

#1331657

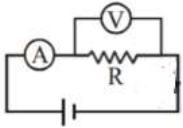
Two Forces P and Q of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the forces Q is doubled, then their resultant also gets doubled. Then, the angle is

- A 30°
- B 60°
- C 90°
- D 120°

Solution

$$\begin{aligned}F^2 + 9F^2 + 12F^2 \cos\theta &= R^2 \\4F^2 + 36F^2 + 24F^2 \cos\theta &= 4R^2 \\4F^2 + 36F^2 + 24F^2 \cos\theta & \\= 4(13F^2 + 12F^2 \cos\theta) &= 52F^2 + 48F^2 \cos\theta \\ \cos\theta &= \frac{12F^2}{24F^2} = -\frac{1}{2}\end{aligned}$$

#1331713



The actual value of resistance R, shown in the b figure is 30Ω . This is measured in an experiment as shown using the standard

Formula $R = \frac{v}{I}$, where V and I are the readings

of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is:

- A 350Ω
- B 570Ω
- C 35Ω
- D 600Ω

Solution

$$\begin{aligned}0.95R &= \frac{RR_u}{R + R_u} \\0.95 \times 30 &= 0.05R_u \\R_u &= 19 \times 30 = 570\Omega\end{aligned}$$

#1331771

An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128g containing 240 g of water a temperature of 8.4°C

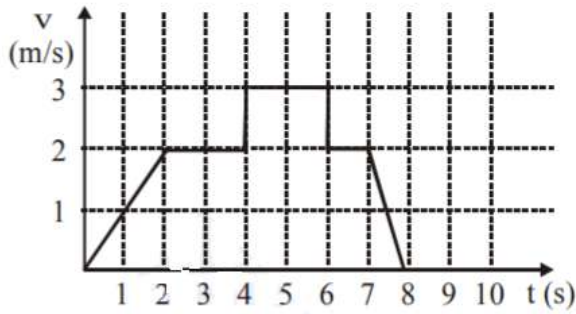
calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C (specific heat of brass is $394\text{J Kg}^{-1}\text{K}^{-1}$)

- A $1232\text{ J kg}^{-1}\text{K}^{-1}$
- B $458\text{ J kg}^{-1}\text{K}^{-1}$
- C $654\text{ J kg}^{-1}\text{K}^{-1}$
- D $916\text{ J kg}^{-1}\text{K}^{-1}$

Solution

$$\begin{aligned}
 &192 \times S \times (100 - 21.5) \\
 &= 128 \times 394 \times (21.5 - 8.4) \\
 &+ 240 \times 4200 \times (21.5 - 8.4) \\
 \implies S &= 916
 \end{aligned}$$

#1331807



A particle starts from the origin at time $t = 0$ and moves along the positive x - axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$?

- A 6 m
- B 9 m
- C 3 m
- D 10 m

Solution

$S = \text{Area under graph}$

$$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9m$$

#1331859

The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is :

- A 437.5 J
- B 637.5 J
- C 740 J
- D 540 J

Solution

$$L \frac{di}{dt} = 25$$

$$L \times \frac{15}{1} = 25$$

$$L = \frac{5}{3} H$$

$$\Delta E = \frac{1}{2} \times \frac{5}{3} (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5 J$$

#1331918

A current of 2mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :

- A $11 \times 10^{-5} W$
- B $11 \times 10^{-4} W$
- C $11 \times 10^5 W$

$$D \quad 11 \times 10^{-3} W$$

Solution

$$P = I^2 R$$

$$4.4 = 4 \times 10^{-6} R$$

$$R = 1.1 \times 10^6 \Omega$$

$$P^1 = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5} W$$

#1331986

The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

A $4260 + 80 \text{ cm}^3$

B $4300 + 80 \text{ cm}^3$

C $4264 + 81.0 \text{ cm}^3$

D $4264 + 81 \text{ cm}^3$

Solution

Thus the correct ans is option A which is $4260 + 80$.

$$V = \pi \frac{d^2}{4} h = 4260 \text{ cm}^3$$

$$\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h}$$

$$\Delta V = 2 \times \frac{0.1V}{12.6} + \frac{0.1V}{34.2}$$

$$= \frac{0.2}{12.6} \times 4260 + \frac{0.1 \times 4260}{34.2} = 80$$

#1332063

At some location on earth the horizontal components of earth's magnetic field is $18 \times 10^{-6} T$. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium to keep this needle horizontal, the vertical force that should be applied at one of its ends is:

A $3.6 \times 10^{-5} N$

B $6.5 \times 10^{-5} N$

C $1.3 \times 10^{-5} N$

D $1.8 \times 10^{-5} N$

Solution

At 45° , $B_H = B_V$

$$F \frac{l}{2} = MB_V = m \times l \times B_V$$

$$F = \frac{2mlB_V}{l} = 3.6 \times 18 \times 10^{-6}$$

$$= 6.5 \times 10^{-5} N$$

#1332090

The modulation frequency of an AM radio station is 250 KHz, which is 10% of the carrier wave. If another AM station approaches you for licence what broadcast frequency will you allot?

- A 2750 KHz
 B 2000 KHz
 C 2250 KHz
 D 2900 KHz

Solution

$$f_{\text{carrier}} = \frac{250}{0.1} = 2500 \text{ KHz}$$

Range of signal = 2250 Hz to 2750 Hz Now check all option : for 2000 KHz

$$f_{\text{mod}} = 200 \text{ Hz}$$

Range = 1800 KHz to 2200 KHz

#1332163

A hoop and a solid cylinder of same cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. they are placed in a uniform magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then:

- A $T_h = 0.5T_c$
 B $T_h = 2T_c$
 C $T_h = 1.5T_c$
 D $T_h = T_c$

Solution

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

$$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$$

$$T_c = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$$

#1332416

The electric field of a plane polarized electromagnetic Wave in free space at time $t = 0$ is given by an expression

$$\vec{E}(x, y) = 10\hat{j}\cos[(6x + 8z)]$$

The magnetic field $\vec{B}(x, z, t)$ is given by ; (c is the velocity of light will be:

- A $\frac{1}{c}(4\hat{k} + 8\hat{i})\cos[(2x - 8z + 20ct)]$
 B $\frac{1}{c}(6\hat{k} - 8\hat{i})\cos[(6x + 8z - 10ct)]$
 C $\frac{1}{c}(5\hat{k} + 8\hat{i})\cos[(6x + 8z - 80ct)]$
 D $\frac{1}{c}(4\hat{k} - 8\hat{i})\cos[(6x + 8z + 70ct)]$

Solution

$$\vec{E} = 10\hat{j}\cos[(6\hat{i} + 8\hat{k}) \cdot (x\hat{i} + z\hat{k})]$$

$$= 10\hat{j}\cos[\vec{K} \cdot \vec{r}]$$

$\vec{K} = 6\hat{i} + 8\hat{k}$; direction of waves travel. i.e direction of 'c'

Direction of \vec{B} will be along

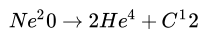
$$\hat{C} \times \vec{E} = \frac{-4\hat{i} + 3\hat{k}}{5}$$

$$\text{Mag. of } \vec{B} = \frac{E}{C} = \frac{10}{C}$$

$$\therefore \vec{B} = \frac{10}{C} \left(\frac{-4\hat{i} + 3\hat{k}}{5} \right) = \frac{(-8\hat{i} + 6\hat{k})}{c}$$

#1332462

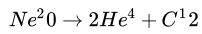
Consider the nuclear fission



Given that the binding energy/nucleon of ${}^{235}\text{Ne}$, ${}^4\text{He}$ and ${}^{12}\text{C}$ are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement:

- A 8.3 MeV energy will be released
- B energy of 12.4 MeV will be supplied
- C energy of 11.9 MeV has to be supplied
- D energy of 3.6 MeV will be released

Solution



$$8.03 \times 202 \times 7.07 \times 4 + 7.86 \times 12$$

$$E_B = (BE)_{\text{react}} - (BE)_{\text{product}} = 9.72 \text{ MeV}$$

#1332552

Two vectors \vec{A} and \vec{B} have equal magnitude. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between \vec{A} and \vec{B} is

- A $\sin^{-1} \left[\frac{n^2 - 1}{n^2 + 1} \right]$
- B $\cos^{-1} \left[\frac{n - 1}{n + 1} \right]$
- C $\cos^{-1} \left[\frac{n^2 - 1}{n^2 + 1} \right]$
- D $\sin^{-1} \left[\frac{n - 1}{n + 1} \right]$

Solution

Handwritten solution for problem #1332552:

$$\begin{aligned} \rightarrow A^2 + A^2 + 2AB \cos \theta &= n^2 (A^2 + A^2 - 2AB \cos \theta) \\ \rightarrow A^2 + A^2 + 2A^2 \cos \theta &= n^2 (A^2 + A^2 - 2A^2 \cos \theta) \\ \rightarrow 2A^2 (1 + \cos \theta) &= 2A^2 n^2 (1 - \cos \theta) \\ \rightarrow 1 + \cos \theta &= n^2 - n^2 \cos \theta \\ \rightarrow \cos \theta (1 + n^2) &= n^2 - 1 \\ \rightarrow \cos \theta &= \frac{n^2 - 1}{n^2 + 1} \\ \rightarrow \theta &= \cos^{-1} \left(\frac{n^2 - 1}{n^2 + 1} \right) \end{aligned}$$

#1332628

A particle executes simple harmonic motion with an amplitude of 5 cm. when the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. then, its periodic time in second is:

- A $\frac{7}{3} \pi$

B $\frac{3}{8}\pi$

C $\frac{4\pi}{3}$

D $\frac{8\pi}{3}$

Solution

$$v = \omega\sqrt{A^2 - x^2} \text{ _____(1)}$$

$$a = -\omega^2 x \text{ _____(2)}$$

$$|v| = |a| \text{ _____(3)}$$

$$\omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$A^2 - x^2 = \omega^2 x^2$$

$$5^2 - 4^2 = \omega^2(4^2)$$

$$\Rightarrow 3 = \omega \times 4$$

Since, $T = 2\pi/\omega$

$$T = 8\pi/3\omega$$

$$v = \omega \sqrt{5^2 - 4^2} = 3\omega$$

$$a = \omega^2 \times 4$$

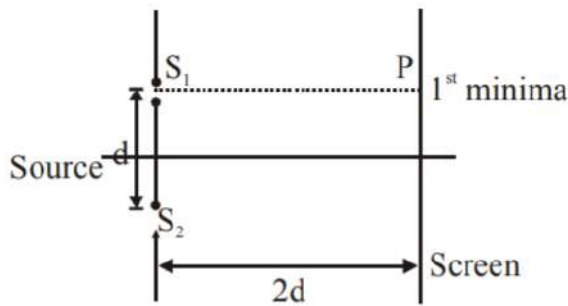
$$|a| = |v|$$

$$4\omega^2 = 3\omega$$

$$\omega = \frac{3}{4} = \frac{2\pi}{T}$$

$$T = \frac{8\pi}{3} \text{ Sec.}$$

#1332680



Consider a young's double slit experiment as shown in figure. What should be the slit separation d in term of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?

A $\frac{\lambda}{2(5 - \sqrt{2})}$

B $\frac{\lambda}{(5 - \sqrt{2})}$

C $\frac{\lambda}{(\sqrt{5} - 2)}$

D $\frac{\lambda}{2(\sqrt{5} - 2)}$

Solution

$$x_1 = 2d$$

$$x_2 = \sqrt{5}d$$

$$\Delta x = x_2 - x_1$$

$$\sqrt{5}d - 2d = \frac{\lambda}{2}$$

$$d = \frac{\lambda}{2(\sqrt{5} - 2)}$$

#1333752

The eye can be regarded as a single refracting surface. The radius of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

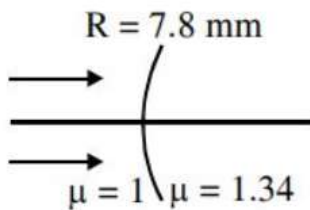
- A 2 cm
B 1 cm
 C 3.1 cm
D 4.0 cm

Solution

$$\frac{1.34}{V} - \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

$$\frac{1.34}{v} = \frac{0.34}{7.8}$$
$$v = \frac{1.34 \times 780}{34}$$

$$\therefore V = 30.7 \text{ mm}$$



#1333764

Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C . Work done by gas close to : (Gas constant $R = 8.31 \text{ J / mol. K}$)

- A 73 J
 B 291 J
C 581 J
D 146 J

Solution

$$\text{Work Done} = P\Delta V = nR\Delta T = 291 \text{ J}$$

#1333776

A metal plate of area $1 \times 10^{-4} \text{ m}^2$ is illuminated by a radiation of intensity 16 mW / m^2 . The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be :

$$[1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

- A 10^{10} and 5 eV
 B 10^{12} and 5 eV
C 10^{14} and 10 eV

D 10^{11} and 5 eV

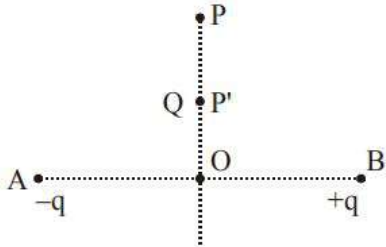
Solution

Maximum Kinetic Energy K . $E_{max} = E - \phi = (10 - 5)eV = 5eV$

$$I = \frac{nE}{At}$$

$$16 \times 10^{-3} = \left(\frac{n}{t}\right)_{Photon} \frac{10 \times 1.6 \times 10^{-19}}{10^{-4}} = 10^{12}$$

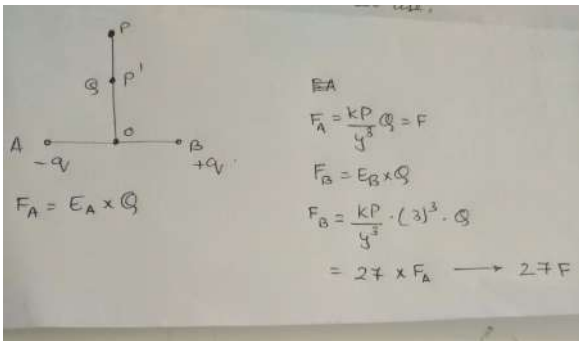
#1333791



Charge $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where $OP = y$ and $y \gg 2a$. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$ the force on Q will be close to L : $\left(\frac{y}{3} \gg 2a\right)$

- A $\frac{F}{3}$
- B $3F$
- C $9F$
- D $27F$

Solution



#1333803

Two stars of masses $3 \times 10^{31}kg$ each, and at distance $2 \times 10^{11}m$ rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (take Gravitational constant $G = 6.67 \times 10^{11}Nm^2kg^{-2}$)

- A $1.4 \times 10^5 m/s$
- B $24 \times 10^4 m/s$
- C $3.8 \times 10^4 m/s$
- D $2.8 \times 10^5 m/s$

Solution

By energy conservation between 0 & ∞

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteorite]

$$\Rightarrow v = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 \text{ m/s}$$

#1333903

A closed organ pipe has a fundamental frequency of 1.5 KHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

- A 7
- B 5
- C 6
- D 4

Solution

For closed organ pipe, resonant frequency is odd multiple of fundamental frequency.

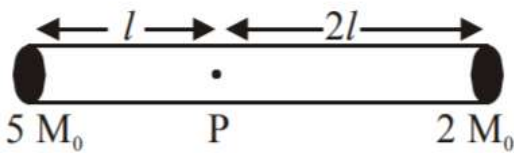
$$(2n + 1) f_0 \leq 20,000$$

(f_0 is fundamental frequency = 1.5 KHz)

$$n = 6$$

Total number of overtone that can be heard is 7. (0 to 6)

#1333983



A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal position, its instantaneous angular acceleration will be:

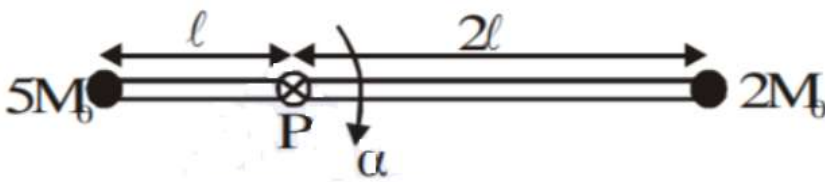
- A $\frac{g}{2l}$
- B $\frac{7g}{3l}$
- C $\frac{g}{13l}$
- D $\frac{g}{3l}$

Solution

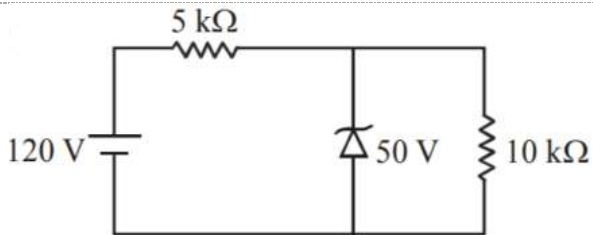
Applying torque equation about point P. $2m_o(2l) - 5M_o l^2 = 13M_o l^2 d$

$$\alpha = \frac{M_o g l}{13M_o \times l^2} \Rightarrow \alpha = -\frac{g}{13 \times l}$$

$$\alpha = \frac{g}{13l} \text{ anticlockwise}$$



#1333998



For the circuit show below, the current through the zener diode is :

- A 5 mA
- B Zero
- C 14 mA
- D 9 mA

Solution

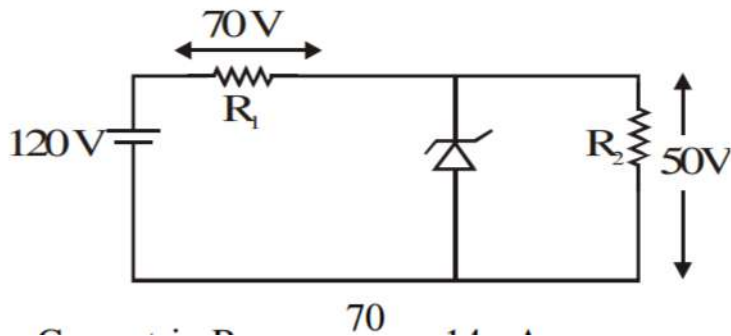
Assuming zener diode does not undergo breakdown, current in circuit = $\frac{120}{15000} = 8mA$

Voltage drop across diode = 80 V > 50 V. The diode undergo breakdown.

Current is $R_1 = \frac{70}{5000} = 14mA$

Current is $R_2 = \frac{50}{10000} = 5A$

current through diode = 9 mA



#1334049

For equal point charges Q each are placed in the xy plane at (0,2), (4,2),(4,-2) and (0,-2). The work required to put a fifth Q at the origin of the coordinate system will be:

- A $\frac{Q^2}{2\sqrt{2}\pi\epsilon_0}$
- B $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{3}}\right)$
- C $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{3}}\right)$
- D $\frac{Q^2}{4\pi\epsilon_0}$

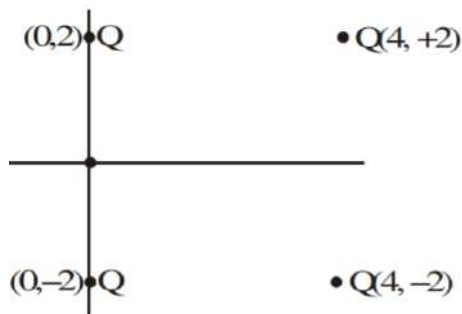
Solution

$$\text{potential at origin} = \frac{kQ}{2} + \frac{kQ}{2} + \frac{kQ}{\sqrt{20}} + \frac{kQ}{\sqrt{20}}$$

(potential at $\infty = 0$)

$$= kQ \left(1 + \frac{1}{\sqrt{5}} \right)$$

work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\epsilon_0 \left(1 + \frac{1}{\sqrt{5}} \right)}$



#1334110

A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω . If the radius of the bottle is 2.5 cm, then ω close to (density of water = 10^3 kg/m^3)

- A 5.00 rads^{-1}
- B 1.25 rads^{-1}
- C 3.75 rads^{-1}
- D None of the above

Solution

Extra Boyant force = $\delta A x g$

$$B_o + B \times mg = ma$$

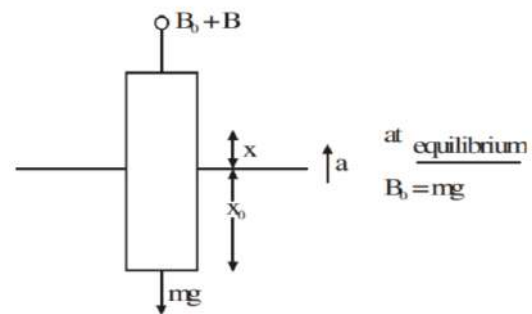
$$B = ma$$

$$a = \left(\frac{\delta A g}{m} \right)$$

$$\omega^2 = \frac{\delta A g}{m}$$

$$\omega = \sqrt{\frac{10^3 \times \pi (2.5)^2 \times 10^{-4} \times 10}{310 \times 10^{-6} \times 10^3}}$$

$$\sqrt{63.30} = 7.95 \text{ rads}^{-1}$$



#1334196

A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

A 692 pJ

B 60 pJ

C 508 pJ

D 560 pJ

Solution

Initial energy of capacitor

$$U_i = \frac{1}{2} \frac{C^2 V^2}{C}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12} = 600 \text{ pJ}$$

Since battery is disconnected so charge remain same.

Final energy of capacitor

$$U_f = \frac{1}{2} \frac{C^2 V^2}{K C}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12 \times 6.5} = 92$$

$$W + U_f = U_i$$

$$W = 508 \text{ J}$$

#1334204

Two kg of a monoatomic gas is at a pressure of $4 \times 10^4 \text{ N/m}^2$. The density of the gas is 8 kg/m^3 . What is the order of energy of the gas due to its thermal motion ?

A 10^3 J

B 10^5 J

C 10^6 J

D 10^4 J

Solution

Thermal energy of N molecule

$$= N \left(\frac{3}{2} kT \right)$$

$$= \frac{N}{N_A} \frac{3}{2} RT$$

$$= \frac{3}{2} (nRT)$$

$$= \frac{3}{2} PV$$

$$= \frac{3}{2} P \left(\frac{m}{\rho} \right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8}$$

$$= 1.5 \times 10^4$$

order will 10^4 .

#1334209

A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

- A 15 J
- B 10 J
- C 12 J
- D 9 J

Solution

$$\begin{aligned} \text{Work done} &= \vec{F} \cdot \vec{d} \\ &= 12J \end{aligned}$$

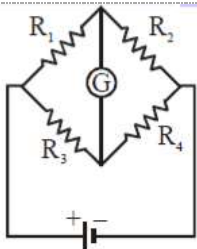
work energy theorem

$$w_{net} = \Delta K = E_f - E_i$$

$$12 = K_f - 3$$

$$K_f = 15J$$

#1334213



The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R1 has the colour code (Orange, Red, Brown). The resistors R2 and R4 are 80 and 40 respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R3, would be

- A Red, Green, Brown
- B Brown, Blue, Brown
- C Grey, Black, Brown
- D Brown, Blue, Black

Solution

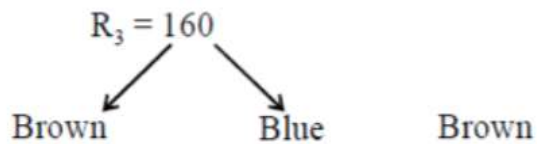
$$R_1 = 3210 = 320$$

for wheat stone bridge

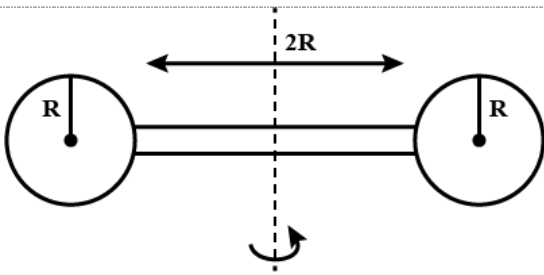
$$\implies \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$$R_3 = 160$$



#1334214



Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:

- A $\frac{152}{15}MR^2$
 B $\frac{17}{15}MR^2$
 C $\frac{137}{15}MR^2$
 D $\frac{209}{15}MR^2$

Solution

For Ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2$$

$$= \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

Irod = for rod $\frac{M(2R)^2}{12} = \frac{MR^2}{3}$

$$I_{system} = I_{Ball} + I_{rod}$$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15}MR^2$$

#1329282

An ideal gas undergoes isothermal compression from $5m^3$ against a constant external pressure of $4Nm^{-2}$. Heat released in this process is used to increase the temperature of $1mole$ of $A1$. If molar heat capacity of $A1$ is $24Jmol^{-1}k^{-1}$, the temperature of $A1$ increased by:

A $\frac{3}{2}K$

B $\frac{2}{3}K$

C $1K$

D $2K$

Solution

Work Done on isothermal irreversible for ideal gas

$$= -P_{ext}(V_2 - V_1)$$

$$= 16 Nm$$

For Isothermal process, $\Delta U = 0$

$$q = -16 J$$

Heat used to increase temperature

$$q = nC_m\Delta T$$

Substituting the Values, we get

$$\Delta T = \frac{2}{3}K$$

#1329440

The 71^{st} electron of an element X with an atomic number of 71 enters into the orbital:

A $4f$

B $6p$

C $6s$

D $5d$

Solution

Electronic Configuration of Element X with atomic number 71 is $[Xe]4f^{14}5d^16s^2$.

The Last electron is in $4f$ orbital

#1329466

The number of $2 - centre - 2 - electron$ and $3 - centre - 2 - electron$ bonds in B_2H_6 respectively,

A 2 and 4

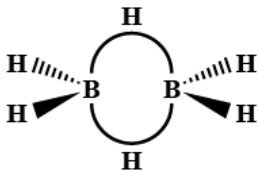
B 2 and 1

C 2 and 2

D 4 and 2

Solution

According to structure, There are 4 2 – centre – 2 – electron bonds and 2 3 – centre – 2 – electron bonds in B_2H_6 .



#1329494

The amount of sugar ($C_{12}H_{22}O_{11}$) required to prepare 2L of its 0.1M aqueous solution is:

A 68.4g

B 17.1g

C 34.2g

D 136.8g

Solution

$$\text{Molarity} = \frac{(n)_{\text{solute}}}{V_{\text{solute}}(\text{in lit})}$$

$$0.1 = \frac{\text{wt}/342}{2}$$

$$\text{wt}(C_{12}H_{22}O_{11}) = 68.4\text{gram}$$

#1329734

Among the following reactions of hydrogen with halogens, the one that requires a catalyst is:

A $H_2 + I_2 \rightarrow 2HI$

B $H_2 + F_2 \rightarrow 2HF$

C $H_2 + Cl_2 \rightarrow 2HCl$

D $H_2 + Br_2 \rightarrow 2HBr$

Solution

The Reaction $H_2 + I_2 \rightarrow 2HI$ is carried out in the presence of Pt Catalyst

So Option A is correct

#1329750

Sodium metal on dissolution in liquid ammonia gives a deep blue solution due to the formation of:

A sodium ion-ammonia complex

B sodamide

C sodium-ammonia complex

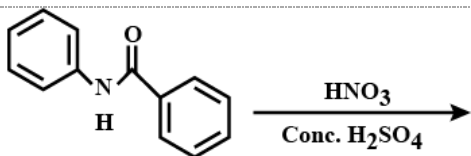
D ammoniated electrons

Solution

The solvated electron is responsible for a great deal of radiation chemistry. Alkali metals dissolve in liquid ammonia giving deep blue solutions which are conducting in nature. The blue colour of the solution is due to ammoniated electrons which absorb energy in the visible region of light

Option D is correct

#1329799



What will be the major product in the following mononitration reaction?

- A
- B
- C
- D

Solution

- The reagent used is a classical reagent for the generation of an Electrophile NO_2^+ . So, it attacks anyone Benzene ring to form Electrophilic substitution reaction.
- The right Benzene ring is deactivated by $\text{C} = \text{O}$ group and hence Electrophile doesn't attack right Benzene ring.
- The left Benzene ring has been activated by Nitrogen lone pair. And hence the Electrophile attacks on a left Benzene ring.
- Now due to steric hindrance, the substitution doesn't take place on ortho position and thus the substitution takes place on para position w.r.t. $-\text{NH}$ group.
- Hence option C shows the appropriate answer.

#1329848

In the cell $\text{Pt}(s), \text{H}_2(g) | 1\text{bar HCl}(aq) | \text{Ag}(s) | \text{Pt}(s)$ the cell potential is 0.92 when 10^{-6} molal HCl solution is used. The standard electrode potential of $(\text{AgCl}/\text{Ag}, \text{Cl}^-)$ electrode is:

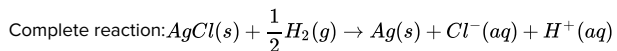
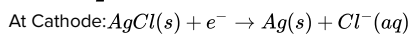
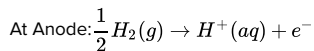
Given: $\frac{2.303RT}{F} = 0.06V$ at $298K$

- A 0.20V
- B 0.076V
- C 0.040V

D 0.94V

Solution

The half-cell reactions are,



We know,

$$E_{cell}^0 = E_{cathode}^0 - E_{anode}^0 = (SRP)_{cathode} - (SRP)_{anode}$$

We know standard hydrogen potential is assumed to be zero.

$$\text{So, } (SRP)_{anode} = 0$$

$$\text{Let, } (SRP)_{cathode} = x$$

So,

$$E_{cell}^0 = x$$

Now we use Nernst equation,

$$E_{cell} = E_{cell}^0 - \frac{2.303RT}{nF} \log(Q)$$

$$\Rightarrow E_{cell} = E_{cell}^0 - 0.06 \times \log([Cl^-][H^+])$$

n=1;

$$0.92 = x - \frac{0.06}{1} \log(10^{-6} \times 10^{-6})$$

$$\Rightarrow x = 0.20V$$

#1329889



The major product of the following reaction is:

- A
- B
- C
- D

Solution

$NaBH_4$ reduces the keto - group to enol - group and it can't reduce the double bonds.

So Option C is correct

#1329948

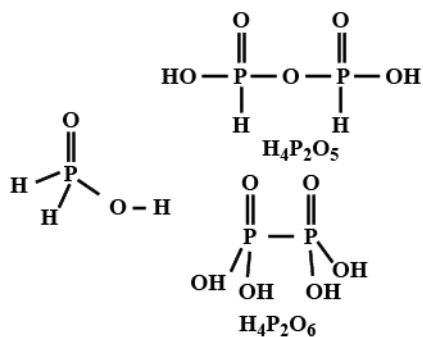
The pair that contains two $P - H$ bonds in each of the oxoacids is:

- A H_3PO_2 and $H_4P_2O_5$
- B $H_4P_2O_5$ and $H_4P_2O_6$
- C H_3PO_3 and H_3PO_2
- D $H_4P_2O_5$ and H_3PO_3

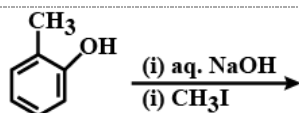
Solution

So H_3PO_2 and $H_4P_2O_5$ contain 2 $P-H$ bonds

Option A is correct



#1329991



The major product of the following reactions is:

- A
- B
- C
- D

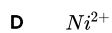
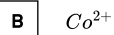
Solution

- We know $NaOH(aq)$ will abstract acidic proton. In the current reaction, it abstracts Phenolic proton. Thus $-OH$ on the ring converts to $-O^-$.
- In the second step, it is reacted with CH_3I which is a classic SN^2 reaction. Thus the $-O^-$ present will form $-OCH_3$.
- Hence option D is a correct answer.

#1330011

The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is:

- A Fe^{2+}



Solution

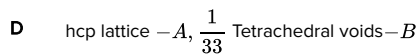
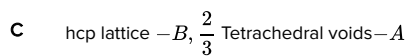
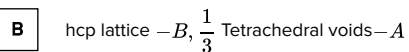
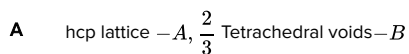
The Difference in number of unpaired electrons of Metal ion in its high-spin and low-spin octahedral complexes is 2.

For the Metal Co^{+2} , the difference of unpaired electrons is $3 - 1 = 2$.

Option B is correct

#1330073

A compound of formula A_2B_3 has the hcp lattice. Which atom forms the hcp lattice and what fraction of tetrahedral voids is occupied by the other atoms?



Solution

A_2B_3 has HCP lattice

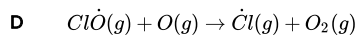
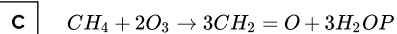
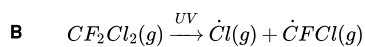
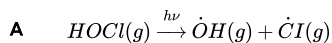
If A form HCP, then 34 of THV must occupied by B to form A_2B_3

If B form HCP, then 13 of THV must occupied by A to form A_2B_3

I

#1330252

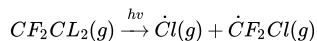
The reaction that is not involved in the ozone layer depletion mechanism is the stratosphere is:



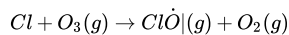
Solution

(1) The upper stratosphere consists of considerable amount of ozone (O_3) which protects us from the harmful UV radiations ($\lambda = 255nm$) coming from the sun. The main reason for depletion is CFCs.

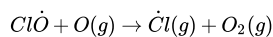
(2) When released in the atmosphere, CFCs mix with the normal atmospheric gases and eventually reach the stratosphere. In stratosphere, they get broken down by powerful UV radiations, releasing chlorine free radical.



(3) The chlorine free radical ($\dot{C}l$) then reacts with stratospheric ozone to form chlorine monoxide radicals ($Cl\dot{O}$) and molecular O_2



Reaction of $Cl\dot{O}$ with atomic oxygen produces more $\dot{C}l$ radicals.



So, Reaction of Methane with Ozone doesn't happen

Option C is correct

#1330312

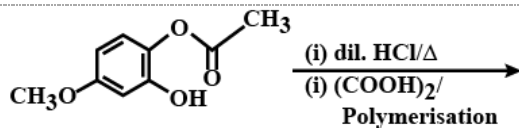
The process with negative entropy change is:

- A dissolution of iodine in water.
- B synthesis of ammonia from N_2 and H_2
- C dissolution of $CaSO_4(s)$ to $CaO(s)$ and $SO_3(g)$
- D sublimation of dry ice

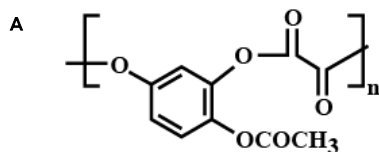
Solution

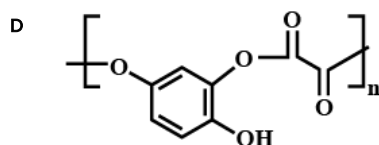
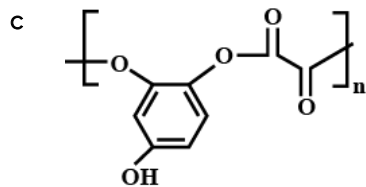
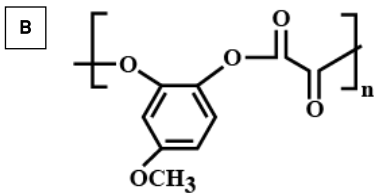
- In option A a solid dissolves to form an aqueous solution and hence entropy increases.
- In option B four moles of gas ($N_2 + 3H_2 \rightarrow 2NH_3$) react to give only two moles of gas and hence entropy decreases.
- In option C a solid is converting into gas and thus entropy increases.
- In option D the reaction is that a solid is converting into gas and thus entropy increases.
- So, option B is the correct answer.

#1330330



The major product of the following reaction is:





#1330366

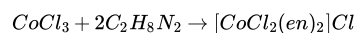
A reaction of cobalt (III) chloride and ethylenediamine in a 1:2 mole ratio generates two isomeric product A (violet coloured) B (green coloured). A can show optical activity, B is optically inactive. What type of isomers does A and B represent?

- A** Geometrical isomers
B Ionisation isomers
C Coordination isomers
D Linkage isomers

Solution

We know Ethylenediamine is a bidentate ligand and Co^{3+} forms an octahedral complex having co-ordination number 6. Here, 2 moles of ethylene diamine can satisfy four co-ordination number. Then the remaining two would be satisfied by existing Chloride ions.

The reaction is,

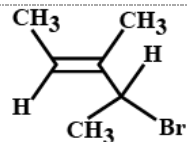


According to a given ratio, the above product can only be formed. As it says there are two products so another product should be an isomer.

Now the possibility is two Cl ions can be either in cis form or in trans-form. And on seeing this in cis form there is no plane of symmetry and hence it is chiral and optically active and the trans will be optically inactive.

Hence they are Geometrical isomers of each other.

#1330425



What is the IUPAC name of the following compound?

- A** 3-Bromo-1,2-dimethylbut-1-ene
B 2-Bromo-3-methylpent-2-ene
C 2-Bromo-3-dimethylpent-3-ene
D 3-Bromo-3-dimethylbut-1, 2-dimethylprop-1-ene

Solution

The Main Chain contains 5 Carbons. Bromine is attached to the 2nd Carbon atom. A Double bond is there between 3rd and the 4th carbon atom. A Methyl group is also attached to the 3rd Carbon atom.

So, the IUPAC name is 2-Bromo-3-methylpent-2-ene

#1330470

Which of the following tests cannot be used for identifying amino acids?

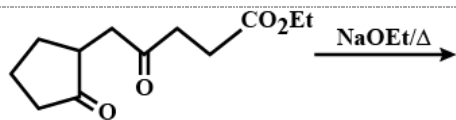
- A Biuret test
- B Xanthoproteic test
- C Barfoed test
- D Ninhydrin test

Solution

Barfoed's test is a chemical test used for detecting the presence of monosaccharides. It is based on the reduction of Copper (II) acetate to Copper (I) oxide, which forms a brick-red precipitate

Biuret test, Xanthoproteic test, Ninhydrin test are used for identifying Amino Acids

#1330485



The major product obtained in the following reaction is:

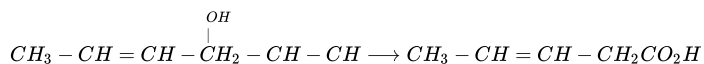
- A
- B
- C
- D

Solution

- We know α -Hydrogen with respect to Carbonyl groups are acidic. In the given reactant there are four α positions.
- $NaOEt$ is a base and in presence of a base, the α hydrogens can be abstracted.
- On looking carefully at the options we can make out that another cyclic compound is getting formed.
- If the α -Hydrogen to the right of outside Carbonyl group is removed there is a possibility of intramolecular cyclisation.
- Hence after the Hydrogen is removed there arises a negative charge on that carbon. That negative charge makes a five-membered ring leaving the charge on the attacked $=O$ as $-O^-$
- Now we can see the ethoxide ion which has abstracted α -Hydrogen forms ethanol and will be present in the solution and a proton can be abstracted from ethanol and $-O^-$ changes to $-OH$.
- Now again hydrogen besides CO_2Et group is removed by $-OEt^-$ ion present. And the negatively charged carbon forms. Now the OH is thrown out.
- Thus option D will be formed as a product.

#1330515

Which is the most suitable reagent for the following transformation?



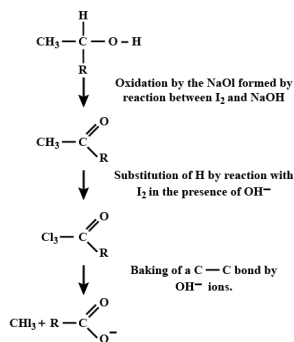
- A alkaline $KMnO_4$
- B $I_2/NaOH$
- C Tollen's reagent
- D CrO_2/CS_2

Solution

Here R is $CH_3 - CH = CH - CH_2$

So, When the reactant gets treated with $I_2/NaOH$, it gives the given product.

So Option B is correct



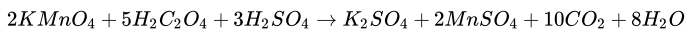
#1330661

In the reaction of oxalate with permanganate in acidic medium, the number of electrons involved in producing one molecule of CO_2 is:

- A 10
- B 2
- C 1
- D 5

Solution

The Reaction of Oxalate with permanganate in Acidic medium is



The Number of electrons involved in producing one molecule of CO_2 is $\frac{2+3+5}{10} = 1$

#1330820

5.1g NH_4SH is introduced in 3.0 L evacuated flask at $327^{\circ}C$. 30% of the solid NH_4SH decomposed to NH_3 and H_2S as gases. The K_p of the reaction at $327^{\circ}C$. is ($R = 0.082 \text{ L atm mol}^{-1}$, molar mass of $S = 32 \text{ gmol}^{-1}$, molar mass of $N = 14 \text{ gmol}^{-1}$).

- A $1 \times 10^{-4} \text{ atm}^2$
B $4.9 \times 10^{-3} \text{ atm}^2$
 C 0.242 atm^2
D $0.242 \times 10^{-4} \text{ atm}^2$

Solution



$$n = \frac{5.1}{51} = .1 \text{ mole}$$

0		
.1(-1 - α)	.1 α	.1 α

$$\alpha = 30\% = .3$$

So number of moles equilibrium

$$.1(1 - .3).1 \times .3 \times .3 = .07 \quad =.03 \quad =.03$$

Now use $PV = nRT$ at equilibrium

$$P_{total} \times 3 \text{ lit} = (.03 + .03) \times .082 \times 600$$

$$P_{total} = .984 \text{ atm}$$

At equilibrium

$$P_{NH_3} = P_{H_2S} = \frac{P_{total}}{2} = .492$$

$$\text{So } k_p = P_{NH_3} \cdot P_{H_2S} = (.492)(.492)$$

$$k_p = .242 \text{ atm}^2$$

#1330881

The electrolytes usually used in the electroplating of gold and silver, respectively are:

- A $[Au(OH)_4]^-$ and $[Ag(OH)_2]^-$
B $[Au(CN)_2]^-$ and $[AgCl_2]^-$
C $[Au(NH_3)_2]^+$ and $[Ag(CN)_2]^-$
 D $[Au(CN)_2]^-$ and $[Ag(CN)_2]^-$

Solution

The Anode is a bar of silver metal, and the electrolyte (the liquid in between the electrodes) is a solution of Silver cyanide, $[Ag(CN)_2]^-$, in water.

Gold plating is done in much the same way, using a gold anode and an electrolyte containing Gold cyanide, $[Au(CN)_2]^-$.

Option D is correct

#1331101

Elevation in the boiling point for 1 molal solution of glucose is 2K. The depression in the freezing point of 2 molal solutions of glucose in the same solvent is 2K. The relation between K_b and K_f is

A $K_b = 0.5K_f$

B $K_b = 2K_f$

C $K_b = 1.5K_f$

D $K_b = K_f$

Solution

$$\frac{\Delta T_b}{\Delta T_f} = \frac{i \cdot m \times k_b}{i \times m \times k_f}$$
$$\frac{2}{2} = \frac{1 \times 1 \times k_b}{1 \times 2 \times k_f}$$

$$K_b = 2K_f$$

#1331102

For an elementary chemical reactions,

$A_2 \xrightleftharpoons[k_{-1}]{k_1} 2A$, the expression for $\frac{d[A]}{dt}$ is:

A $2k_1[A_2] - k_{-1}[A]^2$

B $k_1[A_2] - k_{-1}[A]^2$

C $k_1[A_2] - 2k_{-1}[A]^2$

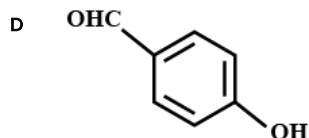
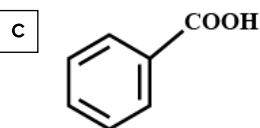
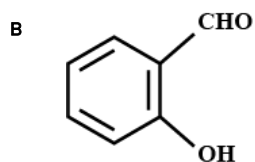
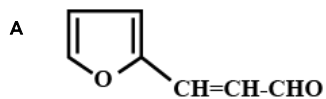
D $k_1[A_2] + k_{-1}[A]^2$

Solution

$$A_2 \xrightleftharpoons[k_{-1}]{k_1} 2A$$
$$\frac{d[A]}{dt} = 2k_1[A_2] - 2k_{-1}[A]^2$$

#1331163

An aromatic compound 'A' having molecular formula $C_7H_6O_2$ on treating with aqueous ammonia heating forms compound 'B'. The compound 'B' on reaction with molecular bromine and potassium hydroxide provides compound 'C'; having molecular formula C_6H_7N . The structure of 'A' is:



Solution

First we find DU of each compound.

$$DU(C_7H_6O_2) = 5$$

$$DU(C_6H_7N) = 4$$

When $C_7H_6O_2$ reacts with aqueous ammonia and heated it forms C_7H_7ON . Now, $DU(C_7H_7ON) = 5$

The reagent $Br_2/NaOH$ is classic Hoffman Bromamide reagent and this gives a clue that B may be an Amide. This consumes one DU and by seeing remaining 6 carbon and 5 Hydrogens we can say B is Benzamide.

So Benzamide on Hoffman Degradation gives Aniline which matches with the formula of C.

We have to know usually $NH_3 + heat$ gives an Amide when a carboxylic acid is used as a reagent.

So among the options, we can see option C which is a Benzoic acid matches the formula of X and hence is the correct answer.

#1331239

The ground state energy of hydrogen atom is -13.6 eV. The energy of second excited state He^+ ion in eV is:

A -6.04

B -27.2

C -54.4

D -3.4

Solution

$$(E)_n^{th} = (E_{GND})_H \frac{Z^2}{n^2}$$

$$E_3^{rd}(He^+) = (-13.6eV) \cdot \frac{2^2}{3^2} = -6.04eV$$

#1331281

Haemoglobin and gold sol are example of:

A negatively charged sols

B positively charged sols

C negatively and positively charged sols, respectively

D positively and negatively charged sols, respectively

Solution

Hemoglobin is a positively charged sol because the reason for coagulation to not occur is Herapin.

Gold sol is negatively charged sol.

Option D is correct

#1331397

Item 'I' (compound)	Item 'II' (reagent)
(A) Lysine	(P)I-naphthol
(B)Furfural	(Q)ninhydrin
(C) Benzyl alcohol	(R) $KMnO_4$
(D)Styrene	(S)ceric ammonium

A (A) \rightarrow (Q), (B) \rightarrow (P), (C) \rightarrow (S), (D) \rightarrow (R)

B (A) \rightarrow (Q), (B) \rightarrow (R), (C) \rightarrow (S), (D) \rightarrow (P)

C (A) \rightarrow (Q), (B) \rightarrow (P), (C) \rightarrow (R), (D) \rightarrow (S)

D (A) \rightarrow (R), (B) \rightarrow (P), (C) \rightarrow (Q), (D) \rightarrow (S)

Solution

Lysine - Ninhydrin

Furfural - I naphthol

Benzyl alcohol - ceric ammonium

Styrene - $KMnO_4$

Option A is correct

#1331840

Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z , then :

A $R(z) > 0$ and $I(z) > 0$

B $R(z) < 0$ and $I(z) > 0$

C $R(z) = 3$

D $I(z) = 0$

Solution

$$z = \left(\frac{\sqrt{3} + i}{2}\right)^5 + \left(\frac{\sqrt{3} - i}{2}\right)^5$$

$$z = (e^{i\pi/6})^5 + (e^{-i\pi/6})^5$$

$$= e^{5\pi/6} + e^{-5\pi/6}$$

$$= \cos\frac{5\pi}{6} + i\frac{\sin 5\pi}{6} + \cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)$$

$$= 2\cos\frac{5\pi}{6} < 0$$

$$I(z) = 0 \text{ and } Re(z) < 0$$

#1331965

Let $a_1, a_2, a_3, \dots, a_{10}$ be in G.P. with $a_1 > 0$ for $i = 1, 2, \dots, 10$ and S be the set of pairs (r, k) , $r, k \in \mathbb{N}$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S , is :

A Infinitely many

B 4

C 10

D 2

Solution

Apply

$$C_3 \rightarrow C_3 - C_2$$

$$C_2 \rightarrow C_2 - C_1$$

Let α be common ratio of GP.

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e (\alpha^{r+k}) & \log_e (\alpha^{r+k}) \\ \log_e a_4^r a_5^k & \log_e (\alpha^{r+k}) & \log_e (\alpha^{r+k}) \\ \log_e a_7^r a_8^k & \log_e (\alpha^{r+k}) & \log_e (\alpha^{r+k}) \end{vmatrix} = 0$$

Which is always true

#1332108

The positive value of λ for which the co-efficient of x^2 in the expression $x^2 \left(\sqrt{x} + \frac{\lambda}{x^2} \right)^{10}$ is 720, is:

A $\sqrt{5}$

B 4

C $2\sqrt{2}$

D 3

Solution

$$x^2 \left({}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{\lambda}{x^2} \right)^r \right)$$

$$x^2 \left[{}^{10}C_r \lambda^r x^{\frac{10-r}{2} - 2r} \right]$$

$$x^2 \left[{}^{10}C_r \lambda^r x^{\frac{10-5r}{2}} \right]$$

$\therefore r = 2$

Hence, ${}^{10}C_2 \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

#1332143

The value of $\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$ is:

A $\frac{1}{256}$

B $\frac{1}{2}$

C $\frac{1}{512}$

D $\frac{1}{1024}$

Solution

$$2 \sin \frac{\pi}{2^{10}} \cos \frac{\pi}{2^{10}} \cdot \dots \cdot \cos \frac{\pi}{2^2}$$

$$\frac{1}{2^9} \sin \frac{\pi}{2} = \frac{1}{512}$$

$$\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \sin \frac{\pi}{2^{10}}$$

$$= \frac{\sin \left(2^9 \cdot \frac{\pi}{2^{10}} \right)}{2^9 \sin \frac{\pi}{2^{10}}} \sin \frac{\pi}{2^{10}} = \frac{1}{2^9} = \frac{1}{512}$$

#1332285

The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[\chi] + [\sin \chi] + 4}$, where $[t]$ denotes the greatest integer less than or equal to t , is:

A $\frac{1}{12}(7\pi + 5)$

B $\frac{3}{10}(4\pi - 3)$

C $\frac{1}{12}(7\pi - 5)$

D $\frac{3}{20}(4\pi - 3)$

Solution

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{[\cos x] + [\sin x] + 4}$$

$$= \int_{-\pi/2}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^0 \frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\pi/2} \frac{dx}{1+0+4}$$

$$= \int_{-\pi/2}^{-1} \frac{dx}{1} + \int_{-1}^0 \frac{dx}{2} + \int_0^1 \frac{dx}{4} + \int_1^{\pi/2} \frac{dx}{5}$$

$$\left(-1 + \frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$$

$$-1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{\pi}{2} + \frac{\pi}{10}$$

$$\frac{-9}{20} + \frac{3\pi}{5}$$

#1332335

If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

A 6

B 5

C 4

D 3

Solution

$$1 - {}^n C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$$

$$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$$

$$n_{\min} = 5$$

#1332380

If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, \dots, x_5 and -50 is equal to :

A 582.5

B 507.5

C 586.5

D 509.5

Solution

$$\bar{x} = 10 \Rightarrow \sum_{i=1}^5 x_i = 50$$

$$S.D. = \sqrt{\frac{\sum_{i=1}^5 x_i^2}{5} - (\bar{x})^2} = 8$$

$$\Rightarrow \sum_{i=1}^5 (x_i)^2 = 109$$

$$\text{Variance} = \frac{\sum_{i=1}^5 (x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^5 \frac{x_i - 50}{6}\right)^2$$

$$= 507.5$$

#1332497

The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

- A $2\sqrt{11}$
- B $3\sqrt{2}$
- C $6\sqrt{3}$
- D $8\sqrt{2}$

Solution

$$x^2 = 4y$$

$$x - \sqrt{2}y + 4\sqrt{2} = 0$$

Solving together we get

$$x^2 = 4\left(\frac{x + 4\sqrt{2}}{\sqrt{2}}\right)$$

$$\sqrt{2}x^2 + 4x + 16\sqrt{2}$$

$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$

$$x_1 + x_2 = 2\sqrt{2}; \quad x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$$

Similarly,

$$(\sqrt{2}y - 4\sqrt{2})^2 = 4y$$

$$2y^2 + 32 - 16y = 4y$$

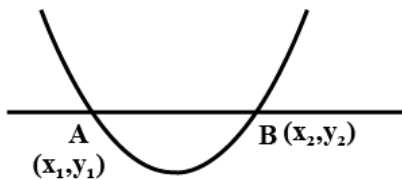
$$2y^2 - 20y + 32 = 0 \quad \begin{matrix} x_1 + y_2 = 10 \\ y_1y_2 = 16 \end{matrix}$$

$$l_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2\sqrt{2})^2 + 64 + (10)^2 - 4(16)}$$

$$= \sqrt{8 + 64 + 100 - 64}$$

$$= \sqrt{108} = 6\sqrt{3}$$



#1332543

Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is:

- A $\sqrt{3}$
- B $-\sqrt{3}$
- C $-2\sqrt{3}$
- D $2\sqrt{3}$

Solution

$$A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix} (b > 0)$$

$$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$$

$$|A| = 2(b^2 + 2) - b^2 - 1$$

$$|A| = b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow b + \frac{3}{b} \geq 2\sqrt{3}$$

$$b + \frac{3}{b} \geq 2\sqrt{3}$$

#1332593

The tangent to the curve, $y = xe^{x^2}$ passing through the point $(1, e)$ also passes through the point:

A $\left(\frac{4}{3}, 2e\right)$

B $(2, 3e)$

C $\left(\frac{5}{3}, 2e\right)$

D $(3, 6e)$

Solution

$$y = xe^{x^2}$$

$$\frac{dy}{dx} \Big|_{(1,e)} = \left(x \cdot e^{x^2} \cdot 2x + e^{x^2} \right) \Big|_{1,e} = 2 \cdot e + e = 3e$$

$$T: y - e = 3e(x - 1)$$

$$y = 3ex - 3e + e$$

$$y = (3e)x - 2e$$

$$\left(\frac{4}{3}, 2e\right) \text{ lies on it}$$

#1332667

The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$

$$x + 4y + 7z = 0$$

$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is :

A One

B Three

C Four

D Two

Solution

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$$

$$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$$

$$14 - 7\sin 3\theta - 14\cos 2\theta = 0$$

$$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$$

$$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$$

$$7\sin\theta[-3 + 4\sin 2\theta + 4\sin\theta] = 0$$

$$\sin\theta = 0$$

$$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$$

$$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$$

$$\sin\theta = \frac{-3}{2}; \sin\theta = \frac{1}{2}$$

Hence, 2 solution in $(0, \pi)$

#1332714

If $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$, then $f'(1/2)$ is:

A $\frac{6}{25}$

B $\frac{24}{25}$

C $\frac{18}{25}$

D $\frac{4}{5}$

Solution

$$\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt \quad f'\left(\frac{1}{2}\right) = ?$$

Differentiate w.r.t. 'x'

$$f(x) = 2x + 0 - x^2 f(x)$$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1 + \frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

#1332731

Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

A Three elements

B One element

C Five elements

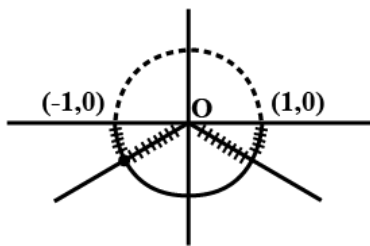
D Two elements

Solution

$f: (-1, 1) \rightarrow \mathbb{R}$

$$f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$$

Non-derivable at 3 points in $(-1, 1)$



#1332786

Let $S = \left\{ (x, y) \in \mathbb{R}^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1 \right\}$, where $r \neq \pm 1$. Then S represents:

- A A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.
- B An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$.
- C A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.
- D An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

Solution

$$\frac{y^2}{1+r} - \frac{x^2}{1-r} = 1$$

$$\text{for } r > 1, \frac{y^2}{1+r} + \frac{x^2}{r-1} = 1$$

$$e = \sqrt{1 - \left(\frac{r-1}{r+1}\right)}$$

$$= \sqrt{\frac{(r+1) - (r-1)}{(r+1)}}$$

$$= \sqrt{\frac{2}{r+1}} = \sqrt{\frac{2}{r+1}}$$

#1333462

If $\sum_{r=0}^{25} ({}^{50}C_r \cdot 50 - r {}^{50}C_{25-r}) = K ({}^{50}C_{25})$, then K is equal to:

- A $2^{25} - 1$
- B $(25)^2$
- C 2^{25}
- D 2^{24}

Solution

$$\begin{aligned}
& \sum_{r=0}^{25} {}^{50}C_r \cdot 50^{-r} {}^{25}C_{25-r} \\
&= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \times \frac{(50-r)!}{(25)!(25-r)!} \\
&= \sum_{r=0}^{25} \frac{50!}{25!25!} \times \frac{25!}{(25-r)!(r!)} \\
&= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = (2^{25}) {}^{50}C_{25} \\
&\therefore K = 2^{25}
\end{aligned}$$

#1333556

Let N be the set of natural numbers and two functions f and g be defined as $f, g: N \rightarrow N$ such that :

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. The fog is:

- A** Both one-one and onto
- B** One-one but not onto
- C** Neither one-one nor onto
- D** onto but not one-one

Solution

$$fx = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ \frac{n}{2} & \text{n is even} \end{cases}$$

$$g(x) = n - (-1)^n \begin{cases} n+1; \text{ n is odd} \\ n-1; \text{ n is even} \end{cases}$$

$$f(g(n)) = \begin{cases} \frac{n}{2}; & \text{n is even} \\ \frac{n+1}{2}; & \text{n is odd} \end{cases}$$

\therefore onto but not one-one

#1333616

The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is:

- A** 2
- B** $\frac{4}{9}$
- C** $\frac{15}{8}$
- D** 1

Solution

$$\alpha + \beta = \lambda - 3$$

$$\alpha\beta = 2 - \lambda$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$$

$$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$$

$$= \lambda^2 - 4\lambda + 5$$

$$(\lambda - 2)^2 + 1$$

$$\lambda = 2$$

#1333637

Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

A Fourth

B Second

C Third

D First

Solution

$$m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$$

$$\Rightarrow b + 4a = 0 \dots(i)$$

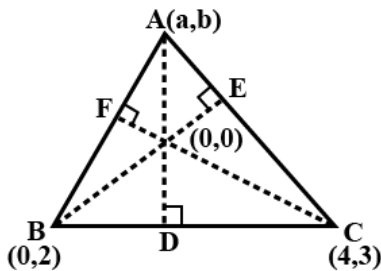
$$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{b-2}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$$

$$\Rightarrow 3b - 6 = -4a \Rightarrow 4a + 3b = 6 \dots(ii)$$

From (i) and (ii)

$$a = \frac{-3}{4}, b = 3$$

\therefore 1st quadrant.



#1333686

Two sides of a parallelogram are along the lines, $x + y = 3$ and $x + y + 3 = 0$. If its diagonals intersect at (2, 4), then one of its vertex is :

A (2, 6)

B (2, 1)

C (3, 5)

D (3, 6)

Solution

Solving

$$x + y = 3 \quad (A(0, 3))$$

$$x - y = -3$$

$$\frac{x_1 + 0}{2} = 2; x_1 = 4 \text{ similarly } y_1 = 5$$

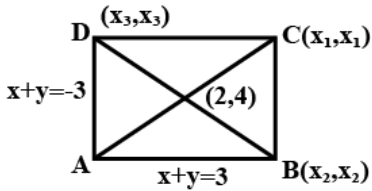
$$C \Rightarrow (4, 5)$$

Now equation of BC is $xy = 1$

and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $xy = -3$

Point D is $(3, 6)$



#1333707

Let $\vec{a} = (\lambda - 2)\vec{a} + b$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors \vec{a} and \vec{b} non-collinear. The value of λ for which vectors \vec{a} and $\vec{\beta}$ are collinear, is :

A -3

B 4

C 3

D -4

Solution

$$\vec{a} = (\lambda - 2)\vec{a} + b$$

$$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$$

$$\frac{\lambda - 2}{4\lambda - 2} = \frac{1}{3}$$

$$3\lambda - 6 = 4\lambda - 2$$

$$\lambda = -4$$

#1333711

The value of $\cot\left(\sum_{n=1}^{19} \cot^{-1}\left(1 + \sum_{p=1}^n 2p\right)\right)$ is:

A $\frac{22}{23}$

B $\frac{23}{22}$

C $\frac{21}{19}$

D $\frac{19}{21}$

Solution

$$\cot\left(\sum_{n=1}^{19} \cot^{-1}(1+n(n+1))\right)$$

$$\cot\left(\sum_{n=1}^{19} \cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19} \tan^{-1}\frac{1}{1+n(n+1)}\right)$$

$$\sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1}n)$$

$$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\text{(Where } \tan A = 20, \tan B = 1) \frac{1\left(\frac{1}{20}\right) + 1}{1 - \frac{1}{20}} = \frac{21}{19}$$

#1333716

With the usual notation, in $\triangle ABC$, if $\angle A + \angle B = 120^\circ$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$ then the ratio $\angle A : \angle B$ is:

- A** 7:1
B 5:3
C 9:7
D 3:1

Solution

$$A + B = 120^\circ$$

$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot\left(\frac{C}{2}\right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})} \cot(30^\circ) = \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1$$

$$\frac{A-B}{2} = 45^\circ$$

$$\Rightarrow A - B = 90^\circ$$

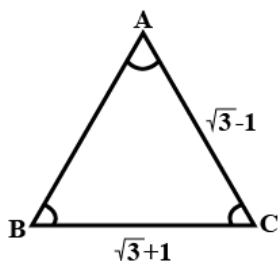
$$A + B = 120^\circ$$

$$2A = 210^\circ$$

$$A = 105^\circ$$

$$B = 15^\circ$$

$$\angle A : \angle B = 7:1$$



#1333734

The plane which bisects the line segment joining the points $(-3, -3, 4)$ and $(3, 7, 6)$ at right angles, passes through which one of the following points?

- A** $(4, 1, 7)$
B $(4, 1, -2)$

C (2, 3, 5)

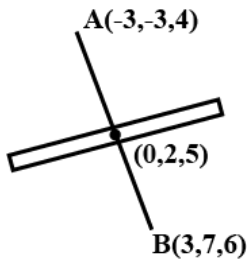
D (2, 1, 3)

Solution

$$\vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$$

$$p: 3(x - 0) + 5(y - 2) + 1(z - 5) = 0$$

$$3x + 5y + z = 15$$



#1333745

Consider the following three statements:

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

A $(P \wedge Q) \vee (\sim R)$

B $(\sim P) \wedge (\sim Q \wedge R)$

C $(\sim P) \vee (Q \wedge R)$

D $P \vee (\sim Q \wedge R)$

Solution

P is True

Q is False

R is True

Option 4) $T \vee (T \wedge F) = T$

#1333757

On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?

A $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

B $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

C $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

D $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Solution

General point on the given line is

$$x = 2\lambda + 4$$

$$y = 2\lambda + 5$$

$$z = \lambda + 3$$

Solving with plane,

$$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$$

$$5\lambda + 12 = 2$$

$$5\lambda = 10$$

$$\lambda = 2$$

#1333770

Let f be a differentiable function such that $f'(x) = 7 - \frac{3f(x)}{4x}$, ($x > 0$) and $f(1) \neq 4$.

Then $\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right)$:

- A Exists and equals 4
- B Does not exist
- C Exist and equals
- D Exists and equals $\frac{4}{7}$

Solution

$$f'(x) = 7 - \frac{3f(x)}{4x} \quad (x > 0)$$

Given $f(1) \neq 4$ $\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right) = ?$

$$\frac{dy}{dx} + \frac{3y}{4x} = 7 \quad (\text{This is LDE})$$

$$\text{IF} = e^{\int \frac{3}{4x} dx} = e^{\frac{3}{4} \ln|x|} = x^{\frac{3}{4}}$$

$$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx$$

$$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$f(x) = 4x + C \cdot x^{-\frac{3}{4}}$$

$$f\left(\frac{1}{x}\right) = \frac{4}{x} + C \cdot x^{\frac{3}{4}}$$

$$\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right) = \lim_{x \rightarrow 0^+} \left(4 + C \cdot x^{\frac{7}{4}}\right) = 4$$

#1333775

A helicopter is flying along the curve given by $y - x^{3/2} = 7$, ($x \geq 0$). A soldier positioned at the point $\left(\frac{1}{7}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this

nearest distance is:

- A $\frac{1}{2}$
- B $\frac{1}{3} \sqrt{\frac{7}{3}}$
- C $\frac{1}{6} \sqrt{\frac{7}{3}}$
- D $\frac{\sqrt{5}}{6}$

Solution

$$y - \frac{3}{x^2} = 7 \quad (x \geq 0)$$

$$\left(\frac{3}{2} \sqrt{x} \right) \left(\frac{7-y}{2-x} \right) = -1$$

$$\left(\frac{3}{2} \sqrt{x} \right) \left(\frac{-x^{\frac{3}{2}}}{2-x} \right) = -1$$

$$\frac{3}{2} x^2 = \frac{1}{2} - x$$

$$3x^2 = 1 - 2x$$

$$3x^2 = 1 - 2x$$

$$3x^2 + 3x - x - 1 = 0$$

$$(x+1)(3x-1) = 0$$

$$\therefore x = -1 \text{ (rejected)}$$

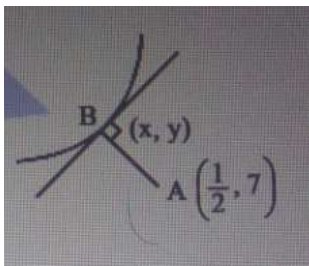
$$x = \frac{1}{3}$$

$$y = 7 + \frac{3}{x^2} = 7 + \left(\frac{1}{3} \right)^{\frac{3}{2}}$$

$$l_{AB} = \sqrt{\left(\frac{1}{3} - \frac{1}{3} \right)^2 - \left(\frac{1}{3} \right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$$

$$= \sqrt{\frac{3+4}{9 \times 12}}$$

$$= \sqrt{\frac{7}{108}} = \frac{1}{6} \sqrt{\frac{7}{3}}$$



#1333781

If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to:

A $-4x^3 - 1$

B $4x^3 + 1$

C $-2x^3 - 1$

D $-2x^3 + 1$

Solution

$$\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$$

$$\text{Put } x^3 = t$$

$$3x^2 dx = dt$$

$$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$$

$$\frac{1}{3} \int t \cdot e^{-4t} dt$$

$$\frac{1}{3} \left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt \right]$$

$$-\frac{e^{-4t}}{48} [4t + 1] + c$$

$$-\frac{e^{-4x^3}}{48} [4x^3 + 1] + c$$

$$\therefore f(x) = -1 - 4x^3$$

From the given options (1) is most suitable

#1333786

The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ which passes through $(1, 1)$ is :

- A A circle with centre on the y-axis
- B A circle with centre on the x-axis
- C An ellipse with major axis along the y-axis
- D A hyperbola with transverse axis along the x-axis

Solution

$$(x^2 - y^2)dx + 2xydy = 0$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Solving we get,

$$\int (v^2 + 1) = -\ln x + C$$

$$(y^2 + x^2) = Cx$$

$$1 + 1 = C \Rightarrow C = 2$$

$$y^2 + x^2 = 2x$$

#1333789

If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to:

- A 20
- B 25
- C 13
- D -25

Solution

$$3\left(\frac{1}{2}r^2 \cdot \sin 120^\circ\right) = 27\sqrt{3}$$

$$\frac{r^2 \sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$

$$r^2 = \frac{108}{3} = 36$$

$$\text{Radius} = \sqrt{25 + 36} = \sqrt{61}$$

$$C = 25$$

∴ Option (2)

