JEE ADVANCED (Paper - 1) PHYSICS

Section 1 (Maximum Marks : 15)

- This section contains **FIVE** questions
- Each question has Four options (A), (B), (C) and (D) only one of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in ORS.
- For each question, marks will be awarded in one of the following categories. : Full Marks : +3 if only the bubble corresponding to the correct option is darkened Zero Marks : 0 if none of the bubbles is darkened. Negative Marks : -1 in all other cases
- 1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. the angle θ of the prism is



2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_o) are given below: $\lambda(\mu m) V_o(Volt)$

0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8 \text{ m s}^{-1}$ and $e = 1.6 \times 10^{-19} \text{C}$, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34} (B)

Ans.

Sol:
$$\frac{hc}{\lambda} = W + eV_o$$

Using reading 1 & 2

 $\frac{hc}{10^{-7}} \left[\frac{1}{3} - \frac{1}{4} \right] = e\left(2 - 1\right)$

We get $h = 6.4 \times 10^{-34}$

Similarly using reading 2 & 3 or 3 & 1 we get the same value

Hence (B) is correct.

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is $4.2 \text{ kJ kg}^{-1}\text{K}^{-1}$ and the density of water is 1000 kg m⁻³) (A) 1600 (B) 2067 (C) 2533 (D) 3933

Ans. (B)

Sol: Total heat generated in 3 hr

 $H = 3 \times 10^3 \times 3 \times 60 \times 60$

Heat used to raise the temperature of cooler

 $Q = m C. \Delta T = 120 \times 4.2 \times 10^{3} \times 20$ Now heat required to eject $Q^{1} = H - Q$ and $P = \frac{Q^{1}}{t} = \frac{3 \times 10^{3} \times 3 \times 60 \times 60 - 120 \times 4.2 \times 10^{3} \times 20}{3 \times 60 \times 60}$ = 3000 - 933.33 = 2067 watt

4. A uniform wooden stick of mass 1.6 kg and length 1 rests in an inclined manner on a smooth, vertical wall of height h(<1) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/1 and the frictional force f at the bottom of the stick are

(g = 10 ms⁻²)
(A)
$$\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$$

(B) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$
(C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$
(D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

Ans. (I

Sol: $\Sigma F_x = 0 \Rightarrow f = \frac{\sqrt{3}}{2}N$...(1)



$$N + \frac{N}{2} = 1.6g$$

$$\Sigma F_{y} = 0 \Rightarrow \frac{3N}{2} = 1.6g \qquad ...(2)$$

From equation (1) & (2)

$$f = \frac{\sqrt{3}}{2} \cdot \frac{1.6g \times 2}{3} = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3}N$$

$$\Sigma \tau = 0 \Rightarrow 1.6g \frac{1}{2} \cos 60^\circ = \frac{\sqrt{3}}{2} Nh + \frac{N}{2} \times \frac{h}{\tan 60^\circ}$$
$$\Rightarrow 1.6g \times \frac{1}{4} = \frac{\sqrt{3}}{2} \times \frac{1.6g \times 2}{3} h + \frac{1}{2} \frac{1.6g \times 2}{3} \times \frac{h}{\sqrt{3}}$$
$$\Rightarrow \frac{1}{4} = \frac{h}{\sqrt{3}} + \frac{h}{3\sqrt{3}}$$
$$\Rightarrow \frac{h}{1} = \frac{3\sqrt{3}}{16}$$
$$\therefore \text{ answer is (D)}$$

5.

An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ . The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?



Ans. (D)

Sol: The situation is similar to discharging of R-C circuit. Hence

$$i = i_0 e^{\frac{-t}{\tau}}$$
 \therefore answer is (D)

Section 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 if only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Zero Marks : 0 if none of the bubbles is darkened.

Negative Marks : -2 in all other cases

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.
- 6. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(A) The refractive index of the lens is 2.5

- (B) The radius of curvature of the convex surface is 45 cm
- (C) The faint image is erect and real
- (D) The focal length of the lens is 20 cm
- Sol. (A, D)



Due to refraction, real image of double size is formed

$$\therefore m = -2$$

$$m = \frac{v}{u} \implies -2 = \frac{v}{-30} \implies v = 60$$

For refraction on spherical surface

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \implies \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$$
$$\implies f = 20 \text{ cm}$$

For reflection on convex surface

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_m} \implies \frac{1}{10} - \frac{1}{30} = \frac{1}{f_m}$$

 $f_m = 15 \text{ cm} \Rightarrow R = 30 \text{ cm} = \text{Radius of curvature of mirror.}$

Now
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

 $\frac{1}{20} = (n-1)\left(\frac{1}{30} - \frac{1}{\infty}\right) \Rightarrow n = 2.5$

Hence, option (A) and (D) are correct.

7. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 As^{-1} . Which of the following statement(s) is(are) true?



- (A) There is a repulsive force between the wire and the loop
- (B) If the loop is rotated at a constant angular speed about the wire, an additional emf of

 $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

- (C) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt
- (D) The induced current in the wire is in opposite direction to the current along the hypotenuse
- Sol. (A, C)

Using the concept of mutual induction, if we take the reverse case i.e. if current is passed in the conducting infinite wire. Magnetic flux with the triangular loop can be calculate as



Here, $\tan 45 = \frac{X}{Y} = 1 \implies Y = X; dY = dX$

The flux through the elemental strip

$$d\phi = B.dA = \frac{\mu_0 I}{2\pi x} \times 2xdY = \frac{\mu_0 I}{\pi} dx$$

$$\therefore \qquad \phi = \int d\phi = \frac{\mu_0 I}{\pi} \int_0^{Y=0.1} dx = \frac{\mu_0 I}{\pi} \times .1 = \frac{\mu_0 I}{10\pi}$$

$$\therefore \qquad \text{induced emf in the loop} = \frac{d\phi}{dt} = \frac{\mu_0}{10\pi} \frac{di}{dt}$$

 $=\frac{\mu_0}{10\pi}{\times}10=\frac{\mu_0}{\pi}$

Therefore induced emf in the wire will be same and hence option C is correct.

As per Lenz's law, if current is increased in the wire towards right, then the induced current in the triangular loop is anticlockwise.

Hence option (D) is incorrect.

As per Lenz's law if current is increased in the straight wire the loop will have a tendency to move from stranger to weaker field.

 \therefore option (A) is correct.

When the loop is rotated about the wire the flux linked through the triangular loop does not change due to the current in the straight wire. So emf induced is zero.

So, option B is incorrect.

8. The position vector $\mathbf{\dot{r}}$ of a particle of mass m is given by the following equation $\mathbf{\dot{r}}_{(1)} = -3\hat{\mathbf{a}}_{(2)}\hat{\mathbf{c}}_{(2)}\hat{\mathbf{c}}_{(2)}$

$$\mathbf{r}(t) = \alpha t^3 \mathbf{i} + \beta t^2 \mathbf{j}$$

where $\alpha = 10/3 \text{ ms}^{-3}$, $\beta = 5 \text{ ms}^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\stackrel{1}{v}$ is given by $\stackrel{r}{v} = (10\hat{i} + 10\hat{j})\text{ms}^{-1}$

(B) The angular momentum $\stackrel{1}{L}$ with respect to the origin is given by $\stackrel{1}{L} = -(5/3)\hat{k}$ Nms

(C) The force $\stackrel{1}{F}$ is given by $\stackrel{1}{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\hat{\tau}$ with respect to the origin is given by $\hat{\tau} = -(20/3)\hat{k}$ N m

Sol.
$$(A, B, D)$$

$$\hat{\mathbf{r}} = \alpha t^{3} \mathbf{i} + \beta t^{2} \mathbf{j}$$

$$\hat{\mathbf{v}} = 3\alpha t^{2} \hat{\mathbf{i}} + 2\beta t \hat{\mathbf{j}}$$

$$\hat{\mathbf{a}} = 6\alpha t \hat{\mathbf{i}} + 2\beta \hat{\mathbf{j}}$$
Now,
$$\hat{\mathbf{P}} = \mathbf{m} \hat{\mathbf{v}} = 0.1(3\alpha t^{2} \hat{\mathbf{i}} + 2\beta t \hat{\mathbf{j}})$$

$$\hat{\mathbf{F}} = \mathbf{m} \hat{\mathbf{a}} = 0.1(6\alpha t \hat{\mathbf{i}} + 2\beta \hat{\mathbf{j}})$$
Now,
$$\mathbf{A} t t = 1s$$

$$\hat{\mathbf{r}} = \alpha \hat{\mathbf{i}} + \beta \hat{\mathbf{j}} = \frac{10}{3} \hat{\mathbf{i}} + 5 \hat{\mathbf{j}}$$

Hence, (A), (B) and (D) are correct.

9. A length-scale (l) depends on the permittivity (ϵ) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is (are) dimensionally correct?

$$(A) 1 = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)} \qquad (B) 1 = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)} \qquad (C) 1 = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)} (D)$$
$$1 = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$$

Ans. (B, D)

Potential energy $U = \frac{q^2}{4\pi\epsilon_0 L} \Rightarrow \frac{q^2}{\epsilon_0} = \frac{1}{4\pi UL}$

$$\left[\frac{q^2}{\varepsilon_0}\right] = UL = JL(J \rightarrow Joule)$$

Average KE / Molecular / degree of freedom = $\frac{1}{2}k_{b}T$

$$[k_{b}T] = J(Joule)$$

$$[n] = L^{-3}$$

$$(A) \ l = \sqrt{\left(n\right)\left(\frac{q^{2}}{\varepsilon}\right)\left(\frac{1}{k_{b}P}\right)} = \frac{1}{L}$$

$$(B) \ l = \sqrt{\left(\frac{\varepsilon}{q^{2}}\right)\left(k_{b}T\right)\left(\frac{1}{n}\right)} = L$$

(C)
$$\sqrt{\left[\frac{q^2}{\epsilon}\right] \left[\frac{1}{k_b T}\right] \left[\frac{1}{n^2 h}\right]} = \sqrt{L}$$

(D) $\sqrt{\left[\frac{q^2}{\epsilon}\right] \left[\frac{1}{k_b T}\right] \left[\frac{1}{n^{1/3}}\right]} = L$

Hence (B) and (D) are correct.

- 10. Two loudspeakers *M* and *N* are located 20 *m* apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point *P*, 1800 *m* away from the midpoint *Q* of the line *MN* and moves towards *Q* constantly at 60 km/hr along the perpendicular bisector of *MN*. It crosses *Q* and eventually reaches a point *R*, 1800 m away from *Q*. Let v(t) represent the beat frequency measured by a person sitting in the car at time *t*. Let v_P , v_Q and v_R be the beat frequencies measured at locations *P*, *Q* and *R*, respectively. The speed of sound in air is 330 ms^{-1} . Which of the following statement(s) is (are) true regarding the sound heard by the person?
 - (A) The plot below represents schematically the variation of beat frequency with time



- (B) The rate of change in beat frequency is maximum when the car passes through Q
- (C) $v_{\rm P} + v_{\rm R} = 2v_{\rm Q}$
- (D) The plot below represents schematically the variation of beat frequency with time





Sol. At position of point P. Beat frequency is given by

$$v_P = v_N \left[\frac{v_S + v_0 \cos \theta_0}{v_S - 0} \right] - v_M \left[\frac{v_S + v_0 \cos \theta_0}{v_S - 0} \right] \qquad \dots \dots \dots (i)$$

at point of R the beat frequency is given by

At position of Q the beat frequency is given by

$$v_Q = |v_N - v_M| \quad \dots \quad (\text{iii})$$

After adding both equations (i) and (ii) we get the result $v_P + v_R = 2 v_Q$

and at any time moment the beat frequency is given by

$$v|_{t=t} = |v_N - v_M| + 50\cos\theta$$
. So, the graph will be





and in this above graph the rate of change of frequency is maximum at position Q.

11. A transparent slab of thickness *d* has a refractive index n(z) that increases with *z*. Here *z* is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement *l*.



Which of the following statement(s) is (are) true ?

(A) *l* is dependent on n(z)

(B) $n_1 \sin \theta_i = (n_2 - n_1) \sin \theta_f$

(C) $n_1 \sin \theta_i = n_2 \sin \theta_f$ (D) *l* is independent of n_2



l is dependent of n(z)

l is independent of n_2

- 12. Highly excited states for hydrogen-like atoms (also called Rybderg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is (are) true?
 - (A) Relative change in the radii of two consecutive orbitals does not depend on Z
 - (B) Relative change in the radii of two consecutive orbitals varies as 1/n
 - (C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$
 - (D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

Ans. (A, B, D)

Sol.
$$r = 0.529 \frac{n^2}{z}$$

$$\frac{\Delta r}{r} \propto \frac{1}{n}$$
 (independent of z)

B is correct, A is also correct

$$E = -13.6 \frac{z^2}{n^2}$$

$$\Delta E = -13.6 z^2 \left(-2n^{-3}.\Delta n\right)$$

$$\frac{\Delta E}{E} = \frac{13.6 z^2 \cdot 2\Delta n}{n^3 \cdot \left(-13.6 \frac{z^2}{n^2}\right)}$$

$$\frac{\Delta E}{E} \propto \frac{1}{n}$$

(C) is not correct

$$L = n \frac{h}{2\pi}$$

$$\frac{\Delta L}{L} = \frac{\Delta n}{n}$$
$$\frac{\Delta L}{L} \propto \frac{1}{n}$$

(D) is correct

- 13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true?
 - (A) The temperature distribution over the filament is uniform
 - (B) The resistance over small sections of the filament decreases with time
 - (C) The filament emits more light at higher band of frequencies before it breaks up
 - (D) The filament consumes less electrical power towards the end of the life of the bulb
- Ans. (C, D)
- Sol. Since there is non uniform evaporation therefore heat dissipation (production i.e., i²R) is non uniform, it implies resistance of different part is different. So temperature can't be same everywhere, also length of filament increases with time so resistance increases. As temperature T rises with time so $\lambda_m T = b$, this implies λ decreases and frequency increases. Before breakup, resistance is maximum so $P = \frac{V^2}{R}$, this implies at the end of life P will decrease.

Section 3 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :
 Full Marks : +3 if only the bubble corresponding to all the correct answer is darkened.

Zero Marks : 0 in all other cases

14. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as -13.6 eV, The number of lines present in the emission spectrum is

Ans. (6)

Sol.
$$\Delta E = \frac{hc}{e\lambda}$$

= 12.752 eV
Q $\Delta E = E_1 - E_2$
 $\Rightarrow E_1 - (-13.6)$
 $E_1 = -0.85$
 $\frac{-13.6}{n^2} = -0.85 \Rightarrow n = 4$
No. Of emission spectrum $= \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$

15. The isotope ${}_{5}^{12}$ B having a mass 12.014 u undergoes β -decay to ${}_{6}^{12}$ C. ${}_{6}^{12}$ C has an excited state of the nucleus ${}_{6}^{12}$ C^{*} at 4.041 MeV above its ground state. If ${}_{5}^{12}$ B decays to ${}_{6}^{12}$ C^{*}, the maximum kinetic energy of the β -particle in units of MeV is (1u = 931.5 MeV/c², where c is the speed of light in vacuum).

Ans. (9)

Sol.
$$K=(12.0144-12) \times 931.5 - 4.041$$

 $K=13.041 - 4.041 = 9$

- 16. Consider two solid spheres P and Q each of density 8 gm cm⁻³ and diameter 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm⁻³ and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm⁻³ and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is
- Ans. (3)

Sol.
$$v_t = \frac{2r^2g(\sigma - \rho)}{9\eta}$$

$$\frac{V_{p}}{V_{Q}} = \frac{r_{p}^{2}}{r_{Q}^{2}} \times \frac{(\sigma_{p} - \rho_{p})}{(\sigma_{Q} - \rho_{p})} \times \frac{\eta_{Q}}{\eta_{P}} = (2)^{2} \left(\frac{8 - 0.8}{8 - 1.6}\right) \frac{2}{3} = 3$$

17. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. the ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

Ans.

(8)

Sol.



18. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

Sol.
$$R = \log_2(P/P_0) (R \rightarrow reading)$$

$$T = 487^{\circ}C \rightarrow 2767^{\circ}C$$

$$R_{1} = 4\log_{2}\left(\frac{T_{1}}{P_{0}}\right) = 1...(i)$$

$$R_{2} = 4\log_{2}\left(\frac{T_{2}}{P_{0}}\right) = x \text{ (say) } ...(ii)$$

equation (i) substrated from equation (ii), we get

$$4\left\{\log_{2}\left(\frac{T_{2}}{T_{1}}\right)\right\} = x - 1$$
$$4\log_{2}\left(\frac{3040}{760}\right) = x - 1$$
$$x = 1 + 4\log_{2}(4) = 9$$