JEE(Advanced) – 2017 TEST PAPER WITH SOLUTION

(HELD ON SUNDAY 21st MAY, 2017)

MATHEMATICS

SECTION–1 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –2 In all other cases.
- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened

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37. Which of the following is(are) NOT the square of a 3×3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. (A,B)

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

(A) $p = 5, h = 4, k = -3$ (B) $p = -1, h = 1, k = -3$

(C) $p = -2, h = 2, k = -4$ (D) $p = 2, h = 3, k = -4$

Ans. (D)

Sol. Equation of chord with mid point (h, k) :

$$k.y - 16\left(\frac{x+h}{2}\right) = k^2 - 16h$$

$\Rightarrow \quad 8x - ky + k^2 - 8h = 0$

Comparing with $2x + y - p = 0$, we get

$k = -4;\ 2h - p = 4$

only (D) satisfies above relation.

39. Let a, b, x and y be real numbers such that a – b = 1 and y ≠ 0. If the complex number z = x + iy satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

(A) $-1-\sqrt{1-y^2}$ (B) $1+\sqrt{1+y^2}$ (C) $1-\sqrt{1+y^2}$ (D) $-1+\sqrt{1-y^2}$

Ans. (A,D)

Sol. $\text{Im}\left(\frac{az+b}{z+1}\right)$ = y and z = x + iy

$\therefore \quad \text{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$\Rightarrow \quad \text{Im}\left(\frac{(ax+b+iay)(x+1-iy)}{(x+1)^2+y^2}\right) = y$

$\Rightarrow \quad -y(ax + b) + ay(x + 1) = y((x + 1)^2 + y^2)$

$\Rightarrow \quad (a - b)y = y((x + 1)^2 + y^2)$

$\because \quad y \neq 0$ and $a - b = 1$

$\Rightarrow \quad (x + 1)^2 + y^2 = 1$

$\Rightarrow \quad x = -1 \pm \sqrt{1-y^2}$

40. Let X and Y be two events such that $P(X) = \frac{1}{3}, P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$. Then

(A) $P(X' \mid Y) = \frac{1}{2}$ (B) $P(X \cap Y) = \frac{1}{5}$

(C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(Y) = \frac{4}{15}$

Ans. (A,D)

Sol. $P(x) = \frac{1}{3}; \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}; \frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

from this information, we get

$P(X \cap Y) = \frac{2}{15}; P(Y) = \frac{4}{15}$

$\therefore \quad P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$

$P(\bar{X}/Y) = \frac{P(\bar{X} \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)}$

$\Rightarrow \quad P(\bar{X}/Y) = 1 - \frac{2/15}{4/15} = \frac{1}{2}$

41. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 1$

Ans. (A,C,D)

Sol. $f(x) = x\cos(\pi x + [x]\pi)$

$\Rightarrow f(x) = (-1)^{[x]}\, x\cos\pi x.$

Discontinuous at all integers except zero.

42. If $2x - y + 1 = 0$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle ?

(A) 2a, 4, 1 (B) 2a, 8, 1 (C) a, 4, 1 (D) a, 4, 2

Ans. (B,C,D)

Sol. The line $y = mx + c$ is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $c^2 = a^2m^2 - b^2$

$\therefore (1)^2 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4}$

$\Rightarrow a = \frac{\sqrt{17}}{2}$

For option (A), sides are $\sqrt{17}, 4, 1$ ($\Rightarrow$ Right angled triangle)

For option (B), sides are $\sqrt{17}, 8, 1$ ($\Rightarrow$ Triangle is not possible)

For option (C), sides are $\frac{\sqrt{17}}{2}, 4, 1$ ($\Rightarrow$ Triangle is not possible)

For option (D), sides are $\frac{\sqrt{17}}{2}, 4, 2$ ($\Rightarrow$ Triangle exist but not right angled)

43. Let $f : \mathbb{R} \to (0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(A) $e^x - \int_0^x f(t)\sin t\,dt$ (B) $x^9 - f(x)$

(C) $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$ (D) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$

Ans. (B,D)

Sol. For option (A),

Let $g(x) = e^x - \int_0^x f(t)\sin t\,dt$

$\therefore\ g'(x) = e^x - (f(x).\sin x) > 0\ \forall\ x \in (0,1)$

$\Rightarrow$ g(x) is strictly incrasing function.

Also, $g(0) = 1$

$\Rightarrow g(x) > 1\ \forall\ x \in (0,1)$

$\therefore$ option (A) is not possible.

For option (B), let

$k(x) = x^9 - f(x)$

Now, $k(0) = -f(0) < 0$ (As $f \in (0,1)$)

Also, $k(1) = 1 - f(1) > 0$ (As $f \in (0,1)$)

$\Rightarrow k(0).\ k(1) < 0$

So, option(B) is correct.

For option (C), let

$$T(x) = f(x) + \int_0^{\frac{\pi}{2}} f(t).\sin t\,dt$$

$\Rightarrow T(x) > 0\ \forall x \in (0,1)$ (As $f \in (0,1)$)

so, option(C) is not possible.

For option (D),

$$\text{Let } M(x) = x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$$

$$\therefore\ M(0) = 0 - \int_0^{\pi/2} f(t).\cos t\,dt < 0$$

$$\text{Also, } M(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t).\cos t\,dt > 0$$

$\Rightarrow M(0).\ M(1) < 0$

$\therefore$ option (D) is correct.

SECTION–2 : (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

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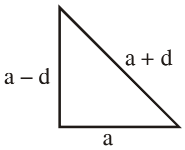
44. The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Ans. 6

Sol.



where $d > 0$, $a > 0$

$\Rightarrow$ length of smallest side = $a - d$

Now $(a + d)^2 = a^2 + (a - d)^2$

$\Rightarrow a(a - 4d) = 0$

$\therefore\ a = 4d$ ...(1)

(As $a = 0$ is rejected)

Also, $\frac{1}{2}a.(a-d) = 24$

$\Rightarrow a(a - d) = 48$ ...(2)

$\therefore$ From (1) and (2), we get $a = 8$, $d = 2$

Hence, length of smallest side

$\Rightarrow (a - d) = (8 - 2) = 6$

45. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Ans. 2

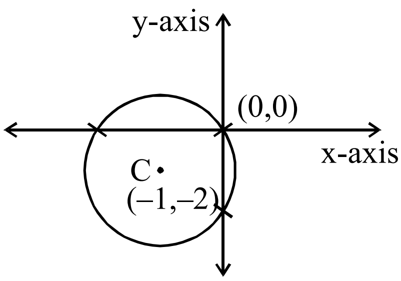
Sol. We shall consider 3 cases.

Case I : When $p = 0$

(i.e. circle passes through origin)

Now, equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$



Case II : When circle intersects x-axis at 2 distinct points and touches y-axis

Now $(g^2 - c) > 0$ & $f^2 - c = 0$

$\Rightarrow$ $1 - (-p) > 0$ & $4 - (-p) = 0$ $\Rightarrow$ $p = -4$

$\Rightarrow$ $p > -1$

$\therefore$ Not possible.

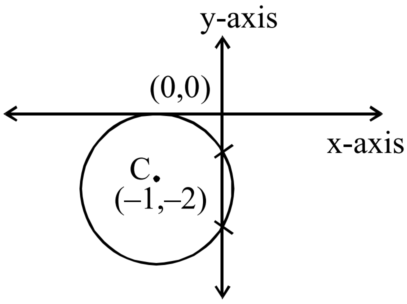
Case III : When circle intersects y-axis at 2 distinct points & touches x-axis.

Now, $g^2 - c = 0$ & $f^2 - c > 0$

$\Rightarrow$ $1 - (-p) = 0$ & $4 - (-p) > 0$

$\Rightarrow$ $p = -1$ $\Rightarrow$ $p > -4$

$\therefore$ $p = -1$ is possible.



$\therefore$ Finally we conclude that $p = 0, -1$

$\Rightarrow$ Two possible values of p.

46. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. 1

Sol. $\Delta = 0 \Rightarrow$ $1(1 - \alpha^2) - \alpha(\alpha - \alpha^3) + \alpha^2(\alpha^2 - \alpha^2) = 0$

$(1 - \alpha^2) - \alpha^2 + \alpha^4 = 0$

$(\alpha^2 - 1)^2 = 0 \Rightarrow \alpha = \pm 1$

but at $\alpha = 1$ No solution so rejected

at $\alpha = -1$ all three equation become

$x - y + z = 1$ (coincident planes)

$\therefore$ $1 + \alpha + \alpha^2 = 1$

47. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}$ =

Ans. 5

Sol. $x = 10!$

$$y = {}^{10}C_1\ {}^{9}C_8\ \frac{10!}{2!}$$

$$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$$

48. Let f : R → R be a differentiable function such that f(0) = 0, $f\left(\frac{\pi}{2}\right)$ = 3 and f'(0) = 1. If

$$g(x) = \int_x^{\frac{\pi}{2}} \left[f'(t)\text{ cosec } t - \cot t \text{ cosec } t\, f(t)\right] dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \to 0} g(x)$ =

Ans. 2

Sol. $$g(x) = \int_x^{\pi/2} (f'(t)\text{cosec}t - f(t)\text{cosec}t\cot t)dt$$

$$= \int_x^{\pi/2} \left(f(t)\text{cosec}t\right)'dt$$

$$= f\left(\frac{\pi}{2}\right)\text{cosec}\left(\frac{\pi}{2}\right) - \frac{f(x)}{\sin x} = 3 - \frac{f(x)}{\sin x}$$

$$\therefore \quad \lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f(x)}{\sin x}; \text{ as } f'(0) = 1$$

$$\Rightarrow \quad \lim_{x\to 0} g(x) = 3 - 1 = 2$$

SECTION–3 : (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –1 In all other cases

Column 1,2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.		
Column 1	**Column 2**	**Column 3**
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only **CORRECT** combination ?

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

Ans. (D)

Sol. $P\left(\sqrt{3}, \frac{1}{2}\right)$; tangent $\sqrt{3}x + 2y = 4$

$\Rightarrow \left(\sqrt{3}\right)x + 4\left(\frac{1}{2}\right)y = 4$ comparing with (II)

$\Rightarrow a = 2 \therefore y = mx + \sqrt{a^2m^2+1}$ is tangent for $m = -\frac{\sqrt{3}}{2}$ i.e (ii)

$\therefore$ point of contact for a = 2, $m = -\frac{\sqrt{3}}{2}$ is R

50. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8,16), then which of the following options is the only **CORRECT** combination ?

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

Ans. (A)

Sol. $y = x + 8$ is tangent $\Rightarrow m = 1$; P(8, 16)

Comparing tangent with (i) of column 2, m = 1 satisfied and a = 8 obtained which matches for point of contact (P) of column 3 and (III) of column I.

51. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1,1), then which of the following options is the only **CORRECT** combination for obtaining its equation ?

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

Ans. (D)

Sol. For $a = \sqrt{2}$ and point (–1,1) only I of column-1 satisfies. Hence equaiton of tangent is $-x + y = 2$ or $y = x + 2 \Rightarrow m = 1$ which matches with (ii) of column 2 and also with Q of column 3

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0,\infty)$.

* Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.

* Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.

* Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1,e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0,1)
(II) $f'(x) = 0$ for some $x \in (1,e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e,e^2)
(III) $f'(x) = 0$ for some $x \in (0,1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0,1)
(IV) $f''(x) = 0$ for some $x \in (1,e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e,e^2)

52. Which of the following options is the only **CORRECT** combination ?

(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

Ans. (D)

53. Which of the following options is the only **CORRECT** combination ?

(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

Ans. (D)

54. Which of the following options is the only **INCORRECT** combination ?

(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

Ans. (D)

Sol. 52. to 54.

$f(x) = x + \ell nx - x\ell nx,\ x > 0$

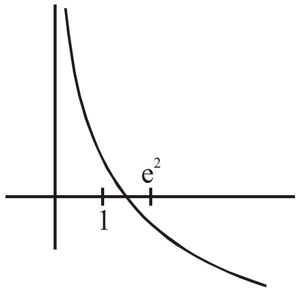
$f'(x) = \cancel{1} + \frac{1}{x} - \ell n x \cancel{-1}$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = \frac{-(x+1)}{x^2}$

(I) $f(1)\, f(e^2) < 0$ so true

(II) $f'(1)\, f'(e) < 0$ so true

(III) Graph of $f'(x)$ so (III) is false



(IV) Is false

As $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x\left[1 + \frac{\ell n x}{x} - \ell n x\right] = -\infty$

$\therefore$ (i) is false (ii) is true

$\lim_{x\to\infty} f'(x) = -\infty$ so (iii) is true

$\lim_{x\to\infty} f''(x) = 0$ so (iv) is true.

(P) $f'(x)$ is positive in $(0,1)$ so true

(Q) $f'(x) < 0$ for in (e,e^2) so true

As $f'(x) < 0\ \forall\ x > 0$ therefor R is false, S is true.

Alternate :

$f(x) = x + \ell nx - x\ell nx$

$f'(x) = \frac{1}{x} - \ell nx = 0$ at $x = x_0$ where $x_0 \in (1,e)$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0\ \forall\ x > 0 \Rightarrow f(x)$ concave down