1. A radioactive nuclei with decay constant 0.5/s is being produced at a constant rate of 100 nuclei/s. If at t=0 there were no nuclei, the time when there are 50 nuclei is :

(1) $\ln\left(\frac{4}{3}\right)s$ (2) ln2s

(3) $2\ln\left(\frac{4}{3}\right)s$ (4) 1s

Ans. (3)

Sol. $\frac{dN}{dt} = 100 - 0.5\ N$

$$\int_0^{50}\frac{dN}{100-0.5N} = \int_0^t dt$$

$$-\frac{1}{0.5}[\ell n(100-0.5N)]_0^{50} = t$$

$$\Rightarrow t = 2\ \ell n\left(\frac{100}{75}\right) = 2\ \ell n\left(\frac{4}{3}\right)$$

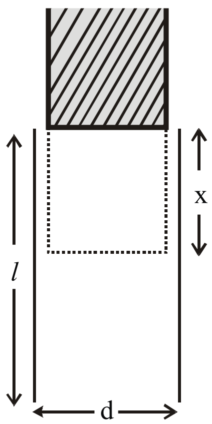
2. Match the List - I (Phenomenon associated with electromagnetic radiation) with List - II (Part of electromagnetic spectrum) and select the correct code from the choices given below the lists :

	List-I		List-II
I	Double of sodium	A	Visible radiation
II	Wavelength corresponding to temperature associated with the isotropic radiation filling all space	B	Microwave
III	Wavelength emitted by atomic hydrogen in intersellar space	C	Short radiowave
IV	Wavelength of radiation arising from two close energy levels in hydrogen	D	X-rays

(1) (I)-(A), (II)-(B), (III)-(B), (IV)-(C)
(2) (I)-(D), (II)-(C), (III)-(A), (IV)-(B)
(3) (I)-(A), (II)-(B), (III)-(C), (IV)-(C)
(4) (l)-(B), (II)-(A), (III)-(D), (IV)-(A)

Ans. (4)

3. A parallel plate capacitor is made of two plates of length l, width ω and separated by distance d. A dielectric slab (dielectric constant K) that fits exactly between the plates is held near the edge of the plates. It is pulled into the capacitor by a force $F = -\frac{\partial U}{\partial x}$ where U is the enerey of the capacitor when dielectric is inside the capacitor up to distance x (See figure). If the charge on the capacitor is Q then the force on the dielectric when it is near the edge is



(1) $\frac{Q^2 d}{2\omega l^2 \varepsilon_0}K$

(2) $\frac{Q^2 \omega}{2 d l^2 \varepsilon_0}(K-1)$

(3) $\frac{Q^2 d}{2\omega l^2 \varepsilon_0}(K-1)$

(4) $\frac{Q^2 \omega}{2 d l^2 \varepsilon_0}K$

Ans. (3)

Sol. $C = C_1 + C_2$

$$= \frac{K(x\omega)\varepsilon_0}{d} + \frac{(1-x)\omega\varepsilon_0}{d}$$

$$C = \frac{\omega\varepsilon_0}{d}[kx + (\ell - x)]$$

$$C = \frac{\omega\varepsilon_0}{d}[\ell + (k-1)x]$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{d.Q^2}{2\omega\varepsilon_0[\ell+(k-1)x]}$$

$$\frac{du}{dx} = -\frac{d.Q^2(K-1)}{2\omega\varepsilon_0[\ell+(K-1)x]^2}$$

$$F = -\frac{du}{dx} = \frac{Q^2.d(K-1)}{2\omega\ell^2\varepsilon_0} \text{ at } (x = 0)$$

4. A cone of base radius R and height h is located in a uniform electric field $\vec{E}$ parallel to its base. The electric flux entering the cone is :-

(1) 4EhR (2) $\frac{1}{2}$EhR

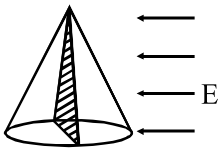
(3) EhR (4) 2EhR

Ans. (3)

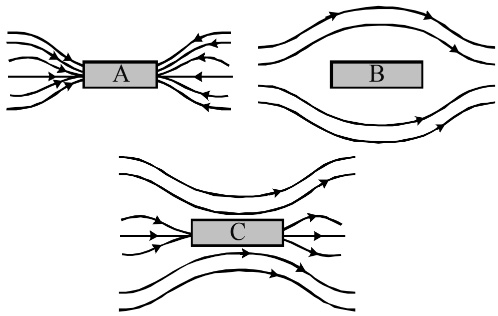
Sol. flux $\phi = E \cdot A_{\perp}$

$= E \cdot \left(\frac{1}{2} h \times 2R\right)$

$= Ehr$



5. Three identical bars A, B and C are made of different magnetic materials. When kept in a uniform magnetic field, the field lines around them look as follows :



Make the correspondence of these bars with their material being diamagnetic (D) ferromagnetic (F) and paramagnetic (P)

(1) A↔P, B↔F, C↔D

(2) A↔F, B↔P, C↔D

(3) A↔D, B↔P, C↔F

(4) A↔F, B↔D, C↔P

Ans. (4)

Sol.

6. The average mass of rain drops is 3.0×10^{-5} kg and their average terminal velocity is 9 m/s. Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.

(1) 9.0×10^{4} J (2) 4.05×10^{4} J

(3) 3.5×10^{5} J (4) 3.0×10^{5} J

Ans. (2)

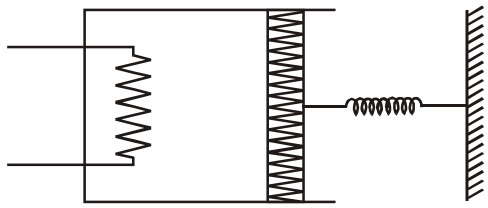
Sol. $E = \frac{1}{2}(1 \times 0 \times 10^{3}) \times 81$

$= 500 \times 81$

$= 40500$ J

$= 4.05 \times 10^{4}$ J

7. An ideal monoatomic gas is confined in a cylinder by a spring loaded piston of cross section 8.0×10^{-3} m². Initially the gas is at 300K and occupies a volume of 2.4×10^{-3} m³ and the spring is in its relaxed state as shown in figure. The gas is heated by a small heater until the piston moves out slowly by 0.1 m. The force constant of the spring is 8000 N/m and the atmospheric pressure is 1.0×10^{5} N/m² The cylinder and the piston are thermally insulated. The piston and the spring are massless and there is no friction between the piston and the cylinder. The final temperature of the gas will be : (Neglect the heat loss through the lead wires of the heater. The heat capacity of the heater coil is also negligible) :-



(1) 500 K (2) 300 K

(3) 800 K (4) 1000 K

Ans. (3)

Sol. $A = 8 \times 10^{-3}$ m²

$T_1 = 300$ K

$V_1 = 2.4 \times 10^{-3}$ m³

$V_2 = V_1 + A\Delta x$

$= 2.4 \times 10^{-3} \times 8 \times 10^{-3} \times 0.1$

$= 3.2 \times 10^{-3}$ m³

$K = 8000$ N/m

$T_2 = ?$

$P_1 = 10^{5}$ N/m²

$P_2 = P_0 + \frac{kx}{A} = 10^{5} + \frac{8000 \times 0.1}{8 \times 10^{-3}}$

$= 2 \times 10^{5}$ N/m²

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{10^{5} \times 2.4 \times 10^{-3}}{300} = \frac{2 \times 10^{5} \times 3.2 \times 10^{-3}}{T_2}$

$T_2 = \frac{3.2 \times 300}{1.2}$

$= 800$ K

8. The angular frequency of the damped oscillator is given by $\omega = \sqrt{\left(\frac{k}{m} - \frac{r^2}{4m^2}\right)}$ where k is the spring constant, m is the mass of the oscillator and r is the damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period compared to the undamped oscillator is approximately as follows :-

(1) Decreases by 1% (2) Increases by 8%
(3) Increases by 1% (4) Decreases by 8%

Ans. (3)

Sol. $\omega = \sqrt{\left(\frac{k}{m} - \frac{r^2}{4m^2}\right)}$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 - \omega = \sqrt{\frac{k}{m}} - \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}$$

$$= \sqrt{\frac{k}{m}}\left(1 - \sqrt{1 - \frac{r^2}{4mk}}\right)$$

$$\frac{\omega_0 - \omega}{\omega_0} = \left[1 - \left(1 - \frac{r^2}{4mk}\right)^{1/2}\right]$$

$$= \left[1 - \left(1 - \frac{r^2}{8mk}\right)\right]$$

$$= \frac{r^2}{8mk} = 1\%$$

9. A coil of circular cross-section having 1000 turns and 4 cm^2 face area is placed with its axis parallel to a magnetic field which decreases by 10^{-2} Wb m^{-2} in 0.01 s. The e.m.f. induced in the coil is :

(1) 200 mV (2) 0.4 mV
(3) 4mV (4) 400 mV

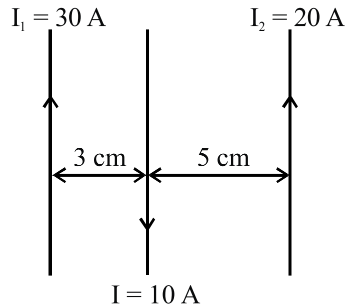
Ans. (4)

Sol. $\varepsilon = -\frac{d\phi}{dt}$

$$\varepsilon = \frac{1000 \times 4 \times 10^{-4} \times 10^{-2}}{.01}$$

$= 4 \times 10^{-1}$ V

10. Three straight parallel current carrying conductors are shown in the figure. The force experienced by the middle conductor of length 25 cm is :



(1) 9 × 10^{-4} toward left
(2) Zero
(3) 6 × 10^{-4} N toward left
(4) 3 × 10^{-4} toward right

Ans. (4)

11. A body of mass 5 kg under the action of constant force $\vec{F} = F_x\hat{i} + F_y\hat{j}$ has velocity at t = 0 s as $\vec{v} = (6\hat{i} - 2\hat{j})$ m/s and at t = 10 s as $\vec{v} = +6\hat{j}$ m/s. The force $\vec{F}$ is :-

(1) $\left(\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right)$N (2) $\left(-3\hat{i} + 4\hat{j}\right)$N
(3) $\left(3\hat{i} - 4\hat{j}\right)$N (4) $\left(-\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$N

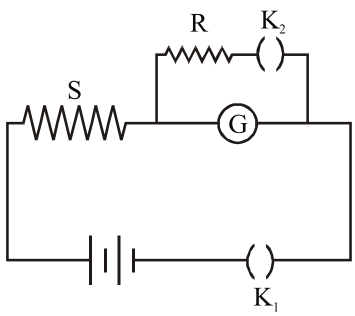
Ans. (2)

Sol. $F = \frac{m(v_f - v_i)}{t}$

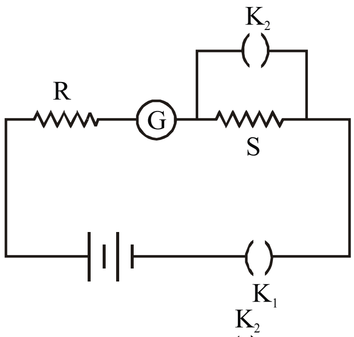
$$= \frac{5(6\hat{j} - 6\hat{i} + 2\hat{j})}{10} = \frac{40\hat{j} - 30\hat{i}}{10} = -3\hat{i} + 4\hat{j}$$

12. In the circuit diagrams (A, B, C and D) shown below, R is a high resistance and S is a resistance the order of galvanometer G. The correct circuit, corresponding to the half deflection method for finding the resistance and figure of merit of the galvanometer, is the circuit labelled as : :-

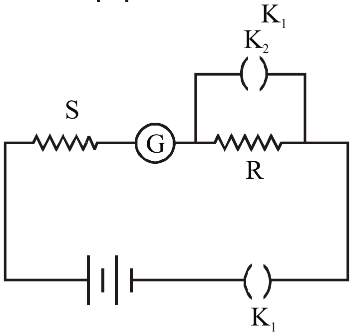
(A)



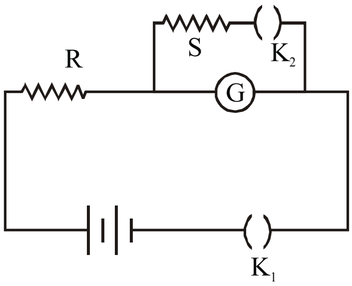
(B)



(C)



(D)



(1) Circuit C with G = S

(2) Circuit B with G = S

(3) Circuit A with $G = \frac{RS}{(R-S)}$

(4) Circuit D with $G = \frac{RS}{R-S}$

Ans. (4)

13. A tank with a small hole at the bottom has been filled with water and kerosene (specific gravity 0.8). The height of water is 3 m and that of kerosene 2 m. When the hole is opened the velocity of fluid coming out from it is nearly :

(take g = 10 ms^{-2} and density of water = 10^3 kg m^{-3})

(1) 7.6 ms^{-1} (2) 9.6 ms^{-1}

(3) 10.7 ms^{-1} (4) 8.5 ms^{-1}

Ans. (2)

Sol. $1000 \times 10 \times 3 + 800 \times 10 \times 2 = \frac{1}{2} \times 1000\, v^2$

$v = \sqrt{92}$

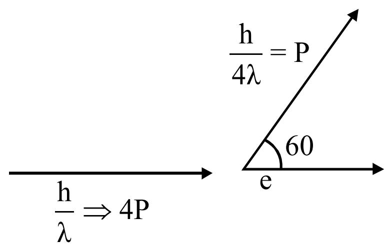
= 9.6 m/s

14. A photon of wavelength λ is scattered from an electron, which was at rest. The wavelength shift Δλ is three times of λ and the angle of scattering θ is 60°. The angle at which the electron recoiled is ϕ. The value of tan ϕ is : (electron speed is much smaller than the speed of light)

(1) 0.28 (2) 0.22 (3) 0.25 (4) 0.16

Ans. (3)

Sol.



$\Delta P = (P\cos 60\hat{i} + P\sin 60\hat{j}) - 4P\hat{i}$

$= -\frac{7P}{2}\hat{i} + \frac{\sqrt{3}P}{2}\hat{j} \Rightarrow \tan\phi = \frac{\sqrt{3}}{7}, \tan\phi = 0.25$

15. The Bulk moduli of Ethanol, Mercury and Water are given as 0.9, 25 and 2.2 respectively in units of 10^9 Nm^{-2}. For a given value of pressure, the fractional compression in volume is $\frac{\Delta V}{V}$. Which of the following statements about $\frac{\Delta V}{V}$ for these three liquids is correct? :-

(1) Water > Ethanol > Mercury

(2) Ethanol > Mercury > Water

(3) Ethanol > Water > Mercury

(4) Mercury > Ethanol > Water

Ans. (3)

16. A hot body, obeying Newton's law of cooling is cooling down drom its peak value 80ºC to an ambient temperature of 30ºC. It takes 5 minutes in cooling down from 80°C to 40°C. How much time will it take to cool down from 62°C to 32°C? (Given ln 2 = 0.693, ln5 = 1.609)

(1) 9.6 minutes (2) 6.5 minutes

(3) 8.6 minutes (4) 3.75 minutes

Ans. (2)

Sol. $\frac{d\theta}{dt} = -C(\theta - \theta_0)$

$$\int_{80}^{40} \frac{1}{\theta - \theta_0}\, d\theta = -C/5$$

$$\int_{62}^{32} \frac{1}{\theta - \theta_0}\, d\theta = -Ct$$

$$\ln\left(\frac{80-30}{40-30}\right) = 5C$$

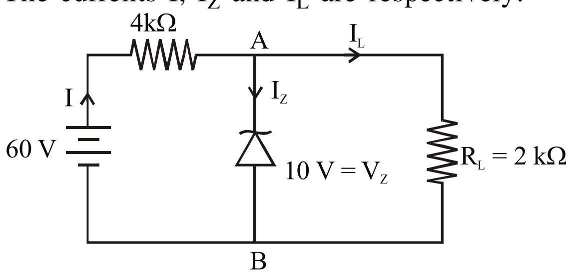
$$\ln\left(\frac{62-30}{40-50}\right) = Ct$$

$\ln 5 = 5c = 1.609$

$\ln 16 = ct = 4 \times 0.693$

$t = 8.6$ min

17. A Zener diode is connected to a battery and a load as shown below :-
The currents I, I_Z and I_L are respectively.



(1) 12.5 mA, 5 mA, 7.5 mA
(2) 15 mA, 7.5 mA, 7.5 mA
(3) 12.5 mA, 7.5 mA, 5 mA
(4) 15 mA, 5 mA, 10 mA

Ans. (3)

Sol. $I_L = \frac{10V}{2k\Omega} = 5mA$

$$I = \frac{(60-10)v}{4k\Omega} = \frac{50}{4k\Omega} = 12.5 \text{ mA}$$

$I_Z = I - I_L = (12.5 - 5)$ mA $= 7.5$ mA

18. An air bubble of radius 0.1 cm is in a liquid having surface tension 0.06 N/m and density 10^3 kg/m^3. The pressure inside the bubble is 1100 Nm^{-2} greater than the atmospheric pressure. At what depth is the bubble below the surface of the liquid? (g = 9.8 ms^{-2}) :-

(1) 0.1 m (2) 0.20 m
(3) 0.15 m (4) 0.25 m

Ans. (1)

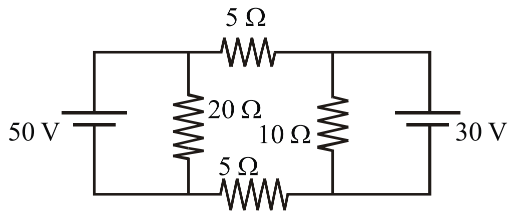
Sol. $h\,dg + \frac{LT}{r} = 1100$

$$h \times 10^3 \times 4.8 = 1100 - \frac{2\times0.06}{0.1\times10^{-2}}$$

$= 980$

$$h = \frac{980}{9.8\times10^3} = 0.1 \text{ m}$$

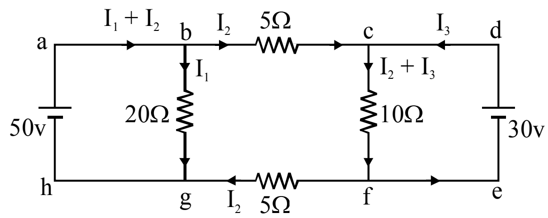
19. In the circuit shown, current (in A) through the 50 V and 30 V batteries are, respectively :-



(1) 3 and 2.5 (2) 3.5 and 2
(3) 4.5 and 1 (4) 2.5 and 3

Ans. (3)

Sol.



KVL in loop abgha

$20 I_1 = 50$

$I_1 = 2.5$ A

KVL in loop abcdefgha

$50 - 5I_2 - 30 - 5I_2 = 0$

$I_2 = 2A$

KVL in loop cdefc

$30 = 10(I_2 + I_3)$

$\Rightarrow I_2 + I_3 = 3$

$I_3 = 3 - 2$

$= 1A$

$\therefore$ Current through 50 V battery is $= I_1 + I_2$

$= 2.5 + 2.0$

$= 4.5$ A

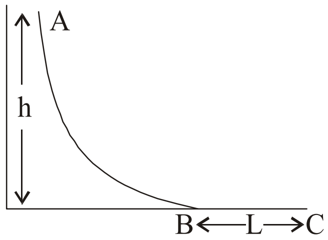
current through 30V battery $= I_3 = 1A$

20. During an adiabatic compression, 830 J of work is done on 2 moles of a diatomic ideal gas to reduce its volume by 50%. The change in its temperature is nearly : ($R = 8.3\ JK^{-1}mol^{-1}$)

(1) 40 K (2) 20 K (3) 33 K (4) 14 K

Ans. (2)

21. A small ball of mass m starts at a point A with speed v_0 and moves along a frictionless track AB as shown. The track BC has coefficient of friction μ. The ball comes to stop at C after travelling a distance L which is :



(1) $\frac{h}{2\mu}+\frac{v_0^2}{2\mu g}$ (2) $\frac{2h}{\mu}+\frac{v_0^2}{2\mu g}$

(3) $\frac{h}{\mu}+\frac{v_0^2}{2\mu g}$ (4) $\frac{h}{2\mu}+\frac{v_0^2}{\mu g}$

Ans. (3)

Sol. $mgh - \mu mgL = 0 - \frac{1}{2}mv_0^2$

$\mu mgL = mgh + \frac{1}{2}mv_0^2$

$L = \frac{h}{\mu}+\frac{v_0^2}{2\mu g}$

22. In a compound microscope the focal length of objective lens is 1.2 cm and focal length of eye piece is 3.0 cm. When object is kept at 1.25 cm in front of objective, final image is formed at infinity. Magnifying power of the compound microscope should be :

(1) 400 (2) 200 (3) 100 (4) 150

Ans. (2)

Sol. $mp = \frac{f_0}{f_0+40}\left(\frac{D}{f_e}\right)$

$= \frac{1.2}{1.2+(-1.25)}\left(\frac{25}{3}\right)$

$= 200$

23. A thin bar of length L has a mass per unit length λ, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_0, then the distance of the centre of mass from the lighter end is :

(1) $\frac{2L}{3}-\frac{\lambda_0 L^2}{6M}$

(2) $\frac{L}{2}-\frac{\lambda_0 L^2}{4M}$

(3) $\frac{L}{3}+\frac{\lambda_0 L^2}{4M}$

(4) $\frac{L}{3}+\frac{\lambda_0 L^2}{8M}$

Ans. (1)

Sol. Mass per unit lengh = $\lambda_0 + kx$

$M = \int_0^L (\lambda_0 + kx)\,dx$

$M = \lambda_0 L + \frac{K \times L^2}{2}$

$\frac{2M - \lambda_0 L}{L^2} = K$

$\frac{2M}{L^2} - \frac{\lambda_0}{L} = K$

$\frac{\int dm(r)}{\int dm} = \frac{\int (\lambda dn)x}{M} = \frac{\int_0^L (\lambda_0 x + kx^2)dx}{M}$

$r_{cm} = \frac{\lambda_0 L + \frac{kL^2}{2}}{M}$

substitute'k'

$r_{cm} = \frac{2L}{3} - \frac{\lambda_0 \ell^2}{6M}$

24. In terms of resistance R and time T, the dimensions of ratio $\frac{\mu}{\varepsilon}$ of the permeability μ and permittivity ε is :-

(1) $[R^2]$ (2) $[R^2T^2]$

(3) $[RT^{-2}]$ (4) $[R^2T^{-1}]$

Ans. (1)

25. The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the centre of a target 700 m away at the same level as the target. How far above the centre of the target the rifle must be aimed in order to hit the target

(1) 4.2 m (2) 6.1 m (3) 1.0 m (4) 9.8 m

Ans. (2)

26. An object is located in a fixed position in front of a screen. Sharp image is obtained on the screen for two positions of a thin lens separated by 10 cm. The size of the images in two situations are in the ratio 3 : 2. What is the distance between the screen and the object ?

(1) 99.0 cm

(2) 124.5 cm

(3) 144.5 cm

(4) 65.0 cm

Ans. (1)

Sol. $\frac{m_1}{m_2} = \frac{3}{2} = \left(\frac{D+10}{D-10}\right)^2$

D = 99 cm

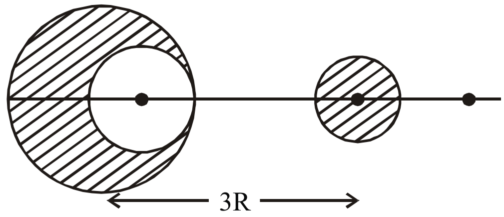
27. Two monochromatic light beams of intensity 16 and 9 units are interfering. The ratio of intensities of bright and dark parts of the resultant pattern is :

(1) $\frac{4}{3}$ (2) $\frac{49}{1}$ (3) $\frac{16}{9}$ (4) $\frac{7}{1}$

Ans. (2)

Sol. $\frac{I_{max}}{Z_{min}} = \frac{(\sqrt{16}+\sqrt{9})^2}{(\sqrt{16}-\sqrt{9})^2} = \frac{49}{1}$

28. From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery. (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two spheres is :

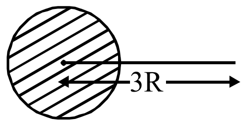


(1) $\frac{59\,GM^2}{450\,R^2}$ (2) $\frac{GM^2}{225\,R^2}$

(3) $\frac{41GM^2}{450\,R^2}$ (4) $\frac{41GM^2}{3600\,R^2}$

Ans. (4)

Sol.



Field at 3R is $\frac{GM}{9R^2}$ at P

Due to cavity field at P is $\frac{GM}{8(2.5R)^2} = \frac{GM}{50R^2}$

29. An electromagnetic wave of frequency 1×10^{14} hertz is propagating along z-axis. The amplitude of electric field is 4 V/m. If $\varepsilon_0 = 8.8 \times 10^{-12}$ $C^2/N\text{-}m^2$, then average energy density of electric field will be :

(1) 35.2×10^{-12} J/m³ (2) 35.2×10^{-13} J/m³

(3) 35.2×10^{-11} J/m³ (4) 35.2×10^{-10} J/m³

Ans. (1)

30. Two factories are sounding their sirens at 800 Hz. A man goes from one factory to other at a speed of 2 m/s. The velocity of sound is 320 m/s. Ihe number of beats heard by the person in one second will be :

(1) 8 (2) 4 (3) 10 (4) 2

Ans. (3)