PART A – CHEMISTRY

1. Identify the **incorrect** statement from the following :
   (1) Ozone absorbs the intense ultraviolet radiation of the sun.
   (2) Depletion of ozone layer is because of its chemical reactions with chlorofluoro alkanes.
   (3) Ozone absorbs infrared radiation.
   (4) Oxides of nitrogen in the atmosphere can cause the depletion of ozone layer.

Sol. **[3]**
Ozone absorbs UV rays from sun and not infrared radiation.

2. When r, P and M represent rate of diffusion, pressure and molecular mass, respectively, then the ratio of the rates of diffusion (r_A/r_B) of two gases A and B, is given as :
   (1) $(P_A/P_B)\ (M_B/M_A)^{1/2}$
   (2) $(P_A/P_B)^{1/2}\ (M_B/M_A)$
   (3) $(P_A/P_B)\ (M_A/M_B)^{1/2}$
   (4) $(P_A/P_B)^{1/2}\ (M_A/M_B)$

Sol. **[1]**

$$r \propto \frac{p}{\sqrt{d}} \propto \frac{p}{\sqrt{M}}$$

$$\frac{r_A}{r_B} = \frac{P_A}{P_B}\left(\frac{M_B}{M_A}\right)^{1/2}$$

3. Consider thiol anion ($RS^{\ominus}$) and alkoxy anion ($RO^{\ominus}$). Which of the following statements is **correct** ?
   (1) $RS^{\ominus}$ is less basic but more nucleophilic than $RO^{\ominus}$
   (2) $RS^{\ominus}$ is more basic and more nucleophilic than $RO^{\ominus}$
   (3) $RS^{\ominus}$ is more basic but less nucleophilic than $RO^{\ominus}$
   (4) $RS^{\ominus}$ is less basic and less nucleophilic than $RO^{\ominus}$

Sol. **[1]**

RO^{Θ}	$R–S^{\Theta}$
size↓	size↑
B.S.↑	B.S.↓
N.S.↓	N.S.↑

4. The change in the optical rotation of freshly prepared solution of glucose is known as :
   (1) racemisation
   (2) specific rotation
   (3) mutarotation
   (4) tautomerism

Sol. **[3]**
Fact

5. The molality of a urea solution in which 0.0100 g of urea, $[(NH_2)_2CO]$ is added to 0.3000 dm^3 of water at STP is :
   (1) $5.55 \times 10^{-4}\ m$
   (2) $33.3\ m$
   (3) $3.33 \times 10^{-2}\ m$
   (4) $0.555\ m$

Sol. **[1]**

$$m = \frac{0.01}{60 \times 0.30} = \frac{1}{60 \times 30} = 5.55 \times 10^{-4}$$

$d_{water} = 1\ kg/dm^3$

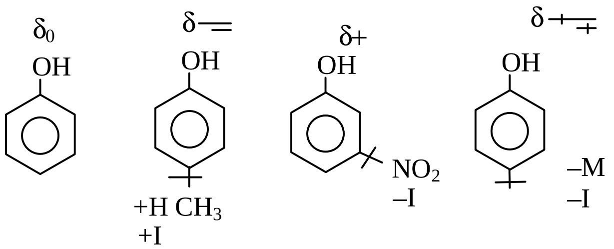
6. The molecular velocity of any gas is :
   (1) inversely proportional to absolute temperature
   (2) directly proportional to square of temperature
   (3) directly proportional to square root of temperature
   (4) inversely proportional to the square root of temperature

Sol. **[3]**

$V_{mp}, V_{rms}, V_{av.} \propto \sqrt{T}$

7. The correct order of acid strength of the following compounds :
   A. Phenol
   B. p-Cresol
   C. m-Nitrophenol
   D. p-Nitrophenol
   is :
   (1) D > C > A > B
   (2) B > D > A > C
   (3) A > B > D > C
   (4) C > B > A > D

Sol. **[1]**



(A) (B) (C) (D)

D > C > A > B

8. The value of enthalpy change (ΔH) for the reaction

$$C_2H_5OH_{(l)} + 3O_{2(g)} \rightarrow 2CO_{2(g)} + 3H_2O_{(l)}$$

at 27°C is –1366.5 kJ mol^{-1}. The value of internal energy change for the above reaction at this temperature will be :
   (1) – 1369.0 kJ
   (2) – 1364.0 kJ
   (3) – 1361.5 kJ
   (4) – 1371.5 kJ

Sol. **[2]**

$\Delta H = \Delta E + \Delta n_g RT$

$\Delta E = \Delta H - \Delta n_g RT$

$= -1366.5 - (-1)\ (8.3 \times 10^{-3}) \times 300$

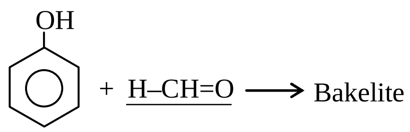
$= -1366.5 + 2.490$

$= -1364.01$

$\simeq -1364$ kJ

9. Thermosetting polymer, Bakelite is formed by the reaction of phenol with :
   (1) CH_3CHO
   (2) HCHO
   (3) HCOOH
   (4) CH_3CH_2CHO

Sol. **[2]**



10. Ozonolysis of an organic compound 'A' produces acetone and propionaldehyde in equimolar mixture. Identify 'A' from the following compounds :

(1) 1 - Pentene

(2) 2 - Pentene

(3) 2 - Methyl - 2 - pentene

(4) 2 - Methyl - 1 - pentene

Sol. **[3]**

$CH_3-\underset{\underset{O}{||}}{C}-CH_3 \qquad CH_3-CH_2-CH=O$

$CH_3-\overset{\overset{CH_3}{|}}{C}=CH-CH_2-CH_3$

2-methyl-2-pentene

11. Consider the reaction :

$4NO_{2(g)} + O_{2(g)} \rightarrow 2N_2O_{5(g)}, \Delta_rH = -111 kJ.$

If $N_2O_{5(s)}$ is formed instead of $N_2O_{5(g)}$ in the above reaction, the Δ_rH value will be : (given, ΔH of sublimation for N_2O_5 is 54 kJ mol^{-1})

(1) +54 kJ

(2) +219 kJ

(3) −219 kJ

(4) −165 kJ

Sol. **[3]**

$4NO_2 + O_2 \rightarrow \underset{(g)}{2N_2O_5} \quad \Delta H = -111 \quad ...(1)$

$4NO_2 + O_2 \rightarrow \underset{(s)}{2N_2O_5} \quad \Delta H = ? \quad ...(2)$

$\underset{(s)}{2N_2O_5} \rightarrow \underset{(g)}{2N_2O_5} \quad \Delta H = 54 \times 2 \quad ...(3)$

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(1) – (3)

$\Delta H = -111 - 108 = -219$ kJ

12. An acid HA ionises as

$HA \rightleftharpoons H^+ + A^-$

The pH of 1.0 M solution is 5. Its dissociation constant would be :

(1) 5

(2) 5×10^{-8}

(3) 1×10^{-5}

(4) 1×10^{-10}

Sol. **[4]**

$pH = \frac{1}{2} pk_a - \frac{1}{2} \log C$

$5 = \frac{1}{2} pk_a - \frac{1}{2} \log 1$

$pk_a = 10$

$k_a = 10^{-10}$

13. The correct order of electron gain enthalpy with negative sign of F, Cl, Br and I, having atomic number 9, 17, 35 and 53 respectively, is :

(1) F > Cl > Br > I

(2) Cl > F > Br > I

(3) Br > Cl > I > F

(4) I > Br > Cl > F

Sol. **[2]**

Cl > F > Br > I

Order of electron gain enthalpy.

14. The frequency of light emitted for the transition n = 4 to n = 2 of He^+ is equal to the transition in H atom corresponding to which of the following ?

(1) n = 2 to n = 1

(2) n = 3 to n = 2

(3) n = 4 to n = 3

(4) n = 3 to n = 1

Sol. **[1]**

$\bar{\nu}_H = \bar{\nu}_{He^+}$

$$\left[RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\right]_H = \left[RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\right]_{He^+}$$

$$\frac{1}{n_1^2}-\frac{1}{n_2^2} = \left[4\left(\frac{1}{4}-\frac{1}{16}\right)\right]$$

$$\frac{1}{n_1^2}-\frac{1}{n_2^2} = \frac{1}{1}-\frac{1}{4}$$

$n_1 = 1$

$n_2 = 2$

15. A 5% solution of cane sugar (molar mass 342) is isotonic with 1% of a solution of an unknown solute. The molar mass of unknown solute in g/mol is :

(1) 171.2

(2) 68.4

(3) 34.2

(4) 136.2

Sol. **[2]**

$\pi_1 = \pi_2$

$c_1 = c_2$

$$\frac{5}{342} = \frac{1}{M}$$

$$M = \frac{342}{5} = 68.4$$

16. In view of the signs of $\Delta_r G^\circ$ for the following reactions :

$PbO_2 + Pb \rightarrow 2\ PbO,\ \Delta_r G^\circ < 0$

$SnO_2 + Sn \rightarrow 2\ SnO,\ \Delta_r G^\circ > 0,$

which oxidation states are more characteristic for lead and tin ?

(1) For lead +2, for tin +2

(2) For lead +4, for tin +4

(3) For lead +2, for tin +4

(4) For lead +4, for tin +2

Sol. **[3]**

$\Delta G^o < 0$ spontaneous reaction

so $\boxed{Pb^{+4} \rightarrow Pb^{+2}}$

$Pb^{+2} > Pb^{+4}$ (stability)

$\Delta G^o > 0$ non-spontaneous reaction

$\boxed{Sn^{+4} \rightarrow Sn^{+2}}$

$Sn^{+4} > Sn^{+2}$ (stability)

Pb^{+2}, Sn^{+4}

17. The K_{sp} for $Cr(OH)_3$ is 1.6×10^{-30}. The molar solubility of this compound in water is :

(1) $\sqrt[4]{1.6 \times 10^{-30}}$

(2) $\sqrt[4]{1.6 \times 10^{-30}/27}$

(3) $1.6 \times 10^{-30}/27$

(4) $\sqrt[2]{1.6 \times 10^{-30}}$

Sol. **[2]**

$K_{sp} = s(3s)^3 = 27s^4 = 1.6 \times 10^{-30}$

$$s = \sqrt[4]{\frac{1.6\times10^{-30}}{27}}$$

18. The products obtained on heating $LiNO_3$ will be :

(1) $Li_2O + NO_2 + O_2$

(2) $Li_3N + O_2$

(3) $Li_2O + NO + O_2$

(4) $LiNO_2 + O_2$

Sol. **[1]**

$$LiNO_3 \xrightarrow{\Delta} Li_2O + NO_2 + O_2$$

19. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.3 S m^{-1}. If resistance of the 0.4 M solution of the same electrolyte is 260 Ω, its molar conductivity is :

(1) 6.25×10^{-4} S m^2 mol^{-1}

(2) 625×10^{-4} S m^2 mol^{-1}

(3) 62.5 S m^2 mol^{-1}

(4) 6250 S m^2 mol^{-1}

Sol. **[1]**

$$\lambda_m = \kappa \times \frac{1000}{M} (\kappa = 1.3 \text{ S m}^{-1} = 1.3 \times 10^{-2} \text{ S cm}^{-1})$$

$$= \frac{1}{\kappa} \times \frac{\ell}{a} \times \frac{1000}{M}$$

$$\kappa = \frac{1}{R} \times \frac{\ell}{a}$$

$$\frac{\ell}{a} = \kappa R = 1.3 \times 10^{-2} \times 50 = 65.0 \times 10^{-2} \text{ cm}^{-1}$$

$$\lambda_m = \frac{1}{260} \times 65 \times 10^{-2} \times \frac{1000}{0.4}$$

$$= 6.25 \text{ S mol}^{-1} \times \text{cm}^2$$

$$= 6.25 \times \frac{1}{100} \times \frac{1}{100} \text{ m}^2$$

$$6.25 \times 10^{-4} \text{ m}^2 \text{ S mol}^{-1}$$

20. Among the ligands NH_3, en, CN^- and CO the correct order of their increasing field strength, is :

(1) NH_3 < en < CN^- < CO

(2) CN^- < NH_3 < CO < en

(3) en < CN^- < NH_3 < CO

(4) CO < NH_3 < en < CN^-

Sol. **[1]**

According to spectrochemical series increasing order of strength of ligands is

NH_3 < en < CN^- < CO

21. Consider the following reaction :

$C_2H_5OH + H_2SO_4 \rightarrow$ Product

Among the following, which one cannot be formed as a product under any conditions ?

(1) Ethylene

(2) Acetylene

(3) Diethyl ether

(4) Ethyl-hydrogen sulphate

Sol. **[2]**

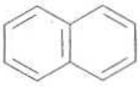
$$C_2H_5OH \xrightarrow[110^\circ C]{H_2SO_4} C_2H_5\text{–O–}SO_3H$$

$$C_2H_5OH \xrightarrow{140^\circ C} C_2H_5\text{–O–}C_2H_5$$

$$C_2H_5OH \xrightarrow{170^\circ C} CH_2{=}CH_2$$

22. The non aromatic compound among the following is :

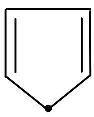
(1) 

(2) 

(3) 

(4) 

Sol. **[4]**



There is no continuous resonance thus it is non-aromatic.

23. The number of types of bonds between two carbon atoms in calcium carbide is :

(1) One sigma, one pi

(2) Two sigma, one pi

(3) Two sigma, two pi

(4) One sigma, two pi

Sol. **[4]**

Calcium carbide is a ionic carbide

$\Rightarrow Ca^{2+} [C\equiv C]^{2-}$

It contains 1σ and 2π bonds.

24. A reactant (A) forms two products :

$A \xrightarrow{k_1} B$, Activation Energy Ea_1

$A \xrightarrow{k_2} C$, Activation Energy Ea_2

If $Ea_2 = 2\ Ea_1$, then k_1 and k_2 are related as :

(1) $k_2 = k_1 e^{Ea_1/RT}$

(2) $k_2 = k_1 e^{Ea_2/RT}$

(3) $k_1 = Ak_2 e^{Ea_1/RT}$

(4) $k_1 = 2k_2 e^{Ea_2/RT}$

Sol. **[3]**

$$k_1 = A_1 e^{\frac{-Ea_1}{RT}}$$

$$k_2 = A_2 e^{\frac{-Ea_2}{RT}}$$

$$\frac{k_2}{k_1} = \frac{A_2}{A_1} e^{-\frac{Ea_2}{RT} + \frac{Ea_1}{RT}}$$

$$\frac{k_2}{k_1} = Ae^{\frac{Ea_1 - Ea_2}{RT}} \qquad (Ea_2 = 2Ea_1)$$

$$\frac{k_2}{k_1} = Ae^{-\frac{Ea_1}{RT}}$$

$$k_2 = Ak_1\ e^{-\frac{Ea_1}{RT}}$$

$$k_1 = Ak_2\ e^{+\frac{Ea_1}{RT}}$$

25. Copper crystallises in fcc lattice with a unit cell edge of 361 pm. The radius of copper atom is :

(1) 108 pm

(2) 128 pm

(3) 157 pm

(4) 181 pm

Sol. **[2]**

$4r = \sqrt{2}a$

$r = \frac{\sqrt{2}}{4} \times 361$

$= 127.61 \simeq 128$ pm

26. The mass of potassium dichromate crystals required to oxidise 750 cm^3 of 0.6 M Mohr's salt solution is : (Given molar mass : potassium dichromate = 294, Mohr's salt = 392)

(1) 0.45 g

(2) 22.05 g

(3) 2.2 g

(4) 0.49 g

Sol. **[2]**

gm eqv. of Mohr's salt = gm. Eqv. of $K_2Cr_2O_7$

$$\frac{NV(ml)}{1000} = \frac{gm}{E.w}$$

$$\frac{(0.6 \times 1 \times 750)}{1000} = \frac{x}{\frac{294}{6}}$$

$$x = \frac{0.6 \times 750}{1000} \times \frac{294}{6} = \frac{75 \times 294}{1000}$$

= 22.05 gm

27. What is the best description of the change that occurs when $Na_2O(s)$ is dissolved in water ?

(1) Oxide ion accepts sharing in a pair of electrons

(2) Oxide ion donates a pair of electrons

(3) Oxidation number of oxygen increases

(4) Oxidation number of sodium decreases

Sol. **[1]**

$Na_2O + H_2O \longrightarrow 2NaOH$

28. Which of the following has maximum number of lone pairs associated with Xe ?

(1) XeF_4

(2) XeF_6

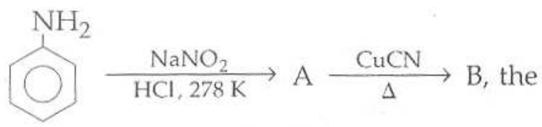
(3) XeF_2

(4) XeO_3

Sol. **[3]**

XeF_2 has maximum number of lone pairs (3 lone pairs)

29. In the chemical reactions



compounds A and B respectively are :

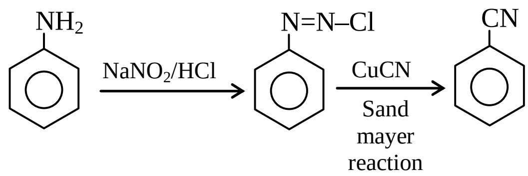
(1) Benzene diazonium chloride and benzonitrile

(2) Nitrobenzene and chlorobenzene

(3) Phenol and bromobenzene

(4) Fluorobenzene and phenol

Sol. **[1]**



30. Which one of the following complex ions has geometrical isomers ?

(1) $[Ni(NH_3)_5Br]^+$

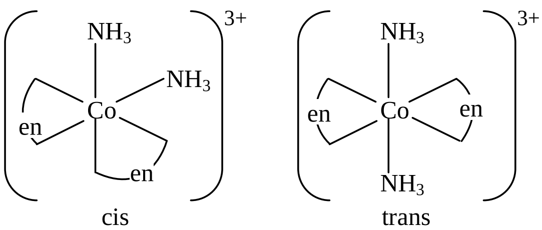
(2) $[Co(NH_3)_2(en)_2]^{3+}$

(3) $[Cr(NH_3)_4(en)]^{3+}$

(4) $[Co(en)_3]^{3+}$

(en = ethylenediamine)

Sol. **[2]**



It has geometrical isomers.

PART B – PHYSICS

31. At time $t = 0s$ a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :

(1) constant

(2) t

(3) $1/\sqrt{t}$

(4) $\sqrt{t}$

Ans.[3]

Sol. K.E. $\propto$ t i.e., $\frac{1}{2}mv^2 \propto t$

$v \propto \sqrt{t}$

$a = \frac{dv}{dt} \propto \frac{1}{2\sqrt{t}}$, $F = ma$

$F \propto \frac{1}{\sqrt{t}}$

32. At two points P and Q on a screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$ respectively. The ratio of intensities at P and Q will be :

(1) 2 : 1

(2) $\sqrt{2}$: 1

(3) 4 : 1

(4) 3 : 2

Ans.[1]

Sol. $I = 4I_0 \cos^2\phi/2$

$I_P = 4I_0 \cos^2 0 = 4I_0$ ....(1)

$I_Q = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{4I_0}{2} = 2I_0$ ....(2)

$\frac{I_P}{I_Q} = 2$

33. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is :

(1) $\sqrt{\frac{Gm}{4R}}$

(2) $\sqrt{\frac{Gm}{3R}}$

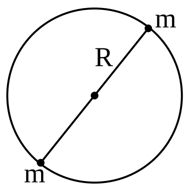
(3) $\sqrt{\frac{Gm}{2R}}$

(4) $\sqrt{\frac{Gm}{R}}$

Ans.[1]

Sol. $\frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$

$v = \sqrt{\frac{GM}{4R}}$



34. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$ then the ratio F_1/F_2 is :

(1) 1

(2) 2

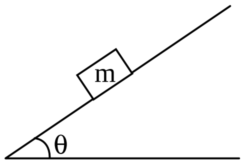
(3) 3

(4) 4

Ans.[3]

Sol. $F_1 = mg\sin\theta + \mu mg\cos\theta$ ...(i)

$F_2 = mg\sin\theta - \mu mg\cos\theta$ ...(ii)



$$\frac{F_1}{F_2} = \frac{\tan\theta + 1}{\tan\theta - 1} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3}{1}$$

35. If 400 Ω of resistance is made by adding four 100 Ω resistances of tolerance 5%, then the tolerance of the combination is :

(1) 5%

(2) 10%

(3) 15%

(4) 20%

Ans.[1]

Sol. $R = R_1 + R_2 + R_3 + R_4 = 400\ \Omega$

$$\%\ \frac{\Delta R}{R} = \frac{\Delta[R_1 + R_2 + R_3 + R_4]}{400} \times 100$$

$$= \frac{20}{400} \times 100 = 5\ \%$$

36. An electric charge $+q$ moves with velocity $\vec{V} = 3\hat{i} + 4\hat{j} + \hat{k}$, in an electromagnetic field given by :

$\vec{E} = 3\hat{i} + \hat{j} + 2\hat{k}$ and

$\vec{B} = \hat{i} + \hat{j} - 3\hat{k}$. The y - component of the force experienced by $+q$ is :

(1) 11 q

(2) 5 q

(3) 3 q

(4) 2 q

Ans.[1]

Sol. Magnetic force $\vec{F} = q(\vec{V} \times \vec{B}) = 10q\,\hat{j}$

and electric force in direction of y-axis $\vec{F} = q\,\hat{j}$

$F_{Net} = 11q$ in direction of y-axis.

37. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} ohm metre and 8×10^{-7} m^2 respectively. The potential gradient will be equal to :

(1) 1 V/m

(2) 0.5 V/m

(3) 0.1 V/m

(4) 0.2 V/m

Ans.[3]

Sol. $\phi = \frac{V_{AB}}{L} = \frac{i \times \rho}{A}$ = 0.1 volt/meter

38. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :

(1) zero

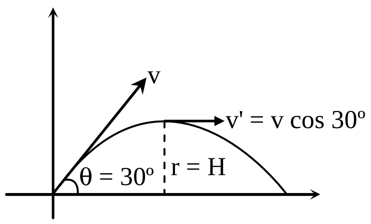
(2) $\dfrac{m v^3}{\sqrt{2}\ g}$

(3) $\dfrac{\sqrt{3}}{16} \dfrac{m v^3}{g}$

(4) $\dfrac{\sqrt{3}}{2} \dfrac{m v^2}{g}$

Ans.[3]

Sol.



$L = mv'r$

$= m(v\cos 30^\circ)\left(\dfrac{v^2 \sin^2 30^\circ}{2g}\right)$

$= \dfrac{mv^3}{2g} \dfrac{\sqrt{3}}{2} \times \dfrac{1}{4}$

$= \dfrac{\sqrt{3} mv^3}{16}$

39. The specific heat capacity of a metal at low temperature (T) is given as :

$$C_p(\text{kJK}^{-1}\text{kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is :

(1) greater than 0.148 kJ

(2) between 0.148 kJ and 0.028 kJ

(3) less than 0.028 kJ

(4) equal to 0.002 kJ

Ans.[2]

Sol. $\Delta Q = mC_p dT$

$= (100 \times 10^{-3}) \times \left[32 \times \left(\dfrac{T}{400}\right)^3\right][100 - 0]$

$= 0.002$ KJ

$\beta = \dfrac{T_2}{T_1 - T_2} = \dfrac{Q_2}{W}$; $T_2 = 300$ K, $Q_2 = 0.002$ KJ

If $T_1 = 20$ K, W = 0.148 KJ

$T_2 = 4$ K, W = 0.028 KJ

i.e., amount of work will be between 0.148 KJ to 0.028 KJ.

40. A wooden cube (density of wood 'd') of side 'l' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :

(1) $2\pi\sqrt{\frac{ld}{\rho g}}$

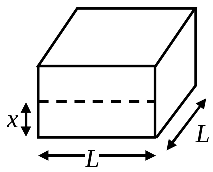
(2) $2\pi\sqrt{\frac{l\rho}{dg}}$

(3) $2\pi\sqrt{\frac{ld}{(\rho - d)g}}$

(4) $2\pi\sqrt{\frac{l\rho}{(\rho - d)g}}$

Ans.[1]

Sol.



Restoring force $F_R = (L^2x)\rho g$ ... (1)

In case of SHM $F_R = m\omega^2 x$ ...(2)

From equation (1) & (2)

$m\omega^2 x = L^2 x\rho g$

$(d)L^3\omega^2 = L^2\rho g$

$dL\omega^2 = \rho g$

$\omega = \sqrt{\frac{\rho g}{dL}}$

$T = 2\pi\sqrt{\frac{dL}{\rho g}}$

41. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :

(1) $\frac{P}{2}, \frac{T}{2}$

(2) P, T

(3) P, $\frac{T}{2}$

(4) $\frac{P}{2}$, T

Ans.[4]

Sol. It is free expansion of Ideal gas which is both adiabatic and isothermal process.

T $\rightarrow$ constant, so for the isothermal process

$P_1V_1 = (P_2V_2)$

$P_1V_1 = P_2(2V_1)$

$P_2 = \frac{P_1}{2}$

42. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2, then the ratio I_1/I_2 is :

(1) 2

(2) 1

(3) 0.5

(4) 4

Ans.[1]

Sol. $I_1 = 4I_0\cos^2\phi/2 = 4I_0$ …. (1)

$I_2 = I_0 + I_0 = 2I_0$ …. (2)

(for non coherent sources)

$$\frac{I_1}{I_2} = \frac{2}{1}$$

43. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a :

(1) NOT gate

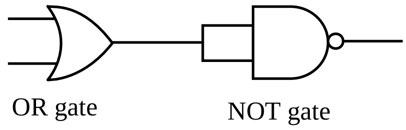
(2) NOR gate

(3) AND gate

(4) OR gate

Ans.[2]

Sol.



NOR gate

44. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is :

(1) zero

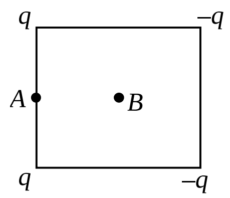
(2) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(3) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

(4) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

Ans.[4]

Sol. $W = q\,(\Delta V) = \Delta KE$



$$V_A = \frac{Kq}{a} + \frac{Kq}{a} - \frac{Kq}{\sqrt{2}a} - \frac{Kq}{\sqrt{2}a}$$

$V_B = 0$

$$W = \frac{2Kq}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

45. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on a (C – R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is :

(1) 10 second

(2) 5 second

(3) 2.5 second

(4) 20 second

Ans.[3]

Sol. $$\frac{0.693 \times R \times 2C = 10}{0.693 \times R \times \frac{C}{2} = T}$$

$$\Rightarrow \quad \frac{2/1}{1/2} = \frac{10}{T}$$

$$\Rightarrow \quad \frac{4}{1} = \frac{10}{T}$$

T = 2.5 sec

46. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is :

(1) $\left(1 + \frac{1}{\mu_1}\right)h_1 - \left(1 + \frac{1}{\mu_2}\right)h_2$

(2) $\left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$

(3) $\left(1 + \frac{1}{\mu_1}\right)h_2 - \left(1 + \frac{1}{\mu_2}\right)h_1$

(4) $\left(1 - \frac{1}{\mu_1}\right)h_2 + \left(1 - \frac{1}{\mu_2}\right)h_1$

Ans.[2]

Sol. Actual depth = $(h_1 + h_2)$

Apparent depth = $\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$

Shift $\Rightarrow (h_1 + h_2) - \left(\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}\right)$

$$\Rightarrow \quad h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

47. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear stress developed in it is :

(1) $Y/\alpha t$

(2) $Y\alpha t$

(3) $1/(Y\alpha t)$

(4) $\alpha t/Y$

Ans.[2]

Sol. $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{L\alpha\Delta t}{L} = \alpha t$$

$$\text{Stress} = Y\alpha t$$

48. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is :

(1) A wave travelling along $+x$ direction.

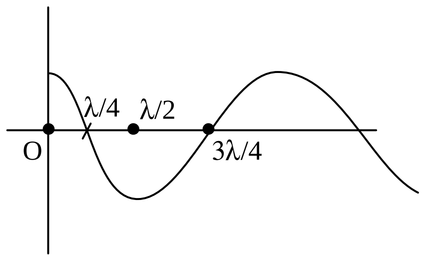
(2) A wave travelling along $-x$ direction.

(3) A standing wave having nodes at $x = \dfrac{n\lambda}{2}$; n = 0, 1, 2 ...

(4) A standing wave having nodes at $x = \left(n+\dfrac{1}{2}\right)\dfrac{\lambda}{2}$; n = 0, 1, 2 ...

Ans.[4]

Sol.



$Y_1 = A\sin(\omega t - kx)$

$Y_2 = A\sin(\omega t + kx)$

$\vec{Y}_R = \vec{Y}_1 + \vec{Y}_2 = 2A\cos kx \sin\omega t$

So, nodes will be at $\dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}$.....

49. A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed ω. The magnetic moment of the disk is :

(1) $\pi R^4\sigma\omega$

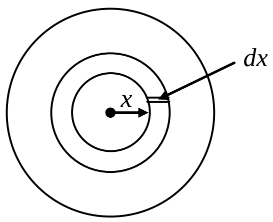
(2) $\dfrac{\pi R^4}{2}\sigma\omega$

(3) $\dfrac{\pi R^4}{4}\sigma\omega$

(4) $2\pi R^4\sigma\omega$

Ans.[3]

Sol.



$$M = i \times A = \frac{q}{T} \times \pi R^2$$

$$dM = \frac{dq}{T} \times \pi x^2$$

$$= \frac{\sigma \times dA}{T} \times \pi x^2$$

$$dM = \frac{\sigma \times 2\pi x\,dx \times \pi x^2}{2\pi}\omega$$

$$M = \sigma \times \omega \int x^3 dx$$

$$= \sigma \times \omega \times \left(\frac{x^4}{4}\right)_0^R = \frac{\sigma \times \omega R^4}{4}$$

50. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23 \times 10^{-6}/°C$) :

(1) 2.89 cc

(2) 9.28 cc

(3) 49.8 cc

(4) 28.9 cc

Ans.[4]

Sol. $\gamma = 3\alpha = 3 \times 23 \times 10^{-6} = 69 \times 10^{-6}/°C$

$$\left[\because \quad V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (10 \times 10^{-2})^3 = \frac{4}{3} \times \pi \times 10^{-3}\right]$$

Change in volume (ΔV)

$V' - V = V\gamma\Delta t$

$$= \left(\frac{4}{3}\pi R^3 \times 69 \times 10^{-6}\right) \times (100 - 0)$$

= 28.9 cc

51. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is :

(1) $4\pi r^2 T$

(2) $2\pi r^2 T$

(3) $2^{8/3}\pi r^2 T$

(4) $2^{5/3}\pi r^2 T$

Ans.[3]

Sol. Volume of big drop = total volume of small drops

$R = 2^{1/3} r$

Surface energy of small drops $U = 2T\,4\pi r^2$

Surface energy of big drop

$U' = T4\pi R^2$

$= T\,4\pi(2^{1/3})^2 r^2$

$= T\,4\pi(2)^{2/3}\,r^2$

$U' = T(2)^2 \pi\,2^{2/3}\,r^2$

$U' = T(2)^{8/3}\,\pi r^2$

52. If a ball of steel (density $\rho = 7.8\ g\ cm^{-3}$) attains a terminal velocity of $10\ cm\ s^{-1}$ when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($\rho = 1.2\ g\ cm^{-3}$, $\eta = 13.2$ Pa.s) would be, nearly :

(1) $6.25 \times 10^{-4}\ cm\ s^{-1}$

(2) $6.45 \times 10^{-4}\ cm\ s^{-1}$

(3) $1.5 \times 10^{-5}\ cm\ s^{-1}$

(4) $1.6 \times 10^{-5}\ cm\ s^{-1}$

Ans.[1]

Sol. $V = \frac{2r^2}{9\eta} g\{d_b - d_\ell\}$

$\frac{V_1}{V_2} = \frac{\eta_2}{\eta_1}\left\{\frac{d-d_1}{d-d_2}\right\}$

$\frac{10}{V} = \frac{13.2}{8.5\times10^{-4}}\left\{\frac{7.8-1}{7.8-1.2}\right\}$

$V = 6.25 \times 10^{-4}$ cm/s

53. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb/m^2. The instantaneous value of the e.m.f induced in the wire will be :

(1) 3 mV

(2) 4.5 mV

(3) 1.5 mV

(4) 6.0 mV

Ans.[1]

Sol. $e = B \times v \times \ell$

$= 0.30 \times 10^{-4} \times 5 \times 20$

= 3 millivolt

54. After absorbing a slowly moving neutron of mass m_N (momentum ~0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5\,m_1$ ($6\,m_1 = M + m_N$), respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, the de Broglie wavelength of the other nucleus will be :

(1) 5 λ

(2) $\lambda/5$

(3) λ

(4) 25 λ

Ans.[3]

Sol. Acceleration to momentum conservation, linear momentum of both point is same So wavelength is also same.

55. Which of the following four alternatives is *not* correct ?

We need modulation :

(1) to reduce the time lag between transmission and reception of the information signal

(2) to reduce the size of antenna

(3) to reduce the fractional band width, that is, the ratio of the signal band width to the centre frequency

(4) to increase the selectivity.

Ans.[1]

Sol. Time is not dependent on modulation.

56. If a spring of stiffness '*k*' is cut into two parts 'A' and 'B' of length $l_A : l_B = 2 : 3$, then the stiffness of spring 'A' is given by :

(1) $3k/5$

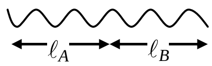
(2) $2k/5$

(3) k

(4) $5/2\ k$

Ans.[4]

Sol.



$\ell = \ell_A + \ell_B$

$\frac{\ell_A}{\ell_B} = \frac{2}{3}$

$K \propto \frac{1}{\ell}$ ... (1)

$K_A \propto \frac{1}{\ell_A}$ ... (2)

From (2)/(1)

$$\frac{K_A}{K} = \frac{\ell}{\ell_A}$$

$$K_A = \frac{K\{\ell_A + \ell_B\}}{\ell_A}$$

$$K_A = K\left\{1 + \frac{\ell_B}{\ell_A}\right\}$$

$$K_A = K\left\{1 + \frac{3}{2}\right\}$$

$$K_A = \frac{5K}{2}$$

57. Statement - 1 :

A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2 but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement - 2 :

To conserve energy and momentum in β-decay at least three particles must take part in the transformation.

(1) Statement-1 is correct but statement-2 is not correct.

(2) Statement –1 and statement–2 both are correct and statement-2 is the correct explanation of statement-1.

(3) Statement-1 is correct, statement-2 is correct and statement-2 is not the correct explanation of statement-1.

(4) Statement-1 is incorrect, statement-2 is correct.

Ans.[2]

Sol. Maximum energy = End point energy $= E_1 - E_2$

Three particles at least required

$$\underbrace{{}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0} + \bar{v}$$

$$\underbrace{{}_1p^1 \rightarrow {}_0n^1 + {}_{+1}e^0} + v$$

58. When monochromatic red light is used instead of blue light in a convex lens, its focal length will :

(1) increase

(2) decrease

(3) remain same

(4) does not depend on colour of light.

Ans.[1]

Sol. $\frac{1}{f} = (\mu - 1)(k)$

$f \propto \lambda$

$f_R > f_V$

so increases.

59. Statement - 1 :

On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.

Statement - 2 :

The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light.

(1) Statement - 1 is true, statement - 2 is false.

(2) Statement - 1 is true, statement - 2 is true, statement - 2 is the correct explanation of statement - 1.

(3) Statement - 1 is true, statement - 2 is true; statement - 2 is not the correct explanation of statement - 1.

(4) Statement - 1 is false, Statement - 2 is true.

Ans.[2]

Sol. Statement-I $\rightarrow$ Polarisation

Statement-II $\rightarrow$ Raylegh's criteria

Scattering $\propto \frac{1}{\lambda^4}$

So statement I and II both correct statement-II is the correct explanation of statement-I.

60. Statement - 1 :

Two longitudinal waves given by equations : $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statements - 2 :

Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

(1) Statement - 1 is true, statement - 2 is false.

(2) Statement - 1 is true, statement - 2 true; statement - 2 is the correct explanation of statement - 1

(3) Statement - 1 is true, statement - 2 is true ; statement - 2 is not correct explanation of statement - 1.

(4) Statement - 1 is false, statement - 2 is true.

Ans.[3]

Sol. $a_1 = 2a,\ a_2 = a$

$\omega_1 = \omega,\ \omega_2 = 2\omega$

$I \propto a^2n^2$ (for same medium)

So $I_1 = I_2$ statement-I (True)

Statement-II

$n \rightarrow$ constant

$I \propto a^2$

$I = ka^2$ (statement -II True)

But no correct explanation, because of frequency.

PART C – MATHEMATICS

61. Let f be a function defined by $f(x) = (x-1)^2 + 1,\ (x \geq 1)$.

Statement - 1 :

The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.

Statement - 2 :

f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}$, $x \geq 1$.

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol. **[2]**

$f(x) = (x-1)^2 + 1 = y$

$(x-1)^2 = y - 1$

$x - 1 = \pm\sqrt{y-1}$

$x = 1 \pm \sqrt{y-1}$

$f^{-1}(y) = 1 \pm \sqrt{y-1}$

$f^{-1}(y) = 1 + \sqrt{y-1} \qquad \because\ y \geq 1$

Statement 2 is true

$f(x) = f^{-1}(x)$

$(x-1)^2 + 1 = 1 + \sqrt{x-1}$

$(x-1)^2 = \sqrt{x-1}$

Solving these we get x = 1, 2

Statement –1 is true.

62. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to :

(1) 0

(2) $-H$

(3) H^2

(4) H

Sol. **[4]**

If H is scalar matrix

$H = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

$H^n = \begin{bmatrix} x^n & 0 \\ 0 & x^n \end{bmatrix}$

$\Rightarrow H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$

63. Let $[\cdot]$ denote the greatest integer function then the value of $\int_0^{1.5} x\left[x^2\right] dx$ is :

(1) 0

(2) $\frac{3}{2}$

(3) $\frac{3}{4}$

(4) $\frac{5}{4}$

Sol. **[3]**

$\int_0^{1.5} x[x^2]dx = \int_0^1 0\ dx + \int_1^{\sqrt{2}} x\ dx + \int_{\sqrt{2}}^{1.5} 2x\ dx$

$= 0 + \left(\frac{x^2}{2}\right)_1^{\sqrt{2}} + \left(x^2\right)_{\sqrt{2}}^{1.5}$

$= \left(1-\frac{1}{2}\right) + (2.25-2) = \frac{3}{4}$

64. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by :

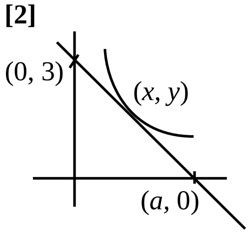
(1) $2y - 3x = 0$

(2) $y = \frac{6}{x}$

(3) $x^2 + y^2 = 13$

(4) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

Sol. **[2]**



$x = \frac{a}{2},\ y = \frac{b}{2}$

$\Rightarrow a = 2x \quad b = 2y$

& $\frac{dy}{dx} = \frac{-b}{a}$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x}$

$\Rightarrow \log y = -\log x + \log c \Rightarrow xy = c$

$\because$ it passes through (2, 3)

$\therefore c = 6$

$\therefore$ equation of curve is $xy = 6$

65. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively :

(1) 32, 2

(2) 32, 4

(3) 28, 2

(4) 28, 4

Sol. **[1]**

There is no change in mean deviation if each observation increased by a constant number while mean increased by that constant number

Hence

A.M. = 30 + 2 = 32

M.D. = 2

66. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval :

(1) $(0, \infty)$

(2) $[1, \infty)$

(3) $(-1, \infty)$

(4) $(-1, 1]$

Sol. **[2]**

$x + y = \pm a$	$x + y = \lvert a \rvert$
$ax - y = 1$	$ax - y = 1$
$(1 + a)x = \lvert a\rvert + 1$	$ax + ay = a\lvert a\rvert$
$x = \frac{\lvert a\rvert + 1}{1+a}$	$(a + 1)y = a\lvert a\rvert - 1$
	$y = \frac{a\lvert a\rvert - 1}{1+a}$

$\frac{|a|+1}{1+a} > 0$, $\frac{a|a|-1}{1+a} > 0$

$\Rightarrow a > 1$

67. If the vectors $p\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ $(p \neq q \neq r \neq 1)$ are coplanar, then the value of $pqr - (p+q+r)$ is :

(1) 2

(2) 0

(3) -1

(4) -2

Sol. **[4]**

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$p(qr - 1) - 1(r - 1) + 1(1 - q) = 0$

$pqr - p - r + 1 + 1 - q = 0$

$pqr - (p + q + r) + 2 = 0$

68. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is :

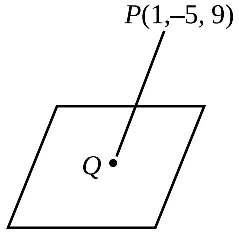
(1) $10\sqrt{3}$

(2) $5\sqrt{3}$

(3) $3\sqrt{10}$

(4) $3\sqrt{5}$

Sol. **[1]**



Equation of PQ

$\Rightarrow \frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

$\Rightarrow Q(\lambda + 1, \lambda - 5, \lambda + 9)$

lies on $x - y + z = 5$

$\lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\lambda = -10$

$Q = (-9, -15, -1)$

$PQ = \sqrt{(-9-1)^2 + (-5+15)^2 + (9+1)^2}$

$= 10\sqrt{3}$

69. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is :

(1) $\vec{a}$

(2) $\vec{c}$

(3) $\vec{0}$

(4) $\vec{a} + \vec{c}$

Sol. **[3]**

$\vec{a} + 3\vec{b} = \lambda\vec{c}$

$\vec{b} + 2\vec{c} = \mu\vec{a}$

$\vec{b} + 2\vec{c} = \mu(\lambda\vec{c} - 3\vec{b})$

$\vec{b} + 2\vec{c} = \lambda\mu\vec{c} - 3\mu\vec{b}$

$(1 - 3\mu)\vec{b} + (2 - \lambda\mu)\vec{c} = 0$

$1 + 3\mu = 0$ $\quad 2 - \lambda\mu = 0$

$\mu = -\frac{1}{3}$ $\quad \lambda\mu = 2$

$\lambda = -6$

$\vec{a} + 3\vec{b} + 6\vec{c} = 0$

70. If A(2, −3) and B(−2, 1) are two vertices of a triangle and third vertex moves on the line $2x+3y=9$, then the locus of the centroid of the triangle is :

(1) $x-y=1$

(2) $2x+3y=1$

(3) $2x+3y=3$

(4) $2x-3y=1$

Sol. **[2]**

A (2, –3) B(–2, 1)

Centroid, (h, k)

Let third vertex is (α, β)

$\frac{2-2+\alpha}{3}=h,\ \frac{-3+1+\beta}{3}=k$

$\Rightarrow \alpha = 3h$

$\Rightarrow \beta = 3k + 2$

$\because\ 2\alpha + 3\beta = 9$

$2(3h) + 3(3k + 2) = 9$

$6h + 9k + 6 = 9$

$6h + 9k = 3$

$2h + 3k = 1$

locus $2x + 3y = 1$

71. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points, then :

(1) $N \le 100$

(2) $100 < N \le 140$

(3) $140 < N \le 190$

(4) $N > 190$

Sol. **[1]**

Max. traingles = ${}^{10}C_3 - {}^{6}C_3$

= 120 – 20

= 100

$\therefore\ N \le 100$

72. Define F(x) as the product of two real functions $f_1(x)=x$, $x \in \mathbb{R}$, and

$$f_2(x) = \begin{cases} \sin\frac{1}{x}, & if\ x \ne 0 \\ 0, & if\ x = 0 \end{cases}$$

as follows :

$$F(x) = \begin{cases} f_1(x).f_2(x), & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Statement - 1 :

F(x) is continuous on $\mathbb{R}$.

Statement - 2 :

$f_1(x)$ and $f_2(x)$ are continuous on $\mathbb{R}$.

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1.

(3) Statement-1 is true, statement 2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol. **[3]**

$f_1(x) = x$ is continuous every R

& $f_2(x) = \begin{cases} \sin\frac{1}{x}; & x \neq 0 \\ 0 & x = 0 \end{cases}$ is discontinuous at $x = 0$

$\Rightarrow$ product of $f_1(x)$ and $f_2(x)$ is continuous

$\therefore$ St. (1) is correct and (2) is false

73. Statement - 1 :

For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7.

Statement - 2 :

For each natural number n, $n^7 - n$ is divisible by 7.

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is *not* a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol. **[1]**

By induction, to proof

$S(n) = n^7 - n$ is divisible by 7

Let n = 1

$S(n = 1) = 1^7 - 1 = 0 \div 7$,

Hence it is true for n = 1

Let the statement is true for n = k

$\Rightarrow P(n = k) \Rightarrow k^7 - k$ is divisible by 7.

$\Rightarrow k^7 - k = 7\lambda'$.

Now to proof the statement is true for n = k + 1

$\Rightarrow P(n = k + 1) \Rightarrow (k + 1)^7 - (k + 1)$ is divisisble by 7

$[1 + {}^7C_1\, k + {}^7C_2\, k^2 + ... + {}^7C_6\, k^6 + k^7] - k - 1$

$= 7.\lambda + (k^7 - k)$

$= 7\lambda + 7\lambda' = 7.(\lambda + \lambda')$

which is divisible by 7.

$\therefore$ Statement - 2 is true

Statement -1

$n^7 - n$ is divisible by 7

$\Rightarrow$ use it put n = n + 1

$(n + 1)^7 - (n + 1) = 7\lambda$ True

$\Rightarrow [(n + 1)^7 - n^7 - 1] + (n^7 - n) = 7\lambda$

= Stanement -1 + $7\lambda' = 7\lambda$

statement-1 is true and statement-2 is also correct explanation of Statement -1.

74. The equation of the circle passing through the points (1, 0) and (0, 1) and having the smallest radius is :

(1) $x^2 + y^2 - 2x - 2y + 1 = 0$

(2) $x^2 + y^2 - x - y = 0$

(3) $x^2 + y^2 + 2x + 2y - 7 = 0$

(4) $x^2 + y^2 + x + y - 2 = 0$

Sol. **[2]**

Points will be end of diameter

$(x - 1)(x - 0) + (y - 0)(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

75. The equation of the hyperbola whose foci are $(-2, 0)$ and $(2, 0)$ and eccentricity is 2 is given by :

(1) $x^2 - 3y^2 = 3$

(2) $3x^2 - y^2 = 3$

(3) $-x^2 + 3y^2 = 3$

(4) $-3x^2 + y^2 = 3$

Sol. **[2]**

ae = 2

e = 2

$a = 1$

$e^2 = 1 + \frac{b^2}{a^2}$

$4 = 1 + b^2$

$b^2 = 3$

$\frac{x^2}{1} - \frac{y^2}{3} = 1$

76. If the trivial solution is the only solution of the system of equations

$$x - ky + z = 0$$

$$kx + 3y - kz = 0$$

$$3x + y - z = 0$$

then the set of all values of k is :

(1) R − {2, −3}

(2) R − {2}

(3) R − {−3}

(4) {2, −3}

Sol. **[1]**

$\Delta \neq 0$ for trivial solution.

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$k^2 + k - 6 \neq 0$

$k \in R - \{-3, 2\}$

77. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are :

(1) 6, 1

(2) 4, 3

(3) −6, −1

(4) −4, −3

Sol. **[1]**

Let quadratic equation be

$x^2 + bx + c = 0$

$\alpha + \beta = 4 + 3 = 7 = -b$

$b = -7$

$\alpha . \beta = 3.2 = 6 = c$

$\therefore$ correct quadratic equation

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

$x = 1, 6$

78. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is :

(1) $\alpha - \beta$

(2) $\frac{\alpha - \beta}{100}$

(3) $\beta - \alpha$

(4) $\frac{\alpha - \beta}{200}$

Sol. **[2]**

$$\sum_{r=1}^{100} a_{2r} = \alpha$$

$$\sum_{r=1}^{100} a_{2r-1} + d = \alpha$$

$$\sum_{r=1}^{100} a_{2r-1} + \sum_{r=1}^{100} d = \alpha$$

$$\beta + 100d = \alpha$$

$$d = \frac{\alpha - \beta}{100}$$

79. Consider the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If $y(1) = 1$, then x is given by :

(1) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$

(2) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

(3) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$

(4) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

Sol. **[3]**

$$\frac{dy}{dx} = -\frac{y^2}{x - \frac{1}{y}}$$

$$\frac{dx}{dy} = -\frac{x - \frac{1}{y}}{y^2}$$

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

$$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

$$x.e^{-1/y} = \int \frac{1}{y^3}.e^{-1/y}dy$$

$$= -\int \underset{I}{t} \cdot \underset{II}{e^t}\, dt \qquad t = -\frac{1}{y}$$

$$= -\left\{te^t - \int 1.e^t dt\right\}$$

$$xe^{-1/y} = -te^t + e^t + c$$

$$xe^{-1/y} = +\frac{1}{y}e^{-1/y} + e^{-1/y} + c$$

$$x = \frac{1}{y} + 1 + ce^{1/y}$$

at $x = 1, y = 1$ $\quad 1 = 2 + ce^{1}$

$c = -1/e$

$$x = \frac{1}{y} + 1 - \frac{1}{e}e^{1/y}$$

80. Let $f : R \rightarrow [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists and $\lim_{x \to 5} \frac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$.

Then $\lim_{x \to 5} f(x)$ equals :

(1) 0

(2) 1

(3) 2

(4) 3

Sol. **[4]**

$$\lim_{x \to 5} \frac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$$

Limit can be zero only when

$\lim_{x \to 5} f(x) = 3$

81. **Statement - 1 :**

Determinant of a skew-symmetric matrix of order 3 is zero.

Statement - 2 :

For any matrix A, $\det(A^T) = \det(A)$ and $\det(-A) = -\det(A)$.

Where det(B) denotes the determinant of matrix B. Then :

(1) Both statements are true.

(2) Both statements are false.

(3) Statement-1 is false and statement-2 is true.

(4) Statement-1 is true and statement-2 is false.

Sol. **[4]**

Statement-1 is true

Statement-2 is false

$|A'| = |A| \Rightarrow$ true

$|-A| = (-1)^n |A| \Rightarrow$ false

Statement-2 is false

82. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are :

(1) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(2) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(3) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

Sol. **[4]**

$2 \sin 4\theta \cos 3\theta + \sin 4\theta = 0$

$\sin 4\theta (2 \cos 3\theta + 1) = 0$

$\sin 4\theta = 0$ $\qquad$ $\cos 3\theta = -\frac{1}{2}$

$\theta = \frac{n\pi}{4}$ $\qquad$ $3\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = (3n \pm 1)\frac{2\pi}{9}$

Put $n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$

83. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is :

(1) $\frac{32}{3}$

(2) $\frac{16}{3}$

(3) $\frac{8}{3}$

(4) 0

Sol. **[4]**

$y^2 = 4ax$ & $x^2 = 4by$

Area bounded by these curve is $\frac{16ab}{3}$

here $a = b = 1$

$\therefore A = \frac{16}{3}$

84. Let f be a function defined by

$$f(x)=\begin{cases}\dfrac{\tan x}{x}, & x\neq 0\\ 1, & x=0\end{cases}$$

Statement - 1 :

$x=0$ is point of minima of f.

Statement - 2 :

$f'(0)=0$.

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is *not* a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol. **[2]**
Since we know in neighbourhood of $x = 0$,
$\dfrac{\tan x}{x} > 1$ & $f(0) = 1$
$\therefore$ $x = 0$ is point of minima
& $f'(a)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

$\Rightarrow f'(0)=\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\dfrac{\frac{\tan}{x}-1}{x}$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{\tan x - x}{x^2}=0$

$\therefore$ St. (1) & (2) both correct but (2) is not correct explanation of (1)

85. The only statement among the followings that is a tautology is :

(1) $A\wedge(A\vee B)$

(2) $A\vee(A\wedge B)$

(3) $[A\wedge(A\to B)]\to B$

(4) $B\to[A\wedge(A\to B)]$

Sol. [3]

A	B	A ∧ B	A ∨ B	A→B	[A ∧ (A→B)]	S→B
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	T	F	T
F	F	F	F	T	F	T

Hence $[A\wedge(A\to B)]\to B$ is tautology

86. Let A, B, C be pairwise independent events with $P(C)>0$ and $P(A\cap B\cap C)=0$. Then $P(A^c\cap B^c|C)$ is equal to :

(1) $P(A)-P(B^c)$

(2) $P(A^c)+P(B^c)$

(3) $P(A^c)-P(B^c)$

(4) $P(A^c)-P(B)$

Sol. [4] $P\left(\dfrac{\bar{A}\cap\bar{B}}{C}\right)$

$=\dfrac{P(\bar{A}\cap\bar{B}\cap C)}{P(C)}$

$=\dfrac{P(C)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)}{P(C)}$

$= 1 - P(A) - P(B)$

$=P(\bar{A})-P(B)$ or $P(\bar{B})-P(A)$

87. Let for $a \neq a_1 \neq 0$,
$f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$
and $p(x) = f(x) - g(x)$.

If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is :

(1) 3

(2) 9

(3) 6

(4) 18

Sol. **[4]**

$0 = (a - a_1) - (b - b_1) + (c - c_1)$ …(1)

$2 = (a - a_1)\, 4 - (b - b_1)2 + (c - c_1)$ …(2)

$p(0) = c - c_1 = 2 \Rightarrow$ $\because$ (x = – 1) is only root

$\therefore D = 0$ of $p(x) = 0 \Rightarrow p(-2) = p(0) = c - c_1 = ?$

To find $(a - a_1)\, 4 + 2\,(b - b_1) + (c - c_1) = p(2)$

Let $a - a_1 = x$ and $b - b_1 = y$

$\therefore\ 2x - 2y + 4 = 0$ …(1)

$4x - 2y + 2 = 2$ …(2)

$\Rightarrow x = 2,\ y = 4$

$= 8 + 8 + 2 = 18$

88. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is :

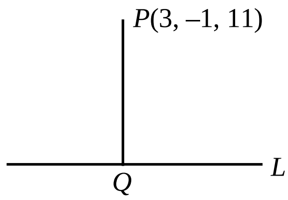
(1) $\sqrt{29}$

(2) $\sqrt{33}$

(3) $\sqrt{53}$

(4) $\sqrt{66}$

Sol. **[3]**



$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$Q = (2\lambda, 3\lambda + 2, 4\lambda + 3)$

D.r^s of PQ $= 2\lambda - 3, 3\lambda + 3, 4\lambda - 8$

$\because L \perp PQ$

$\Rightarrow 2(2\lambda - 3) + 3(3\lambda + 3) + 4\,(4\lambda - 8) = 0$

$\Rightarrow 4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$

$29\lambda = 29 \Rightarrow \lambda = 1$

$\Rightarrow Q \equiv (2, 5, 7)$

$PQ = \sqrt{(3-2)^2 + (-1-5)^2 + (11-7)^2}$

$= \sqrt{1 + 36 + 16}$

$= \sqrt{53}$

89. Consider the following relation R on the set of real square matrices of order 3.

$R = \{(A, B) \mid A = P^{-1}BP$ for some invertible matrix $P\}$.

Statement - 1 :

R is an equivalence relation.

Statement - 2 :

For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

Sol. **[4]**

Statement -1

$A = P^{-1} B P$

$PA = BP$

$P = \{(A, A) \mid PA \neq AP$ in general not reflexive. So not equivalence relation.

Statement-2

$(MN)^{-1} = N^{-1}M^{-1}$ True

from defination.

90. If function $f(x)$ is differentiable at $x = a$ then $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ is :

(1) $-a^2 f'(a)$

(2) $a f(a) - a^2 f'(a)$

(3) $2a f(a) - a^2 f'(a)$

(4) $2a f(a) + a^2 f'(a)$

Sol. **[3]**

$$\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a} \quad \left(\frac{0}{0} \textit{ form}\right)$$

Applying D.L.

$$\lim_{x \to a} 2xf(a) - a^2 f'(x)$$

$$= 2a\, f(a) - a^2 f'(a)$$