:

If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are, respectively:

Options

1. T, F, T
2. F, F, F
3. T, T, T
4. F, T, F

Q27:

If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is:

Options

1. $2\sqrt{5}$
2. 20
3. $2\sqrt{7}$
4. $4\sqrt{7}$

Q28:

If $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then $\lim_{x \to 0} \frac{f'(x)}{x}$

Options

1. Exists and is equal to -2
2. Exists and is equal to 0.
3. Does not exist.
4. Exists and is equal to 2.

Q29:
Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is :
Options
1. $8(2x + y) + 3 = 0$
2. $3(x + y) + 4 = 0$
3. $4(x + y) + 3 = 0$
4. $x + 2y + 3 = 0$

Q30:
n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is:
Options
1. 7
2. 8
3. 9
4. 6

Solutions

Sol 1: (4)

As ω is purely imaginary

$$\omega + \overline{\omega} = 0$$

$$\frac{1 + (1 - 8\alpha)z}{1 - z} + \frac{1 + (1 - 8\alpha)\overline{z}}{1 - z} = 0$$

$$\frac{1 - \overline{z} + (1 - 8\alpha)(z - 1) + a - z + (1 - 8\alpha)(\overline{z} - 1)}{(1 - z)(1 - \overline{z})} = 0$$

$$1 - \overline{z} + \ z - 1 - 8\alpha z + 8\alpha + 1 - z + \overline{z} - 1 - 8\overline{z} - 1 - 8\overline{z}\alpha + 8\alpha = 0$$

$$-8\alpha(z + \overline{z}) + 16\alpha = 0$$

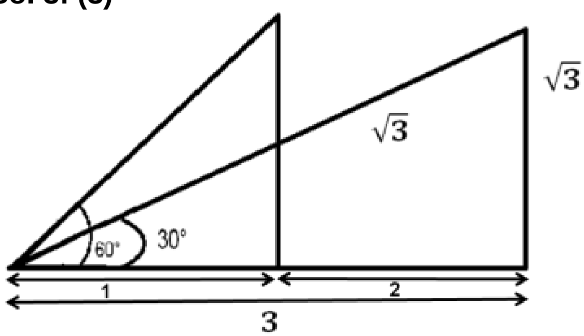
$$8\alpha[2 - (2 + 2)] = 0$$

$$\text{if } \mathrm{Re}(z) \neq 1$$

then $\alpha = 0$

Sol 2: (4)

Sol 3: (3)



M 5 sec = 2km

Speed = 2/5 km/sec

$$= \frac{2}{5} \times 60 \times 60$$

$$= 1440 \frac{\text{km}}{\text{hr}}$$

Sol 4: (4)

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log\left[\left(\frac{2 + \sin x}{2 - \sin x}\right)\right] dx\right.$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left[1 + \log\left(\frac{2 - \sin x}{2 + \sin x}\right) dx\right]$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \; dx$$

$$8I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{3}{2} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 4x}{2} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \cos 2x$$

$$8I = \frac{3}{2}\ \pi$$

$$I = \frac{3\pi}{16}$$

Hint:-

$$\cos^2 x = 2\cos^2 x - 1$$

$$\cos^2 x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x$$

$$4\sin^4 x = (1 - \cos 2x)^2$$

$$\sin^4 x = \frac{1}{4}\ (\cos^2 2x - 2\cos 2x + 1)$$

$$\sin^4 x = \frac{1}{4}\left(\frac{1+\cos 4x}{2} - 2\cos 2x + 1\right)$$

$$\sin^4 x = \frac{1}{4}\left(\frac{3 + \cos 4x - 8\cos 2x}{2}\right)$$

Sol 5: (2)

E.q., of variable plane

$$a(x-3) + b(y-2) + c(2-1) = 0$$

$$A = \frac{2b + c + 3a}{a}$$

$$B = \frac{3a + c + 2b}{b}$$

$$C = \frac{3a + c + 2b}{c}$$

Plane paroled to y^2 plane passing through A

$$x = \frac{2b + c + 3a}{a}$$

$$y = \frac{2b + c + 3a}{b}$$

$$z = \frac{2b + c + 3a}{c}$$

Intersection of the x three

$$\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = 1$$

Sol. 6 (3)

Perpendicular vector to plane

$$\hat{i} + \hat{j} + \hat{k}$$

Parallel vector to line $\begin{vmatrix} \hat{I} & \hat{J} & \hat{k} \\ 3 & 4 & 1 \\ 5 & 8 & 2 \end{vmatrix}$

$= I(8-8) - \hat{J}\,(6-5) + \hat{k}(24-20)$

$= -\hat{J} + 4\hat{k}$

$\sin\theta = \left|\frac{4-1}{\sqrt{17}\sqrt{3}}\right| = \left|\frac{3}{\sqrt{17}\sqrt{3}}\right|$

$\sin\theta = \sqrt{\frac{13}{17}}$

$\theta = \sin^{-1}\left(\sqrt{\frac{13}{17}}\right)$

Sol 7: (2)

Rationalize and get

$\left[\frac{2\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)}{2}\right]^8 \left[\frac{2\sqrt{5x^3+1}-\sqrt{5x^3-1}}{2}\right]^8$

$\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)^8$

${}^8C_r\left(\sqrt{5x^3+1}\right)^{8-r}\left(\sqrt{5x^3-1}\right)^r + {}^8C_{r_1}\left(\sqrt{5x^3+1}\right)^{8-r_1}\left(-\sqrt{5x^3-1}\right)^{r_1}$

Term with $r = 1,3,5,7$ vnish $n = 1,3,5,7$

$2\left[{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 + \cdots {}^8C_8\sqrt{(5x^3+1)}\right)\left(\sqrt{5x^3-1}\right)^8$

Degree $= 12$

Coff $= 2\,[\,{}^8C_0 + {}^8C_r + - {}^8C_8]5^4$

${}^8C_0 + {}^8C_2 + {}^8C_4 + - {}^8C_4 = 2^{4-1}$

Coff $= 2 \quad (2^{8-1}) \quad 5^4$

Coff $= 2 \quad 2^7 5^4$

$= 2 \times 2^3 \times 2^4 \times 5^4$

$= 16 \quad (10)^4$

$= (20)^4$

Sol 8: (1)

$2x\,dx + 6y\,dy = 0$

$2x\,dx = -cy\,dy$

$-\dfrac{dx}{dy} = \dfrac{3y}{x} = \text{stepe of normal}$

$m_1 = \dfrac{3\sqrt{3}\sin\theta}{3\cos\theta} = \sqrt{3}\ \tan\theta$

$m_2 = \dfrac{3\sqrt{3}\cos\theta}{-3\sin\theta} - \sqrt{3}\ \cot\theta$

$\tan\beta = \left[\dfrac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1 + (\sqrt{3}\tan\theta)(-\sqrt{3}\cot\theta)}\right]$

$\tan\beta = \sqrt{3}\left(\left|\dfrac{\tan\theta + \cot\theta}{1-3}\right|\right)$

$= \dfrac{\sqrt{3}}{2}(\tan\theta + \cot\theta)$

We have to find

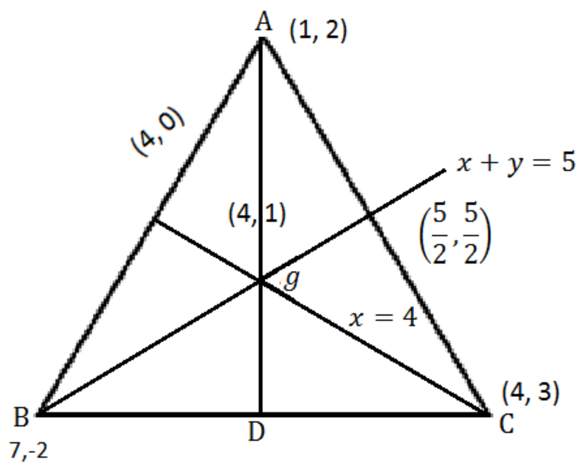
$= \dfrac{2\cot\beta}{\sin 2\theta}$

$= \dfrac{2 \qquad 2}{\sqrt{3}(\tan\theta + \cot\theta)2\sin\theta\ \cos\theta}$

$= \dfrac{2}{\sqrt{3}(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta})\ \sin\theta\ \cos\theta}$

$= \dfrac{2}{\sqrt{3}}$

Sol 9: (3)



centroid = (4, 1)

$A\,g = \sqrt{9+1} = \sqrt{10}$

$A\,D = 2\sqrt{10}$

$$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & -2 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$\frac{1}{2}|-5-2(7-4)+1(29)|$$

$$\frac{1}{2}(-5-6+29)$$

$$= \frac{18}{2}$$

$$= 9$$

Sol 10: (3)

Let observations are x_1 x_2 _______x_{30}

$$\frac{(x_1 + x_2 + \_\_\_ x_{30})}{30} = 75 \ldots (i)$$

For new near

$$\frac{(\lambda x_1 + \lambda x_2 + \_\_\_\_ \lambda x_3) - 30(25)}{30} = 75$$

$$\lambda(x_1 + x_2 + \_\_\_\_\_ x_{30}) - 30(25) = 2250 \text{ from eq(i)}$$

$$\lambda(30)(75) = 750 + 2250$$

$$\lambda(30)(75) = 3000$$

$$\lambda = \frac{100}{75} = \frac{4}{3}$$

Sol 11: (2)

Let A(2, 3) & B(4, 5) be the points through which circle is passing & radius lies on the line y-4x + 3 = 0. Let centre be (n, k), this point must satisfy the line y – 4x - + 3 = 0. Hence

$$k - 4n + 3 = 0$$

$$k = 4n - 3.$$

So, Centre coordinates be 0(n, 4n -3)

Now, OA = OB (both one radius)

A(2, 3) B(4, 5)

$\Longrightarrow OA = OB$

$\Longrightarrow OA^2 = OB^2$

$(n-2)^2 + (4n-3-3)^2 = (n-4)^2 + (4n-3-5)^2$

$n^2 + 4 - 4n + 16n^2 + 36 - 48n = n^2 + 16 - 8n + 16n^2 + 64 - 64n$

$40 - 52n = 80 - 72n$

$72n - 52n = 80 - 40$

$20n = 40$

$n = 2$

Hence $k = 4n - 3$

$k = 8 - 3$

$k = 5$

Centre coordinates $O(2,5)$& $A(2,3)$

radius $= OA = (2-2)^2 + (5-3)^2 = 2$

Hence = radius = 2

Sol 12: (2)

Writing above system of equation in matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$Ax = B$

Now by Rank method, form Augmented matrix

$$[A; B] = \begin{bmatrix} 1 & 1 & 1 & ; 2 \\ 2 & 1 & -1 & ; 3 \\ 3 & 2 & k & ; 4 \end{bmatrix}$$

Applying elementary transformations to make above augment matrix in Echelon form,

$R_3 \rightarrow R_3 - 3R_1 \& R_2 \rightarrow R_2 - 2R_1$

$$[A;B] = \begin{bmatrix} 1 & 1 & 1 & ; 2 \\ 0 & -1 & -3 & ;-1 \\ 0 & -1 & k-3 & ;-2 \end{bmatrix}$$

Apply , $R_3 \rightarrow R_3 - R_2$

$$[A;B] = \begin{bmatrix} 1 & 1 & 1 & ; 2 \\ 0 & -1 & -3 & ;-1 \\ 0 & 0 & k & ;-1 \end{bmatrix}$$

From above matrix, clearly Rank of Augmented matrix will be 3, but Rank of A matrix depends on value of k, i.e if k = 0, Rank of A matrix will be 2 but Rank of Augmented matrix is 3. Hence, in this case No solution is their

So, for vnique solution $\nless \neq 0$

Hence, set S can have an real values except O, S = R – {o}.

Sol 13: (1)

$3x^2 - 10x - 25 = 0,$

$\tan A + \tan B = \frac{10}{3}$

$\tan A + \tan B = -\frac{23}{3}$

$\tan(A+B) = \frac{\tan A + \tan}{1}$

$= \frac{\frac{10}{3}}{1 + \frac{23}{3}}$

$= \frac{10}{28} = \frac{5}{14}$

Divide and multiply by $\cos^2 \times (A+B)$

$3\tan^2(A+B) - 10\tan(A+B) - 25(\cos^2(A+B)$

$3\frac{25}{196} - 10\left(\frac{5}{14}\right) - 25\left(\cos^2(A+B)\right)$

$\frac{75 - 700 - 4500}{196}(\cos^2(A+B))$

$-\frac{5525}{196}\left(\frac{1}{1+\tan^2(A+B)}\right)$

$$-\frac{5525}{196}\left(\frac{1}{1+\frac{25}{196}}\right)$$
$$=\frac{-5521}{221}$$
$$=-25$$

Sol 14: (1)

$x^2+y^2+\sin y=4$

Diff above eqn with respect to x.

$$2x \neq 2y\frac{dy}{dx}+\cos y\cdot\frac{dy}{dx}=0\,.$$

$$\frac{dy}{dx}(2y+\cos y)= -2x$$

$$\frac{dy}{dx}=\frac{-2x}{2y+\cos y}$$

Again differentiate $\frac{d^2y}{dx^2}=\frac{(2y+\cos y)(-2)-(-2x)\left(\frac{2dy}{dx}-\sin y\frac{dy}{dx}\right)}{(2y+\cos y)^2}$

$$\frac{d^2y}{dx^2}=\frac{\frac{-2(2y+\cos y)+(2x)(2-\sin y)(-2x)}{2y+\cos y}}{(2y+\cos y)^2}$$

$$\frac{d^2y}{dx^2}=\frac{-2\left[\frac{(2y+cos)\ +2x^2(2-sin\,y)}{2y+cos\,y}\right]}{(2y+cos\,y)^2}$$

$$\frac{d^2y}{dx^2}=\frac{-2[(2y+cos\,y)^2+2x^2(2-sin\,y)]}{(2y+cos\,y)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-2,0)}=\frac{-2[(2(0)+cos\,0)^2+2(-2)^2(2-sin\,0)]}{(2(0)+cos\,0)^3}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(-2,0)}=\frac{(-2)(1+16)}{(1)^3}= -34$$

Sol 15: (2)

$x_3 = 8$

$x_8 = 20$

$a + 2d = 8$

$a + 7d = 20$

$d = \frac{12}{5} \quad a = \frac{16}{5}$

$x_5 = \quad a + 4d = \frac{16}{5} + 4 \left(\frac{12}{5}\right) = \frac{64}{5}$

$\frac{1}{h_1} \frac{1}{h_2} \frac{1}{h_2} \quad (AP)$

$\frac{1}{h_2} = \frac{1}{8} = a' + d' \ldots \ldots (i)$

$\frac{1}{h_1} \ = \frac{1}{20} = a' + 6d' \ldots . (ii)$

Eq (i) and (ii)

$\frac{1}{h_2} = \frac{1}{8} = a' + d'$

$\frac{1}{h_1} \ = \frac{1}{20} = a' + 6d'$

$\frac{1}{20} - \frac{1}{8} = 5d'$

$\frac{-5 + 2}{40} = 5d'$

$d' = \ -\frac{3}{200}$

$d = \frac{1}{8} + \left(\frac{3}{200}\right)$

$= \frac{1}{8} + \frac{3}{100}$

$= \frac{25 + 3}{200}$

$= \frac{28}{200}$

$x_5 = \frac{64}{5}$

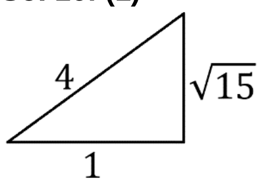
$\frac{1}{h_{10}} \ = \frac{28}{200} - 9\left(\frac{3}{200}\right) = \frac{1}{200}$

$h_{10} = 200$

$x_5 h_{10} = \frac{64}{5} \times 200$

$= 2560$

Sol 16: (1)



$\vec{a} + 2\vec{b} + 2\vec{c} = 0$

$\left|2\vec{b}\right|^2 = |\vec{a} + 2\vec{c}|^2$

$4|b|^2 = |a|^2 + 4|c|^2 + 2\vec{a} \cdot (2\vec{c})$

$4 = 1 + 4 + 4 \cos\theta$

$\cos\theta = \frac{-1}{4}, \sin\theta = \frac{\sqrt{15}}{4}$

$|\vec{a} \times \vec{c}|$

$= (a)|(c) \sin\theta$

$\frac{\sqrt{15}}{4}$

Sol 17: (2)

$4y^2 = x^2 + 1$

Point $4yy_1 = xx_1 + 1$ with $4y_1^2 = x_1^2 + 1$

x axis $\left[\frac{-1}{x}, 0\right]$

y axis $\left[0, \frac{1}{4y_1}\right]$

Mid point $h = \frac{-1}{2x}$, $\quad k = \frac{1}{8y_1}$

$$x_1 = \frac{-1}{2h} y_1 = \frac{1}{8k}$$

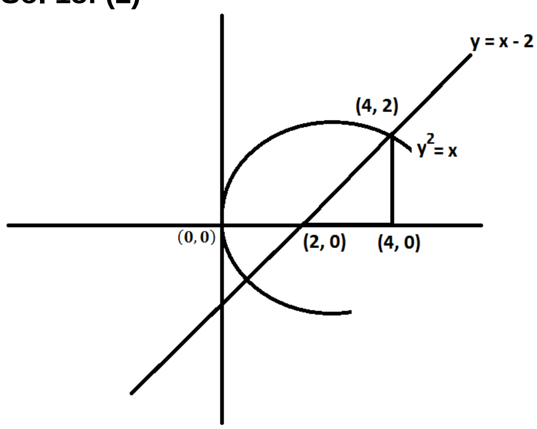
$$4\left(\frac{1}{8k}\right)^2 = \left(\frac{-1}{2h}\right)^2 + 1$$

$$\frac{4}{4k^2} = \frac{1}{4b^2} + 1$$

$$\frac{1}{16y^2} = \frac{1}{4b^2} + 1$$

$$\frac{1}{16y^2} = \frac{1 + 4x^2}{4x^2}$$

Sol 18: (2)



$$\int_0^4 \sqrt{x} = \left(\frac{2\,x^{3/2}}{3}\right)^4$$

$$= \frac{2}{3}(8) = \frac{16}{3}$$

$$\Delta = \frac{1}{2} \times 2 \times = 2$$

$$\text{Area} = \frac{16}{3} - 2 = \frac{10}{3}$$

Sol 19: (3)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} a & 2a + 3b \\ c & 2c + 3d \end{bmatrix}$$

$a = 2c + 3d$

$c = 0$

$2a + 3b = 0$

$ad - bc = 12$

$|3A| = 108$

$|A| = 12$

$$A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

$c = 0$

$d = 2$

$a = 6$

$b = -4$

$$A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$$

Sol 20: (3)

$$f\left(\frac{x - 4}{x + 2}\right) = 2x + 1$$

$$\text{Let } t = \frac{x - 4}{x + 2}$$

$tx + 2t = x - 4$

$2t + 4 = x(1 - t)$

$$x = \frac{2t + 4}{1 - t}$$

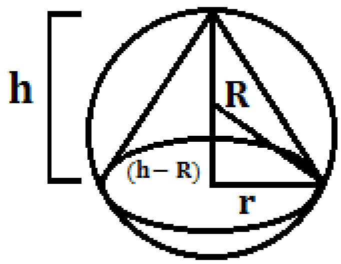
$$f(t) = 2\left(\frac{2t + 4}{1 - t}\right) + 1 = \frac{4t + 8 + 1 - t}{1 - t}$$

$$= \frac{3t + 9}{1 - t}$$

$$\int f(t)\,dt = \int f(t)\;dx = \int \frac{8t+9}{1-t}\,dt$$

$$= \int \frac{3t}{1-t} + \int \frac{9}{1-t}dt = -12\ \text{cm}|1-x| - 5x + c$$

Sol 21: (4)



$$(h-R)^2 + r^2 = R^2$$

$$r^2 = R^2 - (h-R)^2$$

$$R = 3$$

$$r^2 = q - (h-3)^2$$

$$V = \frac{1}{3}\pi[9-(h-3)^2]h$$

$$\frac{dV}{dh} = \frac{1}{3}\pi[9-(h-3)^2 - 2(h-3)h] = 0$$

$$\frac{1}{3}\pi[9 - h^2 - 9 + 6h - 2h^2 + 6h] = 0$$

$$\frac{1}{3}\pi(-3h^2 + 12h) = 0$$

$$h(-3h+12) = 0$$

$$3h = 12, h = 0$$

$$h = 4. \qquad\qquad r = 2\sqrt{2}.$$

$$CSA = \pi r l$$

$$= \pi \times 2\sqrt{2} \times \sqrt{24}$$

$= \pi\ 2 \times \sqrt{48} = 8\pi\sqrt{3}$

Sol 22: (1)

$\frac{6}{1-r} = 5$

$b = 5\ (1 - r)$

$b\ \varepsilon\ (0,10) \qquad [-1 < r < 1]$

Sol 23: (4)

$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$

$b, c\ \epsilon R_1\ \ c, a\ \notin R_1 R_1$ is not symmetric $(b, c), (c, a) \epsilon R_1 (b, a) \notin R_1, R_1$ is not transitive

$R_2 = \{(a, b), (b, a), (c, c), (c, a), (c, a), (a, a), (b, b), (a, c)\}$

$\forall (a, b) \epsilon R_2 (b, a) \times R_2$

Therefore it is symmetric

$(c, a), (a, b) \epsilon R_2 (c, b) \notin R_2$

Therefore R_2 is not transitive

Sol 24: (1)

$f(t) = (|\lambda| e^t - \mu) \sin 2t \qquad t > 0$

$-(|\lambda| \bar{e}^t - \mu) \sin 2t \qquad t < 0$

$f'(t) = 2 \cos 2t\, (|\lambda| e^t - \mu) + |\lambda| e^t \sin 2t \qquad t > 0$

$-2 \cos 2t\, (|\lambda| e^t - \mu) + |\lambda| \bar{e}^t \sin 2t \qquad t > 0$

$f'(t \rightarrow o^t) = 2(|\lambda| - \mu)$

$f'(t \rightarrow \bar{o}) = -2(|\lambda| - \mu)$

for differentiability LHD = RHD

$2(|\lambda| - \mu) = -2(|\lambda| - \mu)$

$|\lambda| = \mu$

$\Longrightarrow \lambda \in R \ \mu \in R_t$

$(\lambda, \mu) \ \ C \quad R \times [0, \infty)$

Sol 25: (1)

$P(E) = P(A)\ P\left(\frac{E}{A}\right) + P(B)\ P\left(\frac{E}{B}\right)$

$= \frac{1}{2}\left[\frac{2.3}{7_{C_2}} + \frac{4.2}{9_{C_2}}\right] = \frac{1}{2}\left[\frac{6}{21} + \frac{8}{36}\right]$

$= \frac{1}{2}\left[\frac{2}{7} + \frac{2}{9}\right] = \frac{16}{63}$

$P\left(\frac{B}{E}\right) = \frac{P(B)P(E/B)}{P(E)} = \frac{1/9}{16/26} = \frac{1}{9} \times \frac{63}{16} = \frac{7}{16}$

Sol 26: (1)

$(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$
$T \rightarrow F = F$
Only possibility for if them
$\left.\begin{array}{l} P \rightarrow F \\ P \rightarrow F \end{array}\right\} q \rightarrow F$
$T \wedge T) \ \wedge (T \wedge r) \rightarrow T$
$r = T$

Sol 27: (1)

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \ \alpha\beta)$

$= \ -(2 - \lambda)((\lambda + \beta)^2 - 3\alpha\beta)$

$= (\lambda - 2)\big((\lambda - 2)^2 + 3(\lambda - 10)\big)$

$= (\lambda - 2)(b^2 - 4\lambda + 4 + 3\lambda - 30)$

Sol 28: (1)

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$R_1 - R_1 - R_3$

$$f(x) = \begin{vmatrix} \cos x - \tan x & 0 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$= \cos x - \tan x\ [x^2 - 2x^2]$

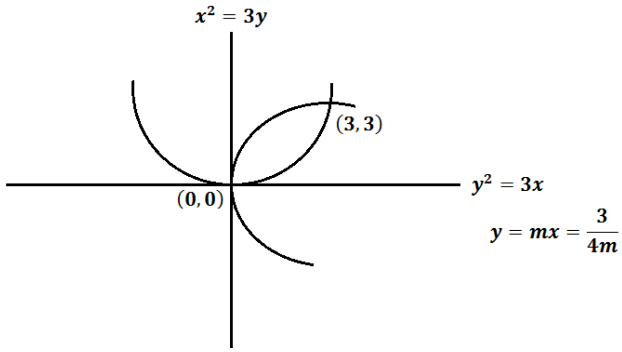
$= x^2 \tan x - x^2 \cos x$

$f'(x) = 2x \tan x\ + x^2 \sec^2 x - 2x \cos x + x^2 \sin x$

$\dfrac{f'(x)}{x} = 2\tan x + x \sec^2 x - 2\cos x + x \sin x$

$\lim\limits_{x \to 0} \dfrac{f'(x)}{x} = 0 + 0 - 2 + 0 = \ -2$

Sol 29: (3)



$x^2 = 3\left(mx + \dfrac{3}{4m}\right)$

$x^2 - 3mx - \dfrac{9}{4m} = 0$

$9m^2 - 4\,(1)(\dfrac{-9}{4m}) = 0$

$9m^2 + \dfrac{9}{m} = 0$

$(m^3 + 1) = 0$

$m = \ -1$

$y = \ -x - \dfrac{3}{4}$

$4y = -4x - 3$

$4(x + y) + 3 = 0$

Sol 30: (1)

For each place we have 3 choices

(i)for $n -$ digits $3 \times 3 \ldots n$ times $= 3^n > 900$

$n = 7$

JEE Main: 2018 (Online CBT)

Answer Key (15/04/2018)

Mathematics

Q. No.	Answer	Q. No.	Answer	Q. No.	Answer
1	4	**11**	2	**21**	4
2	4	**12**	2	**22**	1
3	3	**13**	1	**23**	4
4	4	**14**	1	**24**	1
5	2	**15**	2	**25**	1
6	3	**16**	1	**26**	1
7	2	**17**	2	**27**	1
8	1	**18**	2	**28**	1
9	3	**19**	3	**29**	3
10	3	**20**	3	**30**	1