
#1331078

A body is projected at $t = 0$ with a velocity $10ms^1$ at an angle of 60 with the horizontal.The radius of curvature of its trajectory at $t = 1s$ is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms2$, the value of R is :

A $2.4m$

B $10.3m$

$\boxed{\text{C}}$ $2.8m$

D $5.1m$

Solution

$v_x = 10\cos 60^o = 5m/s$

$v_y = 10\cos 30^o = 5\sqrt{3}m/s$

Velocity after $t = 1$ sec

$v_x = 5m/s$

$v_y = |(5\sqrt{3} - 10)|m/s = 10 - 5\sqrt{3}$

$a_n = \dfrac{v^2}{R} \Rightarrow \dfrac{v_x^2 + v_y^2}{a_n} = \dfrac{25 + 100 + 75 - 100\sqrt{3}}{10\cos\theta}$

$\tan\theta = \dfrac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} \Rightarrow \theta = 15^o$

$R = \dfrac{100(2 - \sqrt{3})}{10\cos 15} = 2.8m$

#1331428

A particle is moving along a circular path with a constant speed of $10ms^1$. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60 around the centre of the circle?

A 0

$\boxed{\text{B}}$ $10m/s$

C $10\sqrt{3}m/s$

D $10\sqrt{2}m/s$

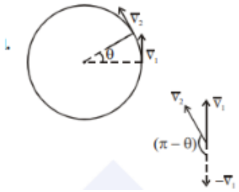
Solution

$|\Delta\bar{c}| = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\pi - \theta)}$

$2v\sin\dfrac{\theta}{2}$ Since $[|\bar{v}_1| = |\bar{v}_2|]$

$= (2 \times 10) \times \sin(30^o)$

$= 10m/s$



#1331470

A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength $980\mathring{A}$. The radius of the atom in the excited state, it terms of Bohr radius a_0, will be :

$(h_c = 12500\ eV - \mathring{A})$

A $9a_0$

B $25a_0$

C $4a_0$

$\boxed{\text{D}}$ $16a_0$

Solution

Energy of photon $=\dfrac{12500}{980}=12.75eV$

$\therefore$ Electron will excite to $n=4$

Since $'R'\propto n^2$

$\therefore$ Radius of atom will be $16a_0$

#1331514

A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be:

A pv^2

$\boxed{\text{B}}$ $\dfrac{3}{4}pv^2$

C $\dfrac{1}{2}pv^2$

D $\dfrac{1}{4}pv^2$

Solution

Momentum per second carried by liquid per

second is ρav^2

net force due to reflected liquid $=2\times\left[\dfrac{1}{4}\rho av^2\right]$

net force due to stopped liquid $=\dfrac{1}{4}\rho av^2$

Total force $=\dfrac{3}{4}\rho av^2$

net pressure $=\dfrac{3}{4}\rho v^2$

#1331578

An electromagnetic wave of intensity $50Wm^2$ enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by :

A $\left(\dfrac{1}{\sqrt{n}},\dfrac{1}{\sqrt{n}}\right)$

$\boxed{\text{B}}$ $\left(\sqrt{n},\dfrac{1}{\sqrt{n}}\right)$

C $(\sqrt{n},\sqrt{n})$

D $\left(\dfrac{1}{\sqrt{n}},\sqrt{n}\right)$

Solution

$C=\dfrac{1}{\sqrt{\mu_0\in_0}}$

$V=\dfrac{1}{k\in_0\mu_0}$ [For transparent medium $\mu_r\approx\mu_0]$

$\therefore\dfrac{C}{V}=\sqrt{k}=n$

$\dfrac{1}{2}\in_0E_0^2C=$ intensity $=\dfrac{1}{2}\in_0kE^2v$

$\therefore E_0^2C=kE^2v$

$\Rightarrow\dfrac{E_0^2}{E^2}=\dfrac{kV}{C}=\dfrac{n^2}{n}\Rightarrow\dfrac{E_0}{E}=\sqrt{n}$

similarly

$\dfrac{B_0^2C}{2\mu_0}=\dfrac{B^2v}{2\mu_0}\Rightarrow\dfrac{B_0}{B}=\dfrac{1}{\sqrt{n}}$

#1331644

An amplitude modulated signal is given by $V(t)=10[1+0.3\cos(2.2\times10^4)]\sin(5.5\times10^5t)$ here t is in seconds. The side band frequencies (in kHz) are , [Given $\pi=22/7$]

A 1785 and 1715

B 892.5 and 857.5

[✓ **C**] 89.25 and 85.75

D 178.5 smf 171.5

Solution

$V(t)=10+\dfrac{3}{2}[2\cos A\sin B]$

$=10+\dfrac{3}{2}[\sin(A+B)-\sin(A-B)]$

$10+\dfrac{3}{2}[\sin(57.2\times10^4t)-\sin(52.8\times10^4t)]$

$\omega_1=57.2\times10^4=2\pi f_1$

$f_1=\dfrac{57.2\times10^4}{2\times\left(\dfrac{22}{7}\right)}=9.1\times10^4$

$\approx 91KHz$

$f_2=\dfrac{52.8\times10^4}{2\times\left(\dfrac{22}{7}\right)}$

$\approx 84KHz$

Side band frequency are

$f_1=f_c-f_w=\dfrac{52.8\times10^4}{2\pi}\approx 85.00kHz$

$f_2=f_c+f_w=\dfrac{57.2\times10^4}{2\pi}\approx 90.00kHz$

#1331750

The force of interaction between two atoms is given by $F=\alpha\beta exp\left(-\dfrac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzman constant and T is temperature and α and β are two constants. The dimension of β is

A $M^2L^2T^{-2}$

[✓ **B**] M^2LT^{-4}

C $M^0L^2T^{-4}$

D MLT^{-2}

Solution

$$F = \alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$$

$$\left[\frac{x^2}{\alpha KT}\right] = m^o L^o T^o$$

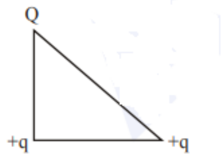
$$\frac{L^2}{[\alpha]ML^2T^{-2}} = M^oL^oT^o \Rightarrow [\alpha] = M^{-1}T^2$$

$$[F] = [\alpha][\beta\}$$

$$MLT^{-2} = M^{-1}T^2[\beta]$$

$$\Rightarrow [\beta] = M^2LT^{-4}$$

#1331792



The charges $Q+q$ and $+q$ are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, it the value of Q is:

A $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

B $-2q$

C $\frac{-q}{1+\sqrt{2}}$

D $+q$

Solution

$$U = K\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right] = 0$$

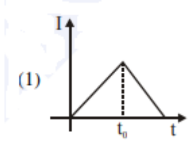
$$\Rightarrow q = -Q\left[1 + \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow Q = \frac{-q\sqrt{2}}{\sqrt{2}+1}$$

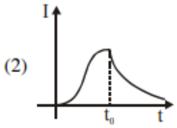
#1331827

In the circuit shown,the switch S_1 is closed at time $t=0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviours of the current I as a function of time $'t'$ is given by :

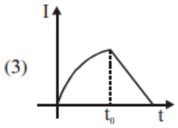
A



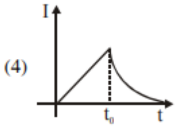
B



C



D

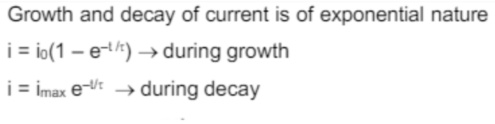


Solution

From time $t = 0$ to $t = t_0$, growth of current takes

place and after that decay of current takes place.

most appropriate is (2)


Growth and decay of current is of exponential nature

$i = i_0(1 - e^{-t/\tau}) \rightarrow$ during growth

$i = i_{max}\, e^{-t/\tau} \rightarrow$ during decay

#1331881

Equation of travelling wave on a stretched string of linear density $5g/m$ is $y = 0.03sin(450t9x)$ where distance and time are measured is SI units. The tension in the string is :

A $10N$

[✓ **B**] $12.5N$

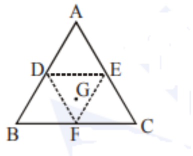
C $7.5N$

D $5N$

Solution

$$y = 0.03\sin(450t - 9x)$$
$$v = \frac{\omega}{k} = \frac{450}{9} = 50m/s$$
$$v\sqrt{\frac{T}{\mu}} \Rightarrow \frac{T}{\mu} = 2500$$
$$\Rightarrow T = 2500 \times 5 \times 10^{-3}$$
$$= 12.5N$$

#1331945



An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. It the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:

A $I = \frac{9}{16}I_0$

B $I = \frac{3}{4}I_0$

C $I = \frac{I_0}{4}$

[✓ **D**] $I = \frac{15}{16}I_0$

Solution

Suppose M is mass and a is side of larger triangle,then $\frac{M}{4}$ and $\frac{a}{2}$ will be mass and side length of smaller triangle

$$\frac{I_{removed}}{I_{original}}=\frac{\frac{M}{4}\left(\frac{a}{2}\right)^2}{(a)^2}$$

$$I_{removed=}\frac{I_0}{16}$$

So,$I=I_0-\frac{I_0}{16}=\frac{15I_0}{16}$

#1331991

There are two long co-axial solenoids of same length l. the inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is :

A $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$

B $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$

C $\frac{n_1}{n_2}$

[✓ D] $\frac{n_2}{n_1}$

Solution

$M=\mu_0 n_1 n_2 \pi r_1^2$

$L=\mu_0 n_1^2 \pi r_1^2$

$\Rightarrow \frac{M}{L}=\frac{n_2}{n_1}$

#1332009

A rigid diatomic ideal gas undergoes an adiabatic process at room temperature,. The relation between temperature and volume of this process is TV^x = constant, then x is :

A $\frac{5}{3}$

[✓ B] $\frac{2}{5}$

C $\frac{2}{3}$

D $\frac{3}{5}$

Solution

For adiabatic process :$TV^{\gamma-1}=$ constant

For diatomic process : $\gamma-1=\frac{7}{5}-1$

$\therefore x=\frac{2}{5}$

#1332029

The gas mixture constists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is:

A $12RT$

B $20RT$

[✓ C] $15RT$

D $4RT$

Solution

$$U = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$$
$$= \frac{5}{2}(3RT) + \frac{3}{2} \times 5RT$$
$$U = 15RT$$

#1332054

In a Young's double slit experiment, the path different, at a certain point on the screen, between two interfering waves is $\frac{1}{8} th$ of wavelength. The ratio of the intensity at this point to that at the centre of a brigth fringe is close to :

A 0.94

B 0.74

$\boxed{\text{C}}$ 0.85

D 0.80

Solution

$$\Delta x = \frac{\lambda}{8}$$
$$\Delta\phi = \frac{(2\pi)}{\lambda} \frac{\lambda}{8} = \frac{\pi}{4}$$
$$I = I_0 \cos^2\left(\frac{\pi}{8}\right)$$
$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{8}\right)$$

#1332073

If the deBroglie wavelenght of an electron is equal to 10^3 times the wavelength of a photon of frequency $6 \times 10^{14} Hz$, then the speed of electron is equal to : (Speed of light = $3 \times 10^8 m/s$ Planck's constant = $6.63 \times 10^{34} J$. Mass of electron = $9.110^{31} kg$)

$\boxed{\text{A}}$ $1.45 \times 10^6 m/s$

B $1.75 \times 10^6 m/s$

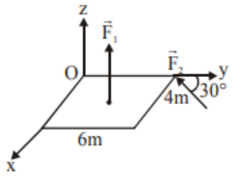
C $1.8 \times 10^6 m/s$

D $1.1 \times 10^6 m/s$

Solution

$$\frac{h}{mv} = 10^{-3}\left(\frac{3 \times 10^8}{6 \times 10^{14}}\right)$$
$$v = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^5}$$

#1332139



A slob is subjected to two force $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ of same magnitude F as shown in the figure .Force $\overrightarrow{F_2}$ is in $XY-$ plane while force $\overrightarrow{F_1}$ acts along $z-$axis at the point $(2\vec{i} + 3\vec{j})$. The moment of these forces about point O will be :

A $(3\hat{i} - 2\hat{j} - 3\hat{k})F$

B $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

C $(3\hat{i} + 2\hat{j} - 3\hat{k})F$

[✓ D] $(3\hat{i} - 2\hat{j} + 3\hat{k})F$

Solution

$\overrightarrow{F_1} = \frac{F}{2}(-\hat{i}) + \frac{f\sqrt{3}}{2}(-\hat{j})$

$\overrightarrow{r_1} = 0\hat{i} + 6\hat{j}$

$\overrightarrow{\tau_{F_1}} = \overrightarrow{r_1} \times \overrightarrow{F_1} = 3F\hat{k}$

Torque for F_2 force

$\overrightarrow{F_2} = F\hat{k}$

$\overrightarrow{r_2} = 2\hat{i} + 3\hat{j}$

$\overrightarrow{\tau_{F_2}} = \overrightarrow{r_2} \times \overrightarrow{F_2} = 3F\hat{i} + 2f(-\hat{j})$

$\overrightarrow{\tau_{net}} = \overrightarrow{\tau_{F_1}} + \overrightarrow{\tau_{F_2}}$

$= 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$

#1332159

A satellite is revolving in a circular orbit at a height h from the earth surface, such that h << R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is :

[✓ A] $\sqrt{gR}(\sqrt{2} - 1)$

B $\sqrt{2gR}$

C $\sqrt{2R}$

D $\sqrt{\frac{gR}{2}}$

Solution

$v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$

$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}$

$\Delta v = v_e - v_0 = (\sqrt{2} - 1)\sqrt{gR}$

#1332172

In an experiment electrons are accelerated,from rest, by applying a voltage of $500V$.Calculate the radius of the path if a magnetic field $100mT$ is then applied.[Charge of the electron =$1.6 \times 10^{-19}C$ Mass of the electron =$9.1 \times 10^{-31}kg$

[✓ A] $7.5 \times 10^{-4}m$

B $7.5 \times 10^{-3}m$

C $7.5m$

D 7.5×10^{-2}m

Solution

$$r = \frac{\sqrt{2mk}}{eb} = \frac{\sqrt{2me\Delta v}}{eB}$$

$$r = \frac{\sqrt{\frac{2m}{e}.\Delta v}}{B} = \frac{\sqrt{\frac{2 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}(500)}}}{100 \times 10^{-3}}$$

$$r = \frac{\sqrt{\frac{9.1}{0.16} \times 10^{-10}}}{10^{-1}} = \frac{3}{4} \times 10^{-4} = 7.5 \times 10^{-4}$$

#1332223

A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210$ s will be :

A 2

B $\frac{1}{9}$

[✓ C] 3

D 1

Solution

$k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

$u = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$

Hence ratio 3

#1332250

Ice at $20C$is added to $50g$ of water at $40C$.When the temperature of the mixture reaches $0C$,it is found that $20g$ of ice is still unmelted. The amount of ice added to the water was close to (specific heat of water $= 4.2J/g/^oC)$

Specific heat of Ice $= 2.1\ J/g/^oC$ Heat of fusion of water at $0^oC = 334j/g$

A $50g$

[✓ B] $40g$

C $60g$

D $100g$

Solution

Let the amount of ice is m gm.

According to the principal of calorimeter

heat taken by ice = heat given by water

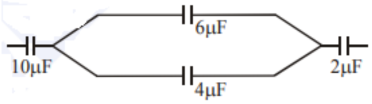
$\therefore 20\times 2.1\times m + (m-20)\times 334$

$= 50\times 4.2\times 40$

$376m = 8400 + 6680$

$m = 40.1$

#1332283



In the figure shown below, the charge on the left plate of the $10\mu F$ capacitor is $30\mu C$. ?The charge on the right plate of the $6\mu F$ capacitor is :

A $-18\mu C$

B $-12\mu C$

C $+12\mu C$

[✓ D] $+18\mu C$

Solution

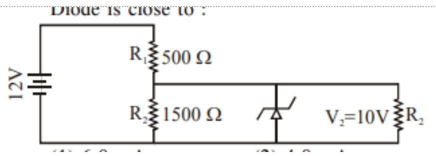
$6\mu F$ & $4\mu F$ are in parallel & total charge on this

combination is $30\mu C$

$\therefore$ Charge of $6\mu F$ capacitor $= \dfrac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+18\mu C$

#1332292



In the given circuit the current through Zener Diode is close to :

A $6.0mA$

B $4.0mA$

C $6.7mA$

[✓ D] $0.0mA$

Solution

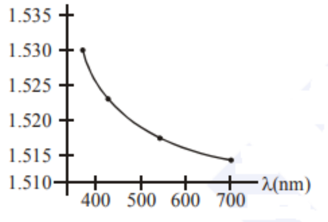
Since voltage across zener diode must be less

than $10V$ therefore it will not work in

breakdown region, & its resistance will be

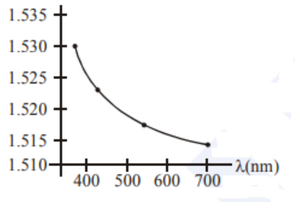
infinite & current through it $= 0$

#1332307

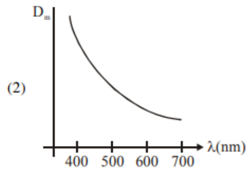


The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?

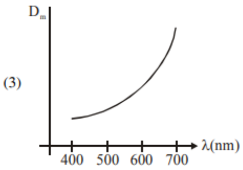
A



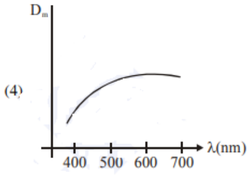
[✓ B]



C



D



Solution

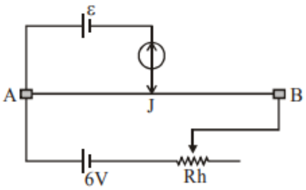
Since $D_m = (\mu 1)A$

& on increasing the wavelength, μ decreases

& hence D_m decreases. Therefore correct

answer is (2)

#1332377



The resistance of the meter bridge AB is given figure is 4Ω. With a cell of emf $\varepsilon = 0.5V$ and rheostat resistance $Rh = 2\Omega$ the null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6\Omega$. The emf ε is;

A $0.6V$

B $0.5V$

[✓ **C**] $0.3V$

D $0.4V$

Solution

Potential gradient with $T_h = 2\Omega$ is $\left(\frac{6}{2+4}\right) \times \frac{4}{L} = \frac{dV}{dL}; L = 100cm$

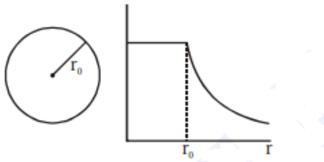
Let null point be at l cm

thus $\varepsilon_1 = 0.5V = \left(\frac{6}{2+4}\right) \times \frac{4}{L} \times l$...(1)

Now with $R_h = 6\Omega$ new potential gradient is $\left(\frac{6}{4+6}\right) \times \frac{4}{L}$ and at null point $\left(\frac{6}{4+6}\right)\left(\frac{4}{L}\right) \times l = \varepsilon_2$...(2)

dividing equation (1) and (2) we get $\frac{0.5}{\varepsilon_2} = \frac{10}{6}$ thus $\varepsilon_2 = 0.3$

#1332384



The given graph shows variation (with distancer from centre) of :

A Potential of a uniformly charged sphere

$\boxed{\text{B}}$ Potential of a uniformly charged spherical shell

C Electric field of uniformly charged spherical shell

D Electric field of uniformly charged sphere

Solution

Potential of a uniformly charged spherical shell is the correct answer of the given graph

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#1332403

Two equal resistance when connected in series to a battery, consume electric power of $60W$. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be

A $60W$

$\boxed{\text{B}}$ $240W$

C $30W$

D $120W$

Solution

In series condition, equivalent resistance is 2R

thus power consumed is $60W = \dfrac{\varepsilon^2}{2R}$ In parallel condition, equivalent resistance is $$R/
2$$ thus new power is

$P' \dfrac{\varepsilon^2}{(R/2)}$
or $P' = 4P = 240W$

#1332456

An object is at a distance of $20m$ from a convex lens of focal length $0.3m$. The lens forms an image of the object. If the object moves away from the lens at a speed of $5m/s$, the speed and direction of the image will be :

A $0.9210^3 m/s$ away from the lens

B $2.26 \times 10^{-3} m/s$ away from the lense

$\boxed{\text{C}}$ $1.16 \times 10^{-3} m/s$ towards the lens

D $3.22 \times 10^{-3} m/s$ towards the lens

Solution

From lens equation

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$
$$\frac{1}{v}-\frac{1}{(-20)}=\frac{1}{(.3)}=\frac{10}{3}$$
$$\frac{1}{v}=\frac{10}{3}-\frac{1}{20}$$
$$\frac{1}{v}=\frac{197}{60};v=\frac{60}{197}$$

Velocity of image wrt. to lens is given by $v_{I/L}=m^2v_O/L$

direction of velocity of image is same as that

of object

$v_{O/L}=5m/s$

$$v_{I/L}=\left(\frac{60\times 1}{197\times 20}\right)^2(5)$$

$1.16\times 10^{-3}m/s$ towards the lens

#1332475

A body of mass 1kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k=1.25106N/m$. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g=10m/s^2$, the value of x will be close to :

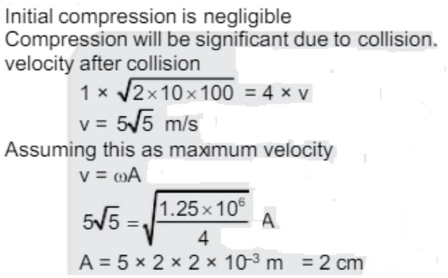
A $4cm$

B $8cm$

C $80cm$

D None of these

Solution



Initial compression is negligible
Compression will be significant due to collision.
velocity after collision

$1\times\sqrt{2\times 10\times 100}=4\times v$

$v=5\sqrt{5}$ m/s

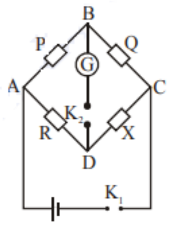
Assuming this as maxmum velocity

$v=\omega A$

$5\sqrt{5}=\sqrt{\frac{1.25\times 10^6}{4}}A$

$A=5\times 2\times 2\times 10^{-3}$ m $=2$ cm

#1332522



In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When $R=400\Omega$, the bridge is equal. When $R=400\Omega$,the bridge is balanced. On inter-changing P and Q, the value of R, for balance, is405Ω.

The value of X is close to :

A $403\,ohm$

B $404.5\,ohm$

C $401.5\,ohm$

D $402.5\,ohm$

Solution

Initiality

$\frac{P}{Q} = \frac{R_1}{X}$_____(1)

After interchanging P and Q

$\frac{Q}{P} = \frac{R_2}{X}$______(2)

From (1) and (2)

$1 = \frac{R_1 \times R_2}{X^2}$

$X = \sqrt{R_1 R_2}$

$X = \sqrt{400 \times 405} = 402.5\Omega$

#1331642

For the cell $Zn(s) | Zn^{2+}(aq) || M^{x+}(aq) | M(s)$, different half cells and their standard electrode potentials are given below :

$M^{x+}(aq/M(s))$	$Au^{3+}(aq)/Au(s)$	$Ag^{+}/Ag(s)$	$Fe^{3+}(aq)/Fe^{2+}(aq)$	$Fe^{2+}(aq)/Fe(s)$
$E^{o}_{M^{x+}/M^{(v)}}$	1.40	0.80	0.77	-0.44

If $E^{o}_{Zn^{2+}/Zn} = -0.76V$, which cathode will give a maximum value of E^{o}_{cell} per electron transferred ?

A Fe^{3+}/Fe^{2+}

[✓ B] Ag^{+}/Ag

C Au^{3+}/Au

D Fe^{2+}/Fe

Solution

We have,

$E^{0}_{cell} = E_{cathode} - E_{anode}$

For a high value of E^{0}_{cell} the value of SRP of cathode should be high.

here the highest value is for Au^{3+}/Au

#1331719

The correct match between items I and II is :

Item-I (mixture)	Item-II (separation method)
H_2O : sugar	Sublimation
H_2O : Aniline	Recrystallization
H_2O: Toluene	Steam distillation
	Differential extraction

[✓ A] A-Q, B-R, C-S

B A-R, B-P, C-S

C A-S, B-R, C-P

D A-Q, B-R, C-P

Solution

(mixture)	(seperation method)
H_2O : Sugar $\Rightarrow$	Recrystallization
H_2O : Aniline $\Rightarrow$	Steam distillation
H_2O : Toluene $\Rightarrow$	Differential extraction

#1331729

If a reaction follows the Arrhenius equation, the plot lnk vs $\frac{1}{(RT)}$ gives a straight line with a gradient $(-y)$ unit. The energy required to activate the reactant is :

[✓ A] y unit

B -y unit

C yR unit

D y/R unit

Solution

We have, $k = Ae^{\frac{-E_a}{RT}}$

$\therefore lnK = ln(Ae^{\frac{-E_a}{RT}})$

$\therefore lnK = lnA - Ea\frac{1}{RT}$

Compare it with $y = mx + c$ we get Slope = $-E_a = -y(Given)$

$\therefore E_a = y$

#1331734

The concentration of dissolved oxygen (DO) in cold water can go upto :

[✓ A] 10 ppm

B 14 ppm

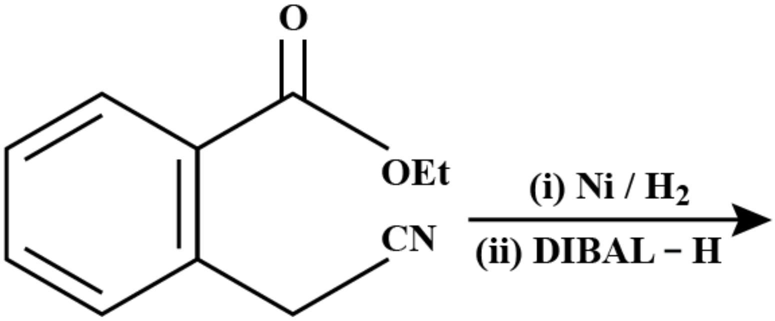
C 16 ppm

D 8 ppm

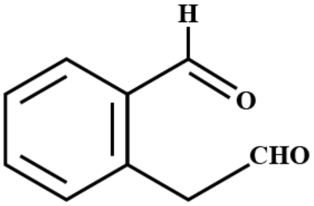
Solution

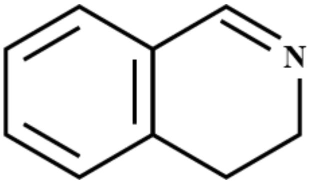
Actual ammount of dissolved oxygen(DO) varies according to temperature. In cold water, dissolved oxygen (DO) can reach a concentration upto 10 ppm.

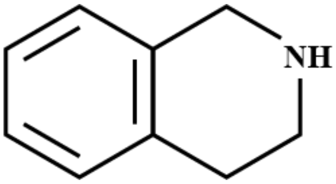
#1331743



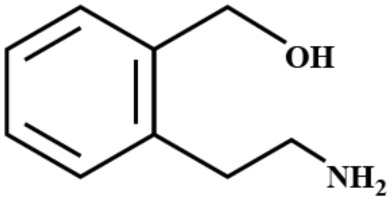
The major product of the following reaction is:

A


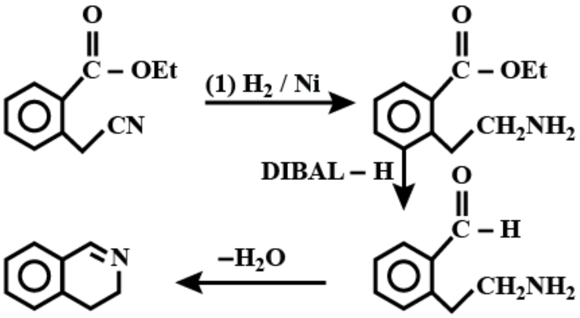
[✓ B]


C


D



Solution



#1331770

The correct statements among (a) to (d) regarding H2 as a fuel are :

(a) It produces less pollutant than petrol

(b) A cylinder of compressed dihydrogen weighs ~ 30 times more than a petrol tank producing the same amount of energy

(c) Dihydrogen is stored in tanks of metal alloys like $NaNi_5$

(d) On combustion, values of energy released per gram of liquid dihydrogen and LPG are 50 and 142 kJ, respectively

A b and d only

[✓ **B**] a, b and c only

C b, c and d only

D a and c only

Solution

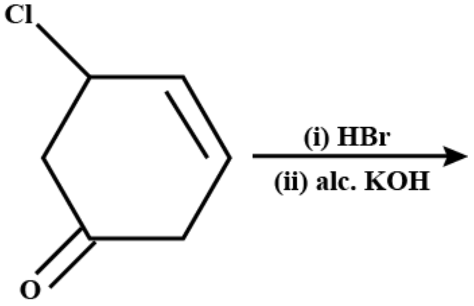
(a) H_2 produces less pollution as compared to petrol because on combustion it does not produce carbon mono oxide.

(b) A cylinder of compressed dihydrogen weighs 30 times more than a petrol tank producing the same amount of energy. It has higher calorific value.

(c) Dihydrogen is stored in tanks of metal alloys

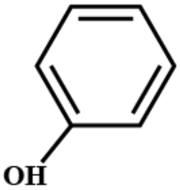
a, b, c are true

#1331777

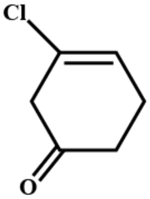


The major poduct of the following reaction is:

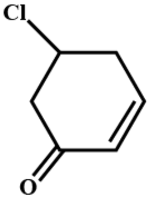
A



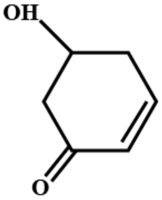
B



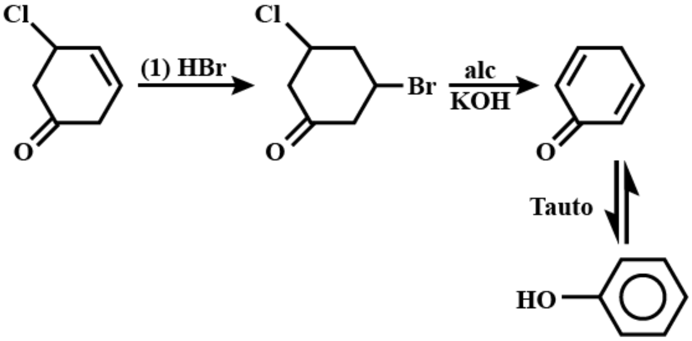
C



D



Solution



#1331800

The element that usually does not show variable oxidation states is :

A V

B Ti

C Sc

D Cu

Solution

Usally Sc(Scandium) does not show variable oxidation states.

Most common oxidation states of :

i) Sc : +3

ii) V : +2, +3, +4, +5

iii) Ti : +2, +3, +4

iv) Cu : +1, +2

#1331876

An organic compound is estimated through Dumus method and was found to evolve 6 moles of CO_2 . 4 moles of H_2O and 1 mole of nitrogen gas. The formula of the compound is :

A $C_{12}H_8N$

B $C_{12}H_8N_2$

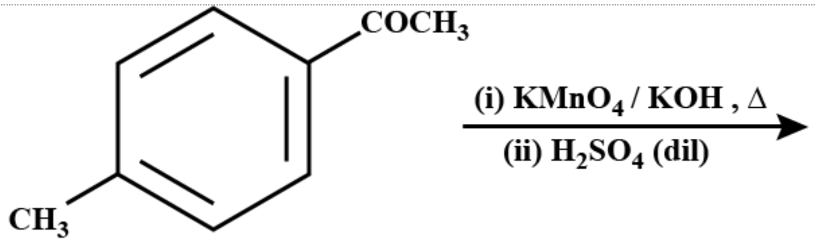
C C_6H_8N

[✓ D] $C_6H_8N_2$

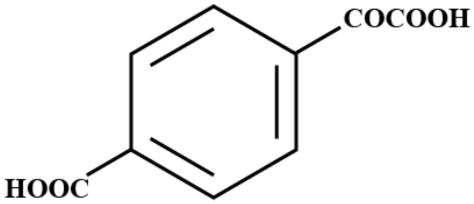
Solution

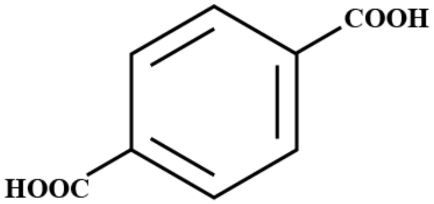
$[C_xH_yN_z] \xrightarrow[Method]{Duma} 6CO_2 + 4H_2O + N_2$ Hence, $C_6H_8N_2$

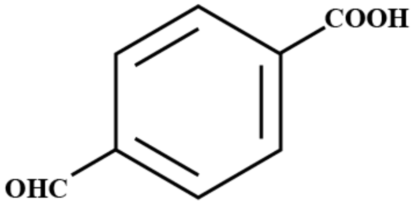
#1331893

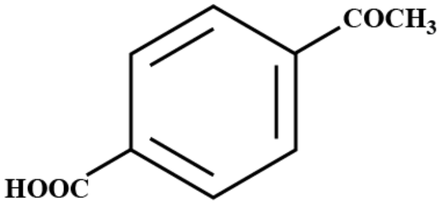


The major product of the following reaction is :

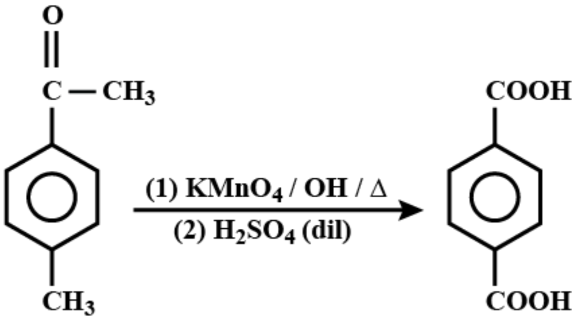
A


[✓ B]


C


D


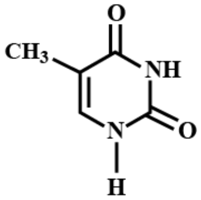
Solution



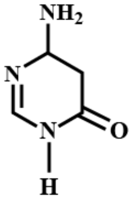
#1331919

Among the following compound which one is found in RNA ?

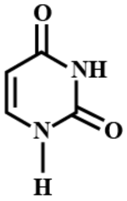
A



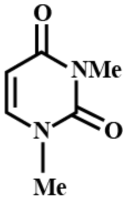
B



C

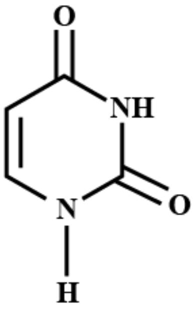


D

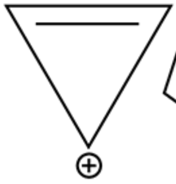


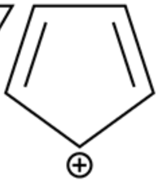
Solution

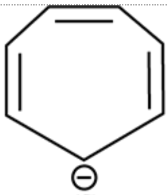
For the given structure 'uracil' is found in RNA

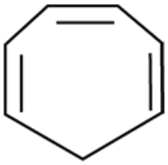


#1331937


(A)


(B)


(C)


(D)

Which compound(s) out of the following is/are not aromatic ?

A C and D

[✓ B] B, C and D

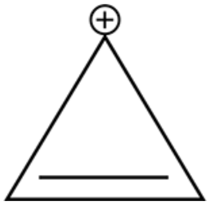
C A and C

D B

Solution

out of the given options only is aromatic.

Hence (B), (C) and (D) are not aromatic



#1331992

The correct match between Item(I) and Item(II)

is :

Item-I	Item-II
Nortehindrone	Anti-biotic
Ofloxacin	Anti-fertility
Equanil	Hypertension
	Analgesics

A A-R, B-P, C-S

[✓ B] A-Q, B-P, C-R

C A-R, B-P, C-R

D A-Q, B-R, C-S

Solution

Norethindrone - Antifertility

Ofloaxacin - Anti-Biotic

Equanil - Hypertension(traiquilizer)

#1332040

Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H-atom is suitable for this purpose ?

$[R_H = 1 \times 10^5\ cm^{-1},\ h = 6.6 \times 10^{-34}\ Js,\ c = 3 \times 10^8 ms^{-1}]$

A Paschen, $5 \rightarrow 3$

[✓ B] Paschen, $\infty \rightarrow 3$

C Lyman, $\infty \rightarrow 1$

D Balmer, ∞ → 2

Solution

We have,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_I^2} - \frac{1}{n_f^2}\right)$$

Here if $n_I = 3$ and $n_f = \infty$

$$\frac{1}{\lambda} = 10^{-7} \times 1^2\left(\frac{1^{2}}{3} - \frac{1}{\infty^2}\right) = \frac{10^{-7}}{9} = 900nm$$

#1332086

Consider the reaction,

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

The equilibrium constant of the above reaction is K_P. If pure ammonia is left to dissociate, the partial pressure of ammonia at equilibrium is given by:

(Assume that $P_{NH_3} << P_{total}$ at equilibrium)

A $\dfrac{3^{\frac{3}{2}}K_P^{\frac{1}{2}}P^2}{4}$

$\boxed{\textbf{B}}$ $\dfrac{3^{\frac{3}{2}}K_P^{\frac{1}{2}}P^2}{16}$

C $\dfrac{K_P^{\frac{1}{2}}P^2}{16}$

D $\dfrac{K_P^{\frac{1}{2}}P^2}{4}$

Solution

$$\begin{array}{lcccccc} & 2NH_3 & \rightleftharpoons & N_2 & + & 3H\_2 \\ t = 0 & P_1 & & 0 & & 0 \\ t = eq & P_{NH_3} & & \frac{P_1}{2} & & \frac{3P_1}{2} \end{array}$$

$$P = P_{total} = \frac{P_1}{2}\frac{3P_1}{2} = 2P_1$$

$$\therefore P_1 = \frac{P}{2}$$

$$k_{eq} = \frac{(P_{N_2})(P_{H_2})}{(P_{NH_3})^2} \Rightarrow \frac{1}{k_p} = \frac{\left(\frac{P_1}{2}\right)\left(\frac{3P_1}{2}\right)^3}{(P_{NH_3})^2}$$

$$\therefore \frac{(P_{NH_3})^2}{K_P} = \frac{P^4}{4^4} \times 3^3$$

$$\therefore P_{NH_3}^2 = K_p\frac{P^4}{4^4} \times 3^3$$

$$P_{NH_3} = \left(K_p\frac{P^4}{4^4}3^3\right)^{\frac{1}{2}} = \frac{(K_p)^{\frac{1}{2}}3^{\frac{1}{2}}P^2}{16}$$

#1332118

Match the ores(Column A) with the metals (column B) :

Column-A Ores	Column-B Metals
Siderite	Zinc
kaolinite	Copper
Malachite	Iron
Calamine	Aluminium

A I-b ; II-c ; III-d ; IV-a

B I-c ; II-d ; III-a ; IV-b

[✓ **C**] I-c ; II-d ; III-b ; IV-a

D I-a ; II-b ; III-c ; IV-d

Solution

Siderite : $FeCO_3$

kaolinite : $Al_2(OH)_4Si_2O_5$

Malachite : $Cu(OH)_2.\ CuCO_3$

Calamine : $ZnCO_3$

#1332133

The correct order of the atomic radii of C, Cs, Al and S is :

A S < C < Al < Cs

B S < C < Cs < Al

C C < S < Cs < Al

[✓ **D**] C < S < Al < Cs

Solution

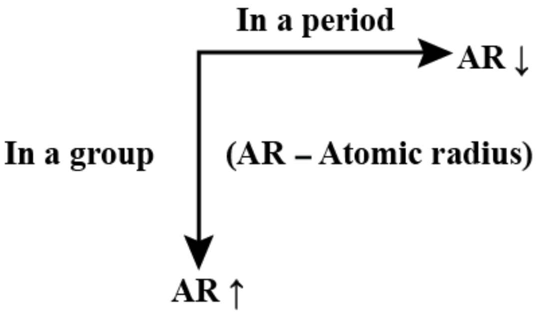
Atomic radii order : C < S < Al < Cs

Atomic radius of C : 170 pm

Atomic radius of S : 180 pm

Atomic radius of Al : 184 pm

Atomic radius of Cs : 300 pm



#1332207

Match the metals (Column I) with the coordination

compound(s) / enzyme(s) (Column II)

	Column-I	Column-II
(A)	Co	Wilkinson catalyst
(B)	Zn	Chlorophyll
(C)	Rh	Vitamin B_{12}
(D)	Mg	Carbonic anhydrase

A A-ii ; B-i ; C-iv ; D-iii

B A-iii ; B-iv ; C-i ; D-ii

C A-iv ; B-iii ; C-i ; D-ii

D A-i ; B-ii ; C-iii ; D-iv

Solution

(i) Wilkinson catalyst : $RhCl(PPh_3)_3$

(ii) Chlorophyll : $C_{55}H_{72}O_5N_4Mg$

(iii) Vitamin B_{12} (also known as cyanocobalamin) contain cobalt.

(iv) Carbonic anhydrase contains a zinc ion.

#1332229

A 10 mg effervescent tablet contianing sodium bicarbonate and oxalic acid releases 0.25 ml of CO_2 at T = 2.98.15 K and p = 1 bar. If molar volume of CO_2 is 25.9L under such condition, what is the percentage of sodium bicarbonate in each tablet ?

[Molar mass of $NaHCO_3$ = 84 $g\,mol^{-1}$]

A 16.8

B 8.4

C 0.84

D 33.6

Solution

$2NaHCO_3 + H_2C_2O_4 \rightarrow Na_2C_2O_4 + 2CO_2 + 2H_2O$

Here, number of moles of $CO_2 = \dfrac{0.25 \times 10^{-3}}{25.9} \approx 10^{-5}$

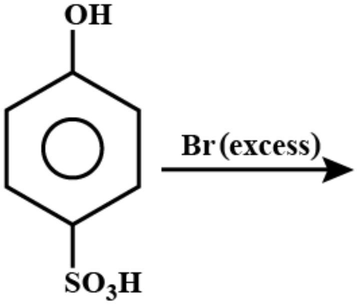
Now, one mole of CO_2 is produced by one mole of $NaHCO_3$.

$\therefore$ the number of moles of $NaHCO_3$ in the given reaction = number of moles of $CO_2 = 10^{-5}$

Now, the weight of $NaHCO_3 = 10^{-5} \times 84 = 84 \times 10^{-5} g$

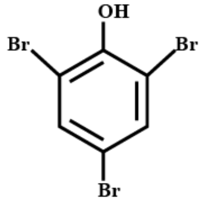
$\therefore$ %Mass $= \dfrac{84 \times 10^{-5}}{10 \times 10^{-3}} \times 100 = 8.4\%$

#1332241

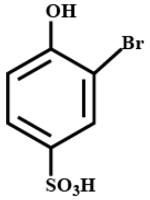


The major product of the following reaction is :

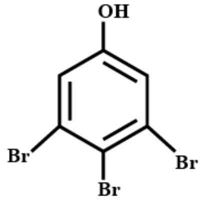
A



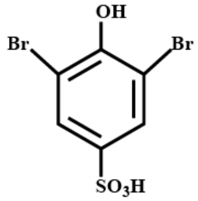
B



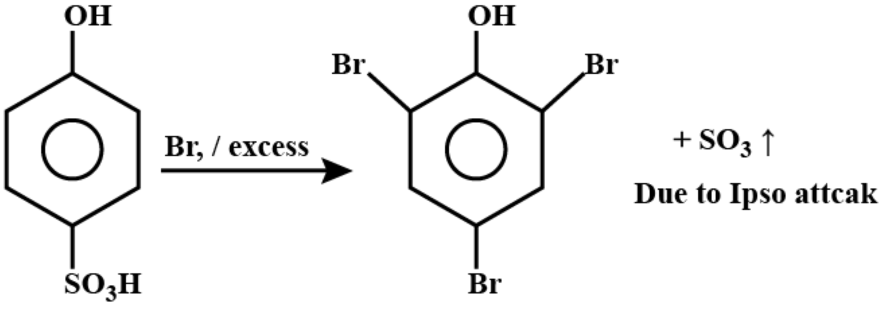
C



D



Solution



#1332275

Two blocks of the same metal having the same mass and at temperature T_1 and T_2, respectively. are brought in contact with each other and allowed to attain thermal equilibrium at constant pressure. The change in entropy, ΔS, for this process is :

A $2C_p ln\left(\frac{T_1 + T_2}{4T_1T_2}\right)$

B $2C_pln\left[\frac{(T_1+T_2)^{\frac{1}{2}}}{T_1T_2}\right]$

[✓ **C**] $C_pln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

D $2C_pln\left[\frac{T_1+T_2}{2T_1T_2}\right]$

Solution

When two blocks are kept in contact with each other, the final temperature will be given as:

$T_f = \frac{T_1+T_2}{2}$

Now, we have $\Delta S_{sys} = \int\frac{dq_{rev}}{T} = nC_p\int\frac{dT}{T}$

For the first block of metal the entropy will be given as:

$\Delta S_1 = nC_p\int_{T_1}^{T_f}\frac{dT}{T} = nC_pln\frac{T_f}{T_1}$

Similarily, $\Delta S_2 = nC_pln\frac{T_f}{T_2}$

Now, total change in entropy $\Delta S_1 + \Delta S_2 = nC_p\frac{T_f^2}{T_1T_2}$

But $T_f = \frac{T_1+T_2}{2}$

$\therefore$ Final entropy will be: $nC_pln[\frac{(T_1+T_2)^2}{4T_1T_2}$

#1332286

The chloride that cannot get hydrolysed is :

A $SiCl_4$

B $SnCl_4$

C $PbCl_4$

[✓ **D**] CCl_4

Solution

CCl_4 cannot get hydrolyzed due to the absence of

vacant orbital at the carbon atom.

#1332299

For the chemical reaction $X \rightleftharpoons Y$, the standard reaction Gibbs energy depends on temperature T (in K) as : $\Delta_rG^o(in\,kJ\,mol^{-1}) = 120 - \frac{3}{8}T$ The major component of the reaction mixture at T is:

A X if $T = 315$ K

[✓ **B**] Y if $T = 350$ K

C Y if $T = 300$ K

D Y if $T = 280$ K

Solution

We have,

$\Delta G = 120 - \dfrac{3}{8}T$

At equilibrium $\Delta G = 0$

$\therefore\ T = 320K$

Here, in the reaction $X \rightleftharpoons Y$, if $T > 320K$ then ΔG becomes negative.

Thus the reaction will proceed in the forward direction and the amount of Y will be higher than its amount on equilibrium.

here only at T=350K temperature is greater than 320 K.

#1332327

The freezing point of a diluted milk sample is found to be -0.2^oC, while it should have been -0.5^oC for pure milk. How much water has been added to pure milk. How much water has been added to pure milk to make the diluted sample ?

A 2 cups of water to 3 cups of pure milk

B 1 cup of water to 3 cups of pure milk

[✓ **C**] 3 cup of water to 2 cups of pure milk

D 1 cup of water to 2 cups of pure milk

Solution

We have,

$T_f = \ -0.5^oC$ (For milk) and $T_f = \ -0.2^oC$ (For diluted solution)

$\therefore\ \Delta T_f = 0.5^oC$ (For milk) and $\Delta T - f = 0.2^oC$ (For diluted solution)

Now, we know,

$\Delta T_f = K_f \times m$

where $m = molality$

We can have,

$\dfrac{(\Delta T_f)_1}{(\Delta T_f)_2} = \dfrac{K_f \times x \times 1000 w_2}{w_1 \times K_f \times x \times 1000}$

\dfrac{0.5}{0.2}=\dfrac{w-2}{w-1}$$

$\therefore\ w_2 = \dfrac{5}{2}w_1$

$\therefore\ W_{water} = \dfrac{5}{2}w_1 - w_1 = \dfrac{2}{3}w_1$

Thus, we required 3 cup of water and 2 cup of milk

#1332341

A solid having density of $9 \times 10^3 kgm^{-3}$ forms face centred cubic crystals of edge length $200\sqrt{2}pm$. What is the molar mass of the solid ?

(Avogadro constant $\cong 6 \times 10^{23} mol^{-1}, \pi \cong 3$)

A $0.0216 kgmol^{-1}$

[✓ **B**] $0.0305 kgmol^{-1}$

C $0.4320 kgmol^{-1}$

D $0.0432 kgmol^{-1}$

Solution

We have,

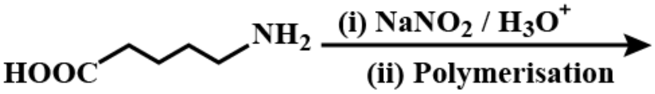
$d = \dfrac{ZM}{a^3 N_A}$

Here, for FCC Z=4,

$9 \times 10^3 = \dfrac{4M}{(200\sqrt{2} \times 10^{-12})^3 \times 6.022 \times 10^{23}}$

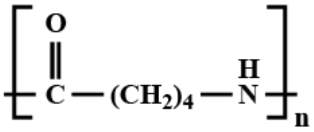
$M = 0.0305 Kg/mol$

#1332353



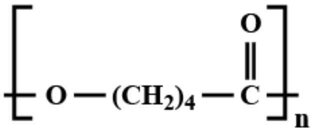
The polymer obtained from the following reactions is :

A



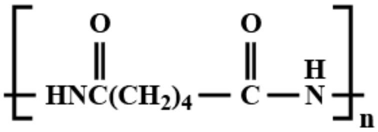
$\left[-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{H}{N}- \right]_n$

[✓ B]



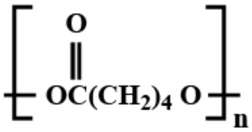
$\left[-O-(CH_2)_4-\overset{O}{\overset{\|}{C}}- \right]_n$

C



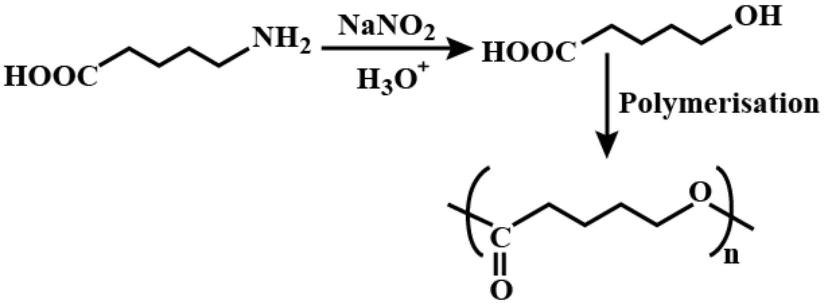
$\left[-HN\overset{O}{\overset{\|}{C}}(CH_2)_4-\overset{O}{\overset{\|}{C}}-\overset{H}{N}- \right]_n$

D



$\left[-O\overset{O}{\overset{\|}{C}}(CH_2)_4\,O- \right]_n$

Solution



#1332354

An example of solid sol is :

A butter

[✓ **B**] gem stones

C paint

D hair cream

Solution

solid sol is the colloidal dispersion which is solid in the state but is little soft.

When they are boiled for 5-6 min, they become softer.

Gemstones are the example of Solid sol.

#1332366

Peoxyacetyl nitrate (PAN), an eye irritant is produced by :

A Acid rain

[✓ **B**] Photochemical smog

C Classical smog

D Organic waste

Solution

Photochemical smog produce chemicals such as formaldehyde, acrolein and peroxyacetyl nitrate (PAN).

#1332373

NaH is an example of :

A Electron-rich hydride

B Molecular hydride

[✓ **C**] Saline hydride

D Metallic hydride

Solution

NaH is an example of ionic hydride which is also known as saline hydride.

#1332389

The amphoteric hydroxide is :

A $Ca(OH)_2$

[✓ **B**] $Be(OH)_2$

C $Sr(OH)^2$

D $Mg(OH)_2$

Solution

$Be(OH)_2$ is amphoteric in nature while rest all alkaline earth metal hydroxide are basic in nature

#1331484

Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|P|$ is?

[✓ A] $\frac{1}{\sqrt{2}}$

B $\frac{1}{\sqrt{5}}$

C $\frac{1}{\sqrt{6}}$

D $\frac{1}{\sqrt{3}}$

Solution

A is orthogonal matrix, since $AA^T = I_3$

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$

#1331618

The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

A $\frac{5}{4}$

[✓ B] $\frac{9}{8}$

C $\frac{3}{4}$

D $\frac{7}{8}$

Solution

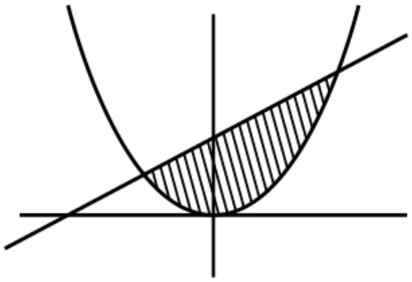
$x = 4y - 2$ & $x^2 = 4y$

Solving the equations,

$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$\therefore x = 2, -1$

so, $\int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx = \frac{9}{8}$



#1331700

The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}$- d each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}$ + d each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals:-

A 2

B $\frac{\sqrt{5}}{2}$

C $\frac{2}{3}$

$\boxed{\text{D}}$ $\sqrt{2}$

Solution

Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

#1331739

The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

A $\frac{4}{9}$

B $\frac{2}{9}$

$\boxed{\text{C}}$ $\frac{2}{3}$

D $\frac{1}{3}$

Solution

$\frac{a}{1-r} = 3$ .....(1)

$\therefore a^3 = 27(1-r)^3$

$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ as $|r| < 1$

#1332007

Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}, \hat{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors.

Then the non-zero vector $\vec{a} \times \vec{c}$ is:

A $-14\hat{i} - 5\hat{j}$

B $-10\hat{i} - 5\hat{j}$

$\boxed{\text{C}}$ $-10\hat{i} + 5\hat{j}$

D $-14\hat{i} + 5\hat{j}$

Solution

$[\vec{a}\,\vec{b}\,\vec{c}] = 0$

$$\begin{array}{ccc}1 & 2 & 4\end{array}$$

$$\Rightarrow \begin{vmatrix}1 & \lambda & 4\\ 2 & 4 & \lambda^2-1\end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3) - 9(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow = 2, 3, \ -3$

so, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

$$\text{Hence } \vec{a} \times \vec{c} = \begin{array}{c}\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k}\end{array}\\ \begin{vmatrix}1 & 2 & 4\\ 2 & 4 & 3\end{vmatrix}\end{array}$$

$= -10\hat{i} + 5\hat{j}$

#1332053

Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x+iy}{27}$ $(i = \sqrt{-1})$, where x and y are real numbers, then $y - x$ equals:

A -85

B 85

C -91

[✓ **D**] 91

Solution

Calculating: $\left(-2 - \frac{i}{3}\right)^3 = \frac{(-6-i)^3}{27}$

$= \frac{-198 - 107i}{27} = \frac{x+iy}{27}$

Hence, $y - x = 198 - 107 = 91$

#1332132

Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then , in the interval $(-2, 2)$, g is:-

A differentiable at all points

B not differentiable at two points

C not continuous

[✓ **D**] not differentiable at one point

Solution

$$|f(x)| = \begin{cases} 1, & -2 \le x < 0 \\ 1 - x^2, & 0 \le x < 1 \\ x^2 - 1, & 1 \le c \le 2 \end{cases}$$

and $f(|x|) = x^2 - 1, x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases} x^2, & x \in [-2, 0) \\ 0, & x \in [0, 1) \\ 2(x^2 - 1), & 1 \le c \le 2 \end{cases}$$

It is not differentiable at $x = 1$

#1332230

Let $f: R \to R$ be defined by $f(x) = \frac{x}{1 + x^2}, x \in R$. Then the range of f is:

A $(-1, 1) - 0$

$\boxed{\text{B}}$ $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

D $R - [-1, 1]$

Solution

$f(0) = 0$ & $f(x)$ is odd.

Further, if $x > 0$ then

$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

#1332269

The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x}\right)^8$ equals 5670 is:

A 6

B 8

$\boxed{\text{C}}$ 0

D 4

Solution

$T_5 = {}^8C_4 \frac{x^{12}}{81} \times \frac{81}{x^4} = 5670$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

Hence, The sum of the real values of x are 0

#1332312

The value of r for which ${}^{20}C_r^{20}C_0 + {}^{20}C_1 + {}^{20}C_{r-2}^{20}C_2 + \ldots . {}^{20}C_0^{20}C_r$ is maximum, is

$\boxed{\text{A}}$ 20

B 15

C 11

D 10

Solution

Given sum = coefficient of x^r in the expansion of $(1+x)^{20}(1+x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when $r = 20$

#1332360

Let $a_1, a_2, \ldots\ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals:

A $2(5^2)$

B $4(5^2)$

$\boxed{\text{C}}$ 5^4

D 5^3

Solution

$a_1, a_2, \ldots, a_{10}$ are in G.P., Let the common ratio be r

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1 r^2}{a_1} = 25$

$\Rightarrow r^2 = 25$

$\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = 5^4$

#1332542

If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2}) + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

A $\frac{-1}{3x^3}$

$\boxed{\text{B}}$ $\frac{-1}{27x^9}$

C $\frac{1}{9x^4}$

D $\frac{1}{27x^6}$

Solution

$$\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)(\sqrt{1-x^2})^m + C$$

$$\int \frac{|x|\sqrt{\frac{1}{x^2-1}}\,\mathrm{dx}}{x^4}$$

Put $\frac{1}{x^2} - 1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case -1 $x \geq 0$

$-\frac{1}{2}\int \sqrt{t}\,dt \Rightarrow -\frac{t^{\frac{3}{2}}}{3} + C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2-1}\right)^{\frac{3}{2}}$

$\Rightarrow \frac{(\sqrt{1-x^2})^3}{-3x^2} + C$

$A(x) = -\frac{1}{3x^3}$ and $m = 3$

$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$

Case-II $x \leq 0$

We get $\frac{\sqrt{(1-x^2)^3}}{-3x^3} + C$

$A(x) = \frac{1}{-3x^3}, m = 3$

$(A(x))^m = \frac{-1}{27x^9}$

#1332612

In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$,where c is the length of the third side of the triangle, then the circumradius of the triangle is :

A $\frac{y}{13}$

$\boxed{\textbf{B}}$ $\frac{c}{\sqrt{3}}$

C $\frac{c}{3}$

D $\frac{3}{2}y$

Solution

Given $a + b = x$ and $ab = y$

If $x^2 - c^2 = y \Rightarrow (a+b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$

$\Rightarrow cosC = -\frac{1}{2}$

$\Rightarrow \angle C = \frac{2\pi}{3}$

$R = \frac{c}{2sinC} = \frac{c}{\sqrt{3}}$

#1332719

The value of the integral $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$(where [x] denotes the greatest integer less than ${}^{20}Cr$ or equal to x) is:

A 4

B $4-\sin 4$

C $\sin 4$

$\boxed{\text{D}}$ 0

Solution

$$I=\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$$

$$I=\int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right]=-1\, as\, x\neq n\pi\right)$$

$$I=\int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx=0$$

#1333504

If the system of linear equations

$2x+2y+3z=a$

$3x-y+5z=b$

$x-3y+2z=c$

Where a,b,c are non-zero real numbers, has more then one solution, then :

$\boxed{\text{A}}$ $b-c-a=0$

B $a+b-c=0$

C $b+c-a=0$

D $b-c+a=0$

Solution

$P_1: 2x+2y+3z=a$

$P_2: 3x-y+5z=b$

$P_3: x-3y+2z=c$

We find

$P_1+P_3=P_2 \Rightarrow a+c=b$

$\therefore b-c-a=0$

#1333523

A square is inscribed inthe circle $x^2 + y^2 - 6x - 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then of the vertex of this square which is nearest to the origin is:-

A 13

B $\sqrt{137}$

C 6

D $\sqrt{41}$

Solution

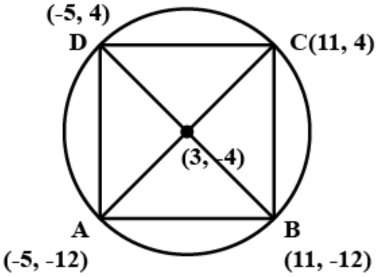
$R = \sqrt{9 + 16 + 103} = 8\sqrt{2}$

$OB = \sqrt{265}$

$OC = \sqrt{137}$

$OD = \sqrt{41}$

Hence OD is nearest to the origin.



#1333551

Let $f_k(x) = \frac{1}{k}(sin^k x + cos^k x)$ for $k = 1, 2, 3, \ldots$. Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to:-

A $\frac{5}{12}$

B $\frac{-1}{12}$

C $\frac{1}{4}$

D $\frac{1}{12}$

Solution

$f_4(x) - f_6(x)$

$$= \frac{1}{4}(sin^4 x + cos^4 x) - \frac{1}{6}(sin^6 x + cos^6 x)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}sin^2 2x\right) = \frac{1}{12}$$

#1333634

Let [x] denote the integer less than or equal to x. Then:-

$$lim_{x \to 0} \frac{tan(\pi sin^2 x) + (|x| - sin(x[x]))^2}{x^2}$$

A equals π

B equals 0

C equals $\pi + 1$

D does not exist

Solution

R.H.L $= lim_{x \to 0^+} \dfrac{tan(\pi sin^2 x) + (|x| - sin(x[x]))^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$lim_{x \to 0^+} \dfrac{tan(\pi sin^2 x)}{\pi sin^2 x} + 1 = \pi + 1$

L.H.L. $= lim_{x \to 0^-} \dfrac{tan(\pi sin^2 x) + (-x + sinx)^2}{x^2}$

(as $x \to 0- \Rightarrow [x] = -1$)

$lim_{x \to 0+} \dfrac{tan(\pi sin^2 x)}{\pi sin^2 x} . \dfrac{\pi sin^2 x}{x^2} + \left(-1 + \dfrac{sin\,x}{x}\right)^2 \Rightarrow \pi$

$R.\,H.\,L \neq L.\,H.\,L.$

#1333665

The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\dfrac{\pi}{4}$ with the plane $y - z + 5 = 0$ is?

A $2\sqrt{3}, 1, -1$

B $2, \sqrt{2}, -\sqrt{2}$

C $2, -1, 1$

$\boxed{\text{D}}$ $\sqrt{2}, 1, -1$

Solution

Let the equation of plane be $a(x - 0) + b(y + 1) + c(z - 0) = 0$ It passes through $(0, 0, 1)$ then $b + c = 0$ .....(1)

Now $cos\dfrac{\pi}{4} = \dfrac{a(0) + b(1) + c(-1)}{\sqrt{2}\sqrt{a^2 + b^2 + c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$ we get $a^2 = 2c^2$

$\Rightarrow a = \pm\sqrt{2}c$

Direction ratio $(a.\ b, c) = (\sqrt{2}, -1, 1)$ or $(\sqrt{2}, 1, -1)$

#1333717

If $xlog_e(log_e x) - x^2 + y^2 = 4(y > 0)$, then dy/dx at $x = e$ is equal to:

A $\dfrac{e}{\sqrt{4 + e^2}}$

B $\dfrac{1 + 2e}{2\sqrt{4 + e^2}}$

$\boxed{\text{C}}$ $\dfrac{2e - 1}{2\sqrt{4 + e^2}}$

D $\dfrac{1 + 2e}{\sqrt{4 + e^2}}$

Solution

When $x = e$,then $0 - e^2 + y^2 = 4, y = \sqrt{e^2 + 4}$

Differentiating with respect to x, We get:

$x. \dfrac{1}{\ell nx}.\dfrac{1}{x} + \ell n(\ell nx) - 2x + 2y.\dfrac{dy}{dx} = 0$ at $x = e$ we get

$1 - 2e + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{2e - 1}{2y}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2e - 1}{2\sqrt{4 + e^2}}$ as $y(e) = \sqrt{4 + e^2}$

#1333719

The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. The the sum of perpendicular distances from A and B on the tangent to the circle at the origin is:

A $\frac{\sqrt{5}}{4}$

[✓ B] $\frac{\sqrt{5}}{2}$

C $2\sqrt{5}$

D $4\sqrt{5}$

Solution

Equation of circle

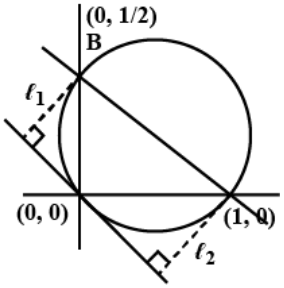
$$(x-1)(x-0)+(y-0)\left(y-\frac{1}{2}\right)=0$$

$$\Rightarrow x^2+y^2-x-\frac{y}{2}=0$$

Equation of tangent of origin is $2x + y = 0$

$$\ell_1+\ell_2=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}}$$

$$=\frac{4+1}{2\sqrt{5}}=\frac{\sqrt{5}}{2}.$$



#1333722

If q is False and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

A $(p \vee r) \rightarrow (p \wedge r)$

B $p \vee r$

C $p \wedge r$

[✓ D] $(p \wedge r) \rightarrow (p \wedge r)$

Solution

Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow q$ is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

#1333723

If $y(x)$ is the differential equation $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0$, where $y(1) = \frac{1}{2}e^{-2}$, then:

A $y(x)$ is decreasing in $(0, 1)$

$\boxed{\textbf{B}}$ $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

C $y(log_e 2) = \frac{log_e 2}{4}$

D $y(log_e 2) = log_e 4$

Solution

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$$

I.F. $= e^{\int\left(\frac{2x+1}{x}\right)dx} = e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x+\ell nx} = e^{2x}.\,x$

So, $y(xe^{2x}) = \int e^{-2x}.\,xe^{2x} + C$

$\Longrightarrow xyw^{2x} = \int x\,dx + C$

$\Longrightarrow 2xye^{2x} = x^2 + 2C$

It passes through $\left(1, \frac{1}{2}e^{-2}\right)$ we get $C = 0$

$$y = \frac{xe^{-2x}}{2}$$

$\Longrightarrow \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$

$\longrightarrow f(x)$ is decreasing in $(\frac{1}{2}, 1)$

$$y(log_e 2) = \frac{(log_e 2)e^{-2(log_e 2)}}{2}$$

$= \frac{1}{8}log_e 2$

#1333725

The maximum value of function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R: x^2 + 30 \leq 11x\}$ is:

$\boxed{\textbf{A}}$ 122

B -222

C -122

D 222

Solution

$S = \{x \in R, x^2 + 30 - 11x \le 0\}$

$= \{x \in R, 5 \le x \le 6\}$

Now $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\Rightarrow f'(x) = 9(x-1)(x-3)$, which is positive in $[5, 6]$

$\Rightarrow f(x)$ increasing in $[5, 6]$

Hence maximum value $= f(6) = 122$

#1333727

If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

A -81

B 100

[✓ **C**] -300

D 144

Solution

$81x^2 + kx + 256 = 0; x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$$

$$\text{Now } -\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27}$$

$\Rightarrow k = \pm 300$

#1333728

Two circles with equal radii are intersecting at the points $(0, 1)$ and $(0, -1)$. The tangent at the point $(0, 1)$ to one of the circle. Then the distance between the centres of these circles is:

A 1

B $\sqrt{2}$

C $2\sqrt{2}$

[✓ **D**] 2

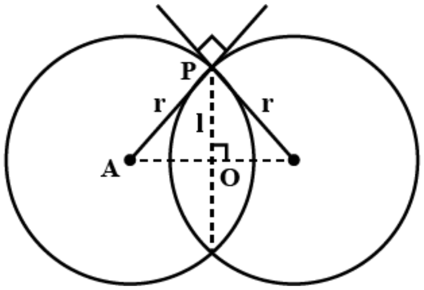
Solution

In ΔAPO

$$\left(\frac{\sqrt{2}r}{2}\right)^2 + 1^2 + r^2$$

$$\Rightarrow r = \sqrt{2}$$

So distance between the centres $= \sqrt{2}r = 2$.



#1333729

Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is:

$\boxed{\text{A}}$ $x + 2y + 4 = 0$

B $x - 2y + 4 = 0$

C $x + y + 1 = 0$

D $4x + 2y + 1 = 0$

Solution

Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \frac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{n}\right) = 2$$

$$\Rightarrow x^2m + \frac{x}{m} - 2 = 0$$

$$D = 0 \Rightarrow m = \frac{1}{2}$$

So tangent is $2y + x + 4 = 0$

#1333730

The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following point?

$\boxed{\text{A}}$ $(2, 0, -2)$

B $(-2, 2, 2)$

C $(0, -2, 2)$

D $(2, 2, 0)$

Solution

The normal vector of required plane

$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is $(-1, 1, 1)$

So required plane is

$-(x-3) + (y+2) + (z-1) = 0$

$\implies -x + y + z + 4 = 0$

Which is satisfied by $(2, 0, -2)$

#1333731

If tangent are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve:

A $\frac{x^2}{2} + \frac{y^2}{4} = 1$

B $\frac{x^2}{4} + \frac{y^2}{2} = 1$

[✓ **C**] $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

D $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

Solution

Equation of general tangent on ellipse

$\frac{x}{a\,sec\theta} + \frac{y}{bcosec\theta} = 1$

$a = \sqrt{2}, b = 1$

$\Rightarrow \frac{x}{\sqrt{2}sec\theta} + \frac{y}{cosec\theta} = 1$

Lt the midpoint be (h,k)

$h = \frac{\sqrt{2}sec\theta}{2} \Rightarrow cos\theta = \frac{1}{\sqrt{2h}}$

and $k = \frac{cosec\theta}{2} \Rightarrow sin\theta = \frac{1}{2k}$.

$\because sin^2\theta + cos^2\theta = 1$

$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$

$\therefore \frac{1}{2x^2} + \frac{1}{4y^2} = 1$

#1333733

Two integers are selected at random from the set $1, 2, . . . , 11$. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:

[✓ **A**] $\frac{2}{5}$

B $\frac{1}{2}$

C $\frac{3}{5}$

D $\frac{7}{10}$

Solution

Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space $={}^5C_2+{}^6C_2$

so required probability $=\dfrac{{}^5C_2}{{}^5C_2+{}^6C_2}=\dfrac{2}{5}$