Paper-1

JEE Advanced, 2016

Part I: Physics

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.

2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.

3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.

4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.

5. Blank spaces are provided within this booklet for rough work.

6. Write your name and roll number in the space provided on the back cover of this booklet.

7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.

8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon – less copy of the ORS.

10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.

11. The ORS will be collected by the invigilator at the end of the examination.

12. You will be allowed to take away the Candidate's Sheet at the end of the examination.

13. Do not tamper with of mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.

16. Darken the bubble ⬭ completely.

17. The correct way of darkening a bubble is as: ⬬

18. The ORS is machine – gradable. Ensure that the bubbles are darkened in the correct way.

19. Darken the bubbles only if you are sure of the answer. There is no way to erase or "un–darken" a darkened bubble.

PART-I : PHYSICS

SECTION–1 : (Maximum Marks : 15)

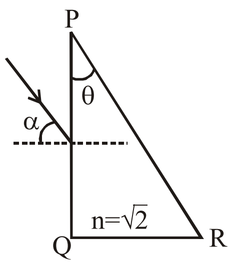
- This section contains **Five** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases

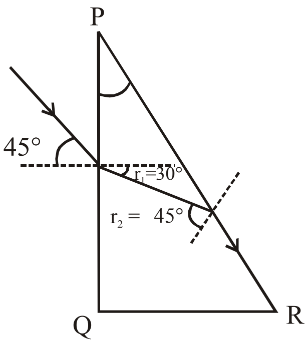
1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is :



(A) 15° (B) 22.5° (C) 30° (D) 45°

Ans. (A)

Sol.



$1 \sin 45° = \sqrt{2} \sin r_1$

$r_2 - r_1 = \theta$

$\theta = 45° - 30°$

$\Rightarrow \theta = 15°$

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

$\lambda(\mu m)$	$V_0(\text{Volt})$
0.3	2.0
0.4	1.0
0.5	0.4

Given that c = 3×10^8 ms^{-1} and e = 1.6×10^{-19}C, Planck's constant (in units of J s) found from such an experiment is :

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Ans. (B)

Sol. $KE_{max} = \frac{hC}{\lambda} - \phi$

$$eV_s = \frac{hC}{\lambda} - \phi$$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \phi \quad \text{... (i)}$$

$$1.6 \times 10^{-19} \times 1 = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - \phi \quad \text{... (ii)}$$

From (ii) $\phi = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - 1.6 \times 10^{-19}$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} + 1.6 \times 10^{-19}$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}}\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{h \times 3 \times 10^8}{10^{-7}}\left[\frac{4-3}{12}\right]$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}} \times \frac{1}{12}$$

$$1.6 \times 4 \times \frac{10^{-19} \times 10^{-7}}{10^8} = h$$

6.4×10^{-34} Js = h

3. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are:

($g = 10\ ms^{-2}$)

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

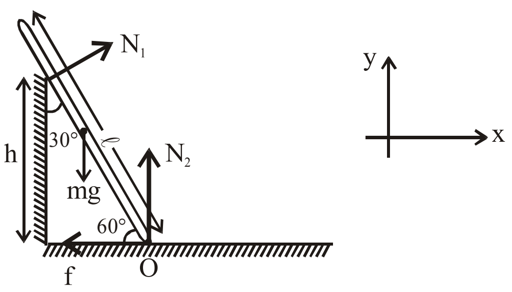
(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

Ans. (D)

Sol.



Force equation in x-direction,

$N_1 \cos 30° - f = 0$ ... (i)

Force equation in y-direction,

$N_1 \sin 30° + N_2 - mg = 0$ ... (ii)

Torque equation about O,

$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0$ ... (iii)

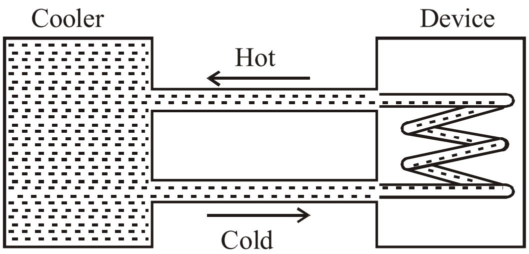
Also, given $N_1 = N_2$ ... (iv)

[Note taking reaction from floor as normal reaction only]

solving (i), (ii), (iii) & (iv) we have

$\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$ & $f = \frac{16\sqrt{3}}{3}$

4. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is :



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

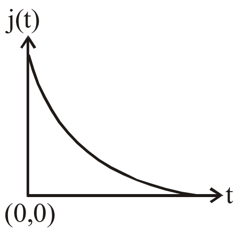
Ans. (B)

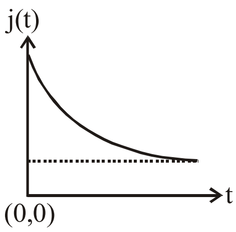
Sol. $3000 - P = (120 \times 1)(4.2 \times 10^3)\dfrac{dT}{dt}$

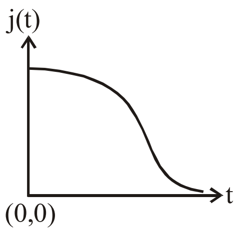
$$\frac{dT}{dt} = \frac{20}{60 \times 60 \times 3}$$

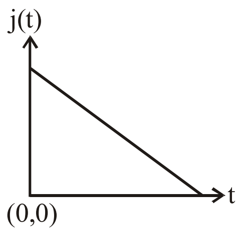
P = 2067 W

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

(A) 

(B) 

(C) 

(D) 

Ans. (A)

Sol. This is the problem of RC circuit where the product RC is a constant.

So due to leakage current, charge & current density will exponentially decay & will become zero at infinite time. So correct answer is (A)

for any small element

Resistance $R = \frac{dr}{\sigma(2\pi r\ell)}$

Capacitance $C = \frac{\in 2\pi r\ell}{dr}$

Product $R \times C = \frac{\in}{\sigma}$ = constant

$q = q_0 e^{-\left(\frac{t\sigma}{\in}\right)}$

$I = \frac{dq}{dt} = \frac{q_0\sigma}{\in} e^{-\left(\frac{t\sigma}{\in}\right)}$

Current density $= \frac{I}{A} = \frac{q_0 \frac{\sigma}{\in} e^{-\frac{t\sigma}{\in}}}{2\pi r\ell}$

$j \propto e^{-\frac{t\sigma}{\in}}$

SECTION–2 : (Maximum Marks : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.

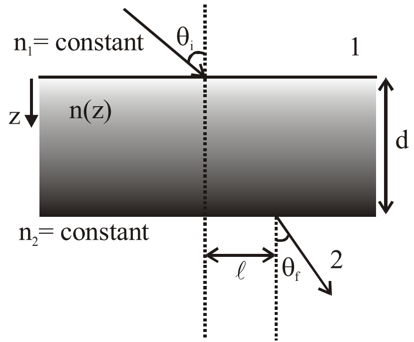
  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –2 In all other cases.
- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened

6. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement ℓ.

Which of the following statement(s) is(are) true ?



(A) ℓ is independent of n_2

(B) ℓ is dependent on n(z)

(C) $n_1\sin\theta_i = (n_2 - n_1)\sin\theta_f$

(D) $n_1\sin\theta_i = n_2\sin\theta_f$

Ans. **(A,B,D)**

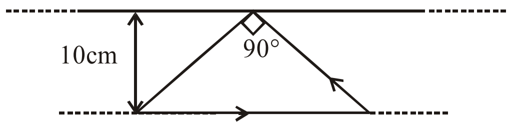
Sol. For parallel slab

$n_1 \sin\theta_i = n_2\sin\theta_f$

And ℓ depends on refractive angle in slab

$\therefore$ ℓ depends on refractive index of slab and independent of n_2

7. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true?



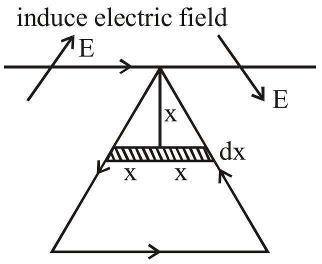
(A) The induced current in the wire is in opposite direction to the current along the hypotenuse.

(B) There is a repulsive force between the wire and the loop

(C) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(D) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

Ans. **(B,D)**

Sol. 

by direction of induced electric field, current in wire is in same direction of current along the hypotenuse.

Flux through triangle if wire have current i $= \int_0^{0.1}\left(\frac{\mu_0 i}{2\pi x}\right)(2x dx) = \frac{\mu_0 i}{10\pi}$

$\Rightarrow$ Mutual inductance = $\frac{\mu_0}{10\pi}$

Induced emf in wire = $\frac{\mu_0}{10\pi}\frac{di}{dt} = \frac{\mu_0}{10\pi}\times 10 = \frac{\mu_0}{\pi}$

8. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

(A) The refractive index of the lens is 2.5

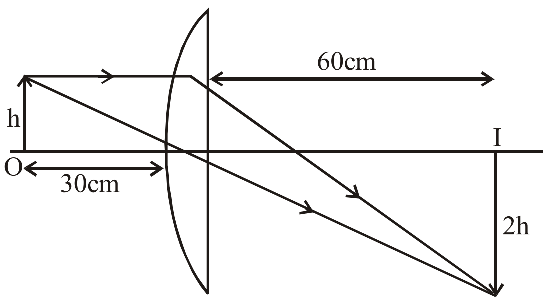
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm.

Ans. (A,D)

Sol. For lens

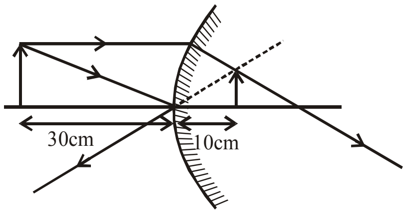


$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f} \Rightarrow f = 20\text{cm} \qquad \text{... (i)}$$

$$\text{Also } \frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(n-1)}{R} \qquad \text{... (ii)}$$

For reflection from convex mirror (curved surface)



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{1}{f}=\frac{2}{R} \qquad \text{... (iii)}$$

R= 30 cm

from (i), (ii) & (iii)

n = 2.5,

faint image erect & virtual

9. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

Ans. (C,D)

Sol. Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as whole increases with time.

Overall resistance increases hence power decreases. At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

10. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant k_B, the absolute temperature T, the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles, Which of the following expressions(s) for ℓ is(are) dimensionally correct?

(A) $\ell=\sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $\ell=\sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell=\sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $\ell=\sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Ans. (B,D)

Sol. We know,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \Rightarrow \frac{q^2}{\varepsilon_0} = (Fr^2)4\pi$$

So, dimension

$$\frac{q^2}{\varepsilon_0} = \dim(Fr^2) = MLT^{-2} \times L^2 = ML^3T^{-2}$$

Similarly; $E = \frac{3}{2}K_BT \Rightarrow \dim(K_BT) = \dim(\text{Energy}) = ML^2T^{-2}$

(A) $\sqrt{\frac{nq^2}{\varepsilon k_B T}} = \sqrt{\frac{L^{-3} \times ML^3T^{-2}}{ML^2T^{-2}}} = \frac{1}{L}$

(B) $\sqrt{\frac{(E) \times vol}{Fr^2}} = \sqrt{\frac{ML^2T^{-2} \times L^3}{MLT^{-2} \times L^2}} = L$

(C) $\sqrt{\frac{Fr^2(vol)^{2/3}}{(K\varepsilon)}} = \sqrt{\frac{MLT^{-2} \times L^2 \times L^2}{ML^2T^{-2}}} = \sqrt{L^3} = L^{3/2}$

(D) $\sqrt{\frac{Fr^2(vol)^{1/3}}{\text{Energy}}} = \sqrt{\frac{MLT^{-2}L^2 \times L}{ML^2T^{-2}}} = L$

[$\therefore$ dimension n = $\dim\left(\frac{1}{vol}\right) = L^{-3}$]

11. Highly excited states for hydrogen like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is (are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive oribitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

Ans. (A, B, D)

Sol. As radius $r \propto \frac{n^2}{z}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{z}\right)^2 - \left(\frac{n}{z}\right)^2}{\left(\frac{n}{z}\right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

as energy $E \propto \frac{z^2}{n^2}$

$$\Rightarrow \frac{\Delta E}{E} = \frac{\frac{z^2}{n^2} - \frac{z^2}{(n+1)^2}}{\frac{z^2}{(n+1)^2}} = \frac{(n+1)^2 - n^2}{n^2.(n+1)^2}.(n+1)^2$$

$$\Rightarrow \frac{\Delta E}{E} = \frac{2n+1}{n^2} \simeq \frac{2n}{n^2} \propto \frac{1}{n}$$

as angular momentum L = $\frac{nh}{2\pi}$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\frac{(n+1)h}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

12. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$, where $\alpha = \frac{10}{3} ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j}) ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right)\hat{k}$ Nms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}$ Nm

Ans. (A, B, D)

Sol. $\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2\hat{i} + 2\beta t\hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\alpha t\hat{i} + 2\beta\hat{j}$$

At t = 1

(A) $\vec{v} = 3 \times \frac{10}{3} \times 1\hat{i} + 2 \times J \times 1\hat{j}$

$= 10\hat{i} + 10\hat{j}$

(B) $\vec{L} = \vec{r} \times \vec{p}$

$= \left(\frac{10}{3} \times 1\hat{i} + 5 \times 1\hat{j}\right) \times 0.1 \ \left(10\hat{i} + 10\hat{j}\right)$

$= -\frac{5}{3}\hat{k}$

(C) $\vec{F} = m \times \left(6 \times \frac{10}{3} \times 1\hat{i} + 2 \times 5\hat{j}\right) = 2\hat{i} + \hat{j}$

(D) $\vec{\tau} = r \times \vec{F}$

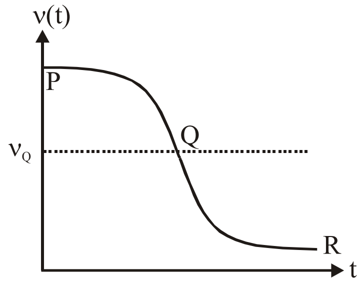
$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(2\hat{i} + \hat{j}\right)$

$= +\frac{10}{3}\hat{k} + 10\left(-\hat{k}\right)$

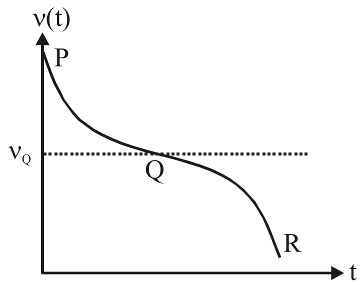
$= -\frac{20}{3}\hat{k}$

13. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person ?

(A) The plot below represents schematically the variation of beat frequency with time



(B) The plot below represents schematically the variations of beat frequency with time

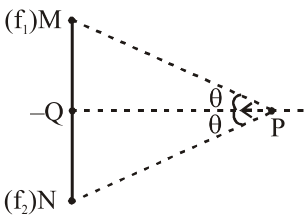


(C) The rate of change in beat frequency is maximum when the car passes through Q

(D) $\nu_P + \nu_R = 2\nu_Q$

Ans. (A, C, D)

Sol. $f_M = \frac{C + V\cos\theta}{C} f_1$



$f_N = \frac{C + V\cos\theta}{C} f_2$

$\Delta f = f_N - f_M$

$= \frac{C + V\cos\theta}{C}(f_2 - f_1)$

$$\frac{d(\Delta f)}{dt} = -\frac{V}{C}(f_2 - f_1)\sin\theta\frac{d\theta}{dt}$$

$\therefore$ & $\frac{d(\Delta f)}{dt}$ is maximum when $\theta = 90°$

[$\therefore$ C is correct]

$$\nu_P = \left(1+\frac{V}{C}\cos\theta\right)\Delta f$$

$$\nu_Q = \Delta f$$

$$\nu_R = \left(1-\frac{V}{C}\cos\theta\right)\Delta f$$

$$\therefore\ \nu_P + \nu_r = 2\nu_Q$$

SECTION–3 : (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

14. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is.

Ans. 3

Sol. $V_T \propto \dfrac{r^2\left[d_m - d_L\right]}{n}$

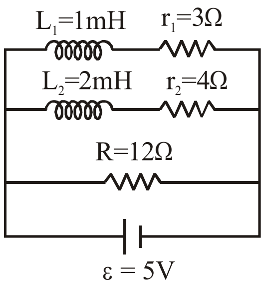
$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{r_P}{r_Q}\right)^2 \times \frac{n_{L_2}}{n_{L_1}} \times \left[\frac{d_m - d_{L_1}}{d_m - d_{L_2}}\right]$$

$$\frac{V_{TP}}{V_{TQ}} = \left(\frac{2}{1}\right)^2 \times \frac{2}{3} \times \left[\frac{8-0.8}{8-1.6}\right]$$

$$\frac{V_{TP}}{V_{TQ}} = 3$$

15. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is.

Ans. 8

Sol. 

$I_{max} = \frac{\varepsilon}{R} = \frac{5}{12}A$ (Initially at t = 0)

$I_{min} = \frac{\varepsilon}{R_{eq}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right)$ (finally in steady state)

$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right)$

$= \frac{10}{3}A$

$\frac{I_{max}}{I_{min}} = 8$

16. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus (${}^{12}_{6}C^*$) at 4.041 MeV above its ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$, the maximum kinetic energy of the β-particle in units of MeV is (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

Ans. 9

Sol. ${}^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}e + \bar{v}$

Mass defect = (12.014 – 12) u

∴ Released energy = 13.041 MeV

Energy used for excitation of ${}^{12}_{6}C$ = 4.041 MeV

∴ Energy converted to KE of electron

= 13.041 – 4.041 = 9 MeV

17. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking hc/e = 1.237 × 10^{-6} eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

Ans. 6

Sol. $\frac{hc}{\lambda} = \frac{12370}{970}$

$-13.6 + 12.7 = -\frac{13.6}{n^2}$

$n^2 = 16$

$n = 4$

Number of lines $= {}^{n}C_2 = 6$

18. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ?

Ans. 9

Sol. $P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1 \qquad \text{...(i)}$$

$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x \qquad \text{...(ii)}$$

(ii) – (i)

$$\log_2 \left(\frac{3040}{760}\right)^4 = x - 1$$

$\therefore\ x = 9$