IIT - JEE ADVENCED - 2012

PAPER-1 [Code – 8]

PART - I: PHYSICS

SECTION I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and ℓ are the same.

(B) due to the error in the measurement of d is twice that due to the error in the measurement of ℓ.

(C) due to the error in the measurement of ℓ is twice that due to the error in the measurement of d.

(D) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ.

Sol. **(A)**

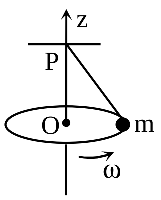
$$\text{L.C.} = \frac{0.5}{100} = 0.005\,\text{mm}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta(d)}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\times10^{-3}}{0.25\times10^{-3}} = \frac{1}{50}$$

$$2\frac{\Delta(d)}{d} = \frac{2\times0.005\times10^{-3}}{0.5\times10^{-3}} = \frac{1}{50}$$

2. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



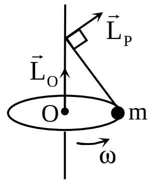
(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

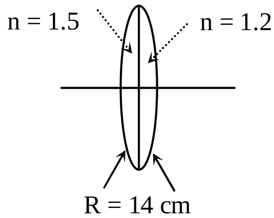
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

Sol. **(C)**



3. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Sol. **(B)**

$$P_T = (1.5-1)\left(\frac{1}{14}-0\right)+(1.2-1)\left(0-\frac{1}{-14}\right) = \frac{0.5}{14}+\frac{0.2}{14}=\frac{1}{20}$$

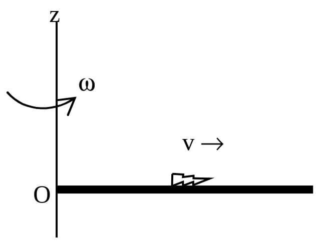
f = +20 cm

$$\frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

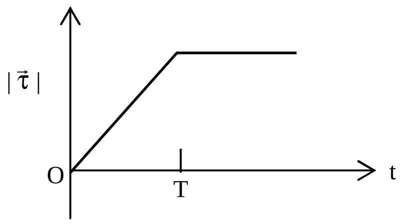
$$\frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}$$

∴ v = 40 cm

4. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) about O, as a function of time is best represented by which plot?



(A)



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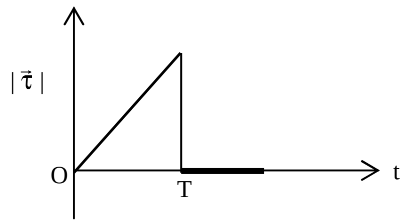
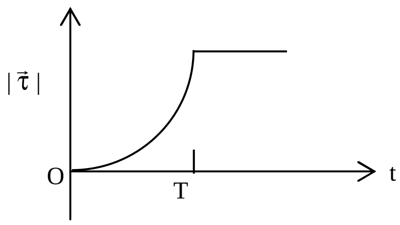
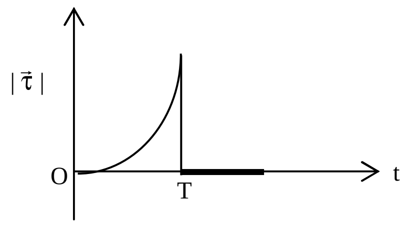


(C)



(D)



Sol. **(B)**

$$\tau = \omega\frac{dI}{dt} = \omega\frac{d}{dt}(C+mv^2t^2)$$

$= m\omega v^2\, 2t.$

5. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Sol. **(D)**

$v_{rms} = \sqrt{\frac{3RT}{M}}$

Required ratio = $\sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10}$

= 3.16.

6. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45^0 to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Sol. **(C)**

qE = mg

q(V/d) = mg

$V = \frac{mgd}{q}$

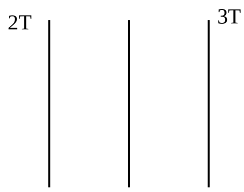
$= \frac{1.67\times10^{-27}\times10\times10^{-2}}{1.6\times10^{-19}}$

$= \frac{10^{-28}}{10^{-19}} = 10^{-9}\,V$

7. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{1/4}T$ (B) $\left(\frac{97}{4}\right)^{1/4}T$ (C) $\left(\frac{97}{2}\right)^{1/4}T$ (D) $(97)^{1/4}T$

Sol. **(C)**



$\sigma A\,(2T)^4 + \sigma A\,(3T)^4 = \sigma\, 2A(T')^4$

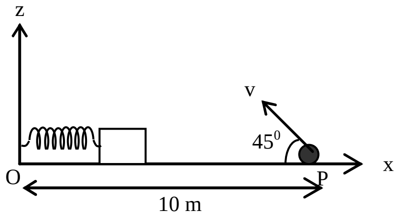
$16T^4 + 81T^4 = 2\,(T')^4$

$97\,T^4 = 2\,(T')^4$

$(T')^4 = \frac{97}{2}\,T^4$

$\therefore\ T' = \left(\frac{97}{2}\right)^{1/4} T$

8. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v form point P at an angle of 45^0 as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Sol. **(A)**

$$\frac{2v\sin 45^0}{g} = 1$$

$\therefore$ v = $\sqrt{50}$ m/s.

9. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

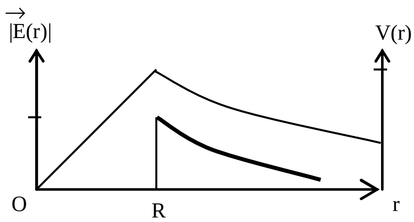
Sol. **(D)**

$\lambda_R > \lambda_G > \lambda_B$

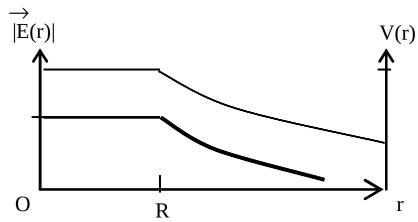
$\therefore \beta_R > \beta_G > \beta_B$

10. Consider a thin spherical shell of radius R with centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

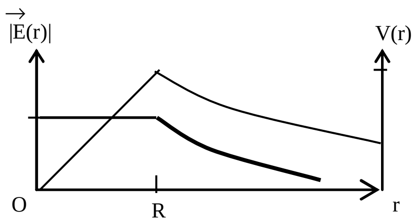
(A)



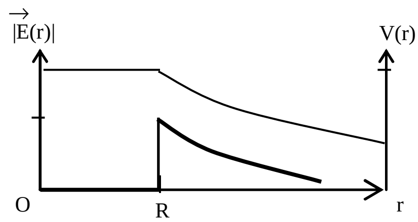
(B)



(C)



(D)



Sol. **(D)**

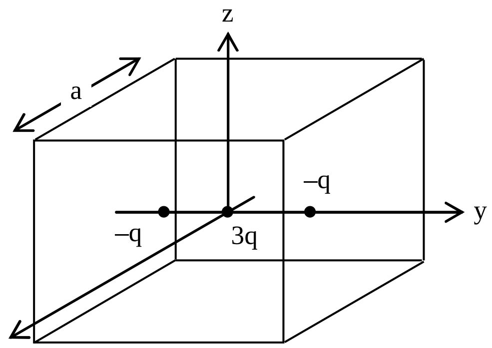
SECTION II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

11. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0 \hat{j}$ and $\vec{B} = B_0 \hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time t > 0?
(A) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.
(B) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.
(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.
(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

Sol. **(C, D)**
If $\theta = 90°, \vec{B}$ exerts no force on q.
If $\theta = 0°, 10°$; the charge particle moves in helix with increasing pitch due to $\vec{E}$ along y-axis.

12. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, −q at (0, −a/4, 0), +3q at (0, 0, 0) and −q at (0, +a/4, 0). Choose the correct options(s)



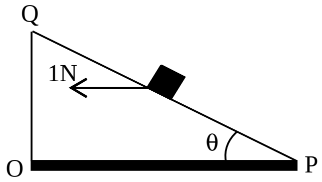
(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = −a/2
(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = −a/2.
(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.
(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

Sol. **(A, C, D)**
Net flux through the cubical region $= \frac{-q + 3q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$
The flux passing through the faces $x = \frac{-a}{2}$, $x = +\frac{a}{2}$ and $z = +\frac{a}{2}$ are same due to symmetry.

13. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,
(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(B) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(C) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

Sol. **(B, D)**
At the open end, the phase of a pressure wave changes by π radian due to reflection. At the closed end, there is no change in the phase of a pressure wave due to reflection.

14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10\ m/s^2$)



(A) θ = 45°

(B) θ > 45° and a frictional force acts on the block towards P.

(C) θ > 45° and a frictional force acts on the block towards Q.

(D) θ < 45° and a frictional force acts on the block towards Q.

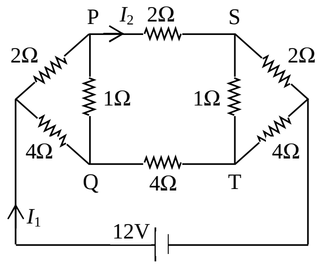
Sol. **(A, C)**

At $\theta = 45°$, mg sin $\theta = 1 \times \cos\theta$

At $\theta > 45°$, mg sin $\theta > 1 \times \cos\theta$ (friction acts upward)

At $\theta < 45°$, mg sin $\theta < 1 \times \cos\theta$ (friction acts downward)

15. For the resistance network shown in the figure, choose the correct option(s)



(A) The current through PQ is zero.

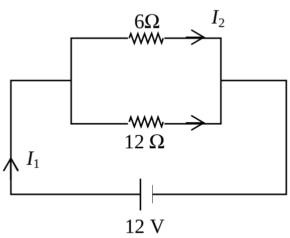
(B) $I_1 = 3A$

(C) The potential at S is less than that at Q.

(D) $I_2 = 2A$

Sol. **(A, B, C, D)**

Nodes P and Q are equipotential and nodes S and T are equipotential from wheatstone bridge, no current passes through PQ and ST.



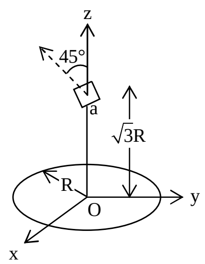
$$I_1 = \frac{12}{4} = 3A$$

$$I_2 = I_1\left(\frac{12}{6+12}\right) = 2A$$

SECTION III : Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

16. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a(a<<R) having two turns is placed with its centre at z = $\sqrt{3}$R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is

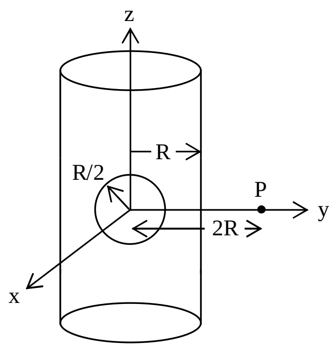


Sol. **(7)**

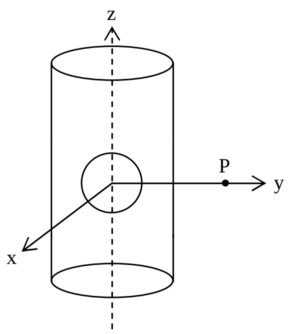
$$M = \frac{N\phi}{I} = \frac{2\left[\frac{\mu_0 IR^2}{2(8R^3)}\right]a^2 \cos 45°}{I} \quad \frac{\mu_0 a^2}{8R2^{1/2}} = \frac{\mu_0 a^2}{R2^{7/2}}$$

So P = 7

17. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Sol. **(6)**



$$\vec{E} = \frac{\lambda(\hat{j})}{2\pi\varepsilon_0(2R)} + \frac{K\left(\rho\frac{4}{3}\pi\frac{R^3}{8}\right)(-\hat{j})}{4R^2}$$

$$\vec{E} = \frac{\rho\pi R^2(\hat{j})}{4\pi\varepsilon_0 R} + \frac{K\pi\rho R(-\hat{j})}{24}$$

$$\vec{E} = K\rho\pi R(\hat{j}) + \frac{K}{24}\rho\pi R(-\hat{j})$$

$$\vec{E} = K\rho\pi R\frac{23}{24}(\hat{j}) = \frac{23}{96\varepsilon_0}\rho R(\hat{j})$$

$\Rightarrow$ k = 6

18. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ J.s / C; $\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$ m/F; 1 fm = 10^{-15})

Sol. **(7)**

$$0+\frac{1}{2}mv^2 = \frac{K(Q)e}{10\times10^{-15}} = \frac{K(120e)e}{10\times10^{-15}}$$

$$\frac{1}{2}\times\frac{5}{3}\times10^{-27}v^2 = \frac{9\times10^{9}\times120\times(1.6\times10^{-19})^2}{10\times10^{-15}}$$

$$v = \frac{9\times6\times10^{9}\times120\times2.56\times10^{-38}}{50\times10^{-42}}$$

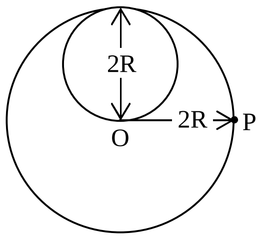
$$v = \sqrt{331.776\times10^{13}}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{4.2\times10^{-15}\times1.6\times10^{-19}}{\frac{5}{3}\times10^{-27}\times\sqrt{331.776\times10^{13}}} = \frac{4.2\times4.8\times10^{-34}}{57.6\times5\times10^{-21}} = 0.07\times10^{-13}$$

$$\lambda = 7\times10^{-15} = 7\text{fm}$$

19. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing though O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P / I_O to the nearest integer is



Sol. **(3)**

$$I_p = \left[\frac{4mR^2}{2} + m\left(4R^2\right)\right] - \left[\frac{m}{4}\frac{R^2}{2} + \frac{m}{4}5R^2\right]$$

$$I_p = mR^2\left[(2+4) - \left(\frac{1}{8}+\frac{5}{4}\right)\right]$$

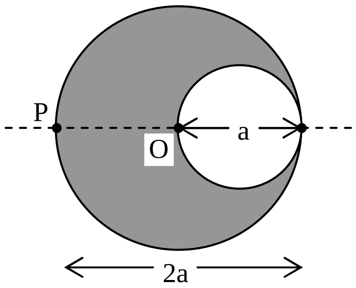
$$I_p = mR^2\left(6-\frac{11}{8}\right) = \frac{37}{8}mR^2 \quad \text{...(1)}$$

$$I_0 = \left(\frac{4mR^2}{2}\right) - \left(\frac{m}{4}\frac{R^2}{2} + \frac{m}{4}R^2\right)$$

$$I_0 = mR^2\left[2-\left(\frac{1}{8}+\frac{1}{4}\right)\right] = mR^2\left[2-\frac{3}{8}\right] = mR^2\left(\frac{13}{8}\right) \quad \text{...(2)}$$

So $\frac{I_p}{I_0} = \frac{37/8}{13/8} \approx 3$ (Nearest Integer)

20. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinity long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Sol. **(5)**

$$B = \frac{\mu_0 (J\pi a^2)}{2\pi a} - \frac{\mu_0 (J\pi a^2 / 4)}{2\pi\left(\frac{3a}{2}\right)}$$

$$B = \frac{5\mu_0 Ja}{12} = \frac{\mu_0 NJa}{12}$$

So N = 5