## PART A - PHYSICS

1. The velocity of water in a river is $18 \mathrm{~km} / \mathrm{hr}$ near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The co-efficient of viscosity of water $=10^{-2}$ poise.
(1) $10^{-1} \mathrm{~N} / \mathrm{m}^{2}$
(2) $10^{-4} \mathrm{~N} / \mathrm{m}^{2}$
(3) $10^{-2} \mathrm{~N} / \mathrm{m}^{2}$
(4) $10^{-3} \mathrm{~N} / \mathrm{m}^{2}$

Ans. (4)
Sol. $\mathrm{F}=-\mathrm{nA} \mathrm{dv} / \mathrm{dx}$

$$
=-\left(10^{-3}\right)\left(\frac{5}{5}\right)=-10^{-3}
$$

now stress $=\mathrm{F} / \mathrm{A}=10^{-3}$
2. Four bulbs $\mathrm{B}_{1}, \mathrm{~B}_{2}, \mathrm{~B}_{3}$ and $\mathrm{B}_{4}$ of 100 W each are connected to 220 V main as shown in the figure. Ihe reading in an ideal ammeter will be

(1) 1.35 A
(2) 0.45 A
(3) 1.80 A
(4) 0.90 A

Ans. (1)
Sol. Resistance of any bull
$\mathrm{R}=\frac{\mathrm{V}^{2}}{\mathrm{P}}=\frac{(220)^{2}}{100}=484 \Omega$
Net resistance of the cks
$\mathrm{R}_{\mathrm{eq}}=\frac{\mathrm{R}}{4}=\frac{484}{4}=121 \Omega$
$\mathrm{v}=\mathrm{i} \mathrm{R}_{\mathrm{eq}}$
$220 \times \mathrm{i} \times 121$
$\mathrm{i}=1.81 \mathrm{Amp}$. (total current supplied by the battery)
Current in each branch in $\mathrm{i}=\frac{1.81}{4} \mathrm{amp}$.

$$
=0.45 \mathrm{amp}
$$

Reading of ammeter $=3 \times 0.45=1.3575 \mathrm{amp}$.
3. In a Young's double slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width. Then the number of intensity maxima observed within the central maximum of the single slit diffraction pattern is
(1) 12
(2) 24
(3) 3
(4) 6

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Ans. (1)
Sol. $\mathrm{d}=6.1 \mathrm{a}$
width of central maxima
$\mathrm{B}_{0}=2 \frac{\mathrm{D} \lambda}{\mathrm{a}}$
$\mathrm{n} \times \frac{\mathrm{D} \lambda}{\mathrm{d}}=\frac{2 \mathrm{D} \lambda}{\mathrm{a}}$
$\mathrm{n}=6.1 \times 2$ $=12$
4. A body is in simple harmonic motion with time period half second ( $\mathrm{T}=0.5 \mathrm{~s}$ ) and amplitude one $\mathrm{cm}(\mathrm{A}=1 \mathrm{~cm})$. Find the average velocity in the interval in which it moves from equilibrium position to half of its amplitude.
(1) $6 \mathrm{~cm} / \mathrm{s}$
(2) $4 \mathrm{~cm} / \mathrm{s}$
(3) $12 \mathrm{~cm} / \mathrm{s}$
(4) $16 \mathrm{~cm} / \mathrm{s}$

Ans. (3)
5. A bullet looses $\left(\frac{1}{n}\right)^{\text {th }}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be :
(1) $n$
(2) $\frac{n^{2}}{2 n-1}$
(3) Infinte
(4) $\frac{2 n^{2}}{n-1}$

Ans. (2)

Sol.

$\left(1-\frac{1}{\mathrm{n}}\right)^{2} \mathrm{~V}^{2}=\mathrm{V}^{2}-2 \mathrm{as}$

$$
\begin{aligned}
& 2 \mathrm{as}= \mathrm{V}^{2}\left(1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{2}\right)=\mathrm{V}^{2}\left(\frac{2 \mathrm{n}-1}{\mathrm{n}^{2}}\right) \\
& \mathrm{O}=\mathrm{V}^{2}-2 \mathrm{ans} \\
& \mathrm{n}=\frac{\mathrm{V}^{2}}{2 \mathrm{as}}=\frac{\mathrm{V}^{2}}{\mathrm{~V}^{2}\left(\frac{2 \mathrm{n}-1}{\mathrm{n}^{2}}\right)}=\frac{\mathrm{n}^{2}}{2 \mathrm{n}-1}
\end{aligned}
$$

6. Long range radio transmission is possible when the radiowaves are reflected from the ionosphere. For this to happen the frequency of the radiowaves must be in the range :
(1) $150-500 \mathrm{kHz}$
(2) $8-25 \mathrm{MHz}$
(3) $80-150 \mathrm{MHz}$
(4) $1-3 \mathrm{MHz}$

Ans. (2)
7. A ray of light is incident from a denser to a rarer medium. The critical angle for total internal reflection is $\theta_{\mathrm{iC}}$ and the Brewster's angle of incidence is $\theta_{\mathrm{iB}}$, such that $\sin \theta_{\mathrm{iC}} / \sin \theta_{\mathrm{iB}}=\eta=1.28$. The relative refractive index of the two media is :
(1) 0.4
(2) 0.9
(3) 0.8
(4) 0.2

Ans. (3)
8. If denote microwaves, $X$ rays, infrared, gamma rays, ultra-violet, radio waves and visible parts of the electromagnetic spectrum by $\mathrm{M}, \mathrm{X}, \mathrm{I}, \mathrm{G}, \mathrm{U}$, R and V , the following is the arrangement in ascending order of wavelength :
(1) M, R, V, X, U, G and I
(2) R, M, I, V, U, X and G
(3) G, X, U, V, I, M and R
(4) I, M, R, U, V, X and G

Ans. (3)
Sol. The desending order of energy for following waves $\mathrm{E}_{\mathrm{y}}>\mathrm{E}_{\mathrm{x}}>\mathrm{E}_{\mathrm{uv}}>\mathrm{E}_{\text {vissible }}>\mathrm{V}_{\mathrm{iR}}>\mathrm{V}_{\mathrm{MW}}$
9. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of $C^{14}$ is 5730 years, then age of the wooden piece placed in the museum is approximately :
(1) 19039 years
(2) 10439 years
(3) 39049 years
(4) 13094 years

Ans. (1)
Sol. use, $\mathrm{A}=\frac{\mathrm{A}_{0}}{2^{\mathrm{n}}}, \mathrm{n}=\frac{\mathrm{t}}{\mathrm{T}}=\frac{\mathrm{t}}{5730}$
$2=\frac{20}{2^{\mathrm{n}}}, \quad \frac{\mathrm{t}}{\mathrm{T}} \log 2=\log 10$
or $t=\frac{5730}{0.3010}=19039$ year
10. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plane of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of $10 \mathrm{~ms}^{-1}$ (see figure). The e.m.f induced at the time the left arm of the frame is at $\mathrm{x}=10 \mathrm{~cm}$ from the wire is :

(1) $1 \mu \mathrm{~V}$
(2) $0.5 \mu \mathrm{~V}$
(3) $2 \mu \mathrm{~V}$
(4) $0.75 \mu \mathrm{~V}$

Ans. (1)
Sol. $\mathrm{e}=\mathrm{B} \times \mathrm{v} \times \ell$

$$
\begin{aligned}
& =\left(\frac{\ell \mathrm{wi}}{2 \pi \mathrm{x}}\right) \times \mathrm{v} \times \ell \\
& =\frac{4 \pi \times 10^{-7} \times 1 \times 10 \times 10 \times 10^{-2}}{2 \pi \times 10 \times 10^{-2}} \\
& =2 \times 10^{-5} \times 10 \times 10^{-2} \\
& =2 \times 10^{-6} \mathrm{volt} \\
& =2 \mu \mathrm{v}
\end{aligned}
$$

11. 



Given : A and B are input terminals.
Logic $1=>5 \mathrm{~V}$
Logic $0=<1 \mathrm{~V}$
Which logic gate operation, the following circuit does?
(1) OR Gate
(2) XOR Gate
(3) AND Gate
(4) NOR Gate

Ans. (BONUS)
12. The gravitational field in a region is given by $\overrightarrow{\mathrm{g}}=5 \mathrm{~N} / \mathrm{kg} \hat{\mathrm{i}}+12 \mathrm{~N} / \mathrm{kg} \hat{\mathrm{j}}$. The change in the gravitational potential energy of a particle of mass 2 kg when it is taken from the origin to a point ( $7 \mathrm{~m},-3 \mathrm{~m}$ ) is :
(1) 1 J
(2) $13 \sqrt{58} \mathrm{~J}$
(3) -71 J
(4) 71 J

Ans. (BONUS)
Sol. $\Delta U=-\int \overrightarrow{\mathrm{E}}_{\mathrm{g}}-\overrightarrow{\mathrm{dr}}$

$$
\begin{aligned}
& =-\int(5 \hat{\mathrm{i}}+12 \hat{\mathrm{j}}) \cdot(\mathrm{dx} \hat{\mathrm{i}}+\mathrm{dy} \hat{\mathrm{j}}) \\
& =-\int_{0}^{7} 5 \mathrm{dx}-\int_{0}^{-3} 12 \mathrm{dy} \\
& =-5\left[7^{\circ}\right]-12[-3] \\
& =1 \mathrm{~J}
\end{aligned}
$$

$$
\text { Energy }=1 \times 2=2 \mathrm{~J}
$$

13. Match List-I (Event) with List-II (Order of the time interval for happening of the event) and select the correct option from the options given below the lists.

| List-I |  | List-II |  |
| :--- | :--- | :--- | :--- |
| (a) | Rotation period of earth | (i) | $10^{5} \mathrm{~s}$ |
| (b) | Revolution period of <br> earth | (ii) | $10^{7} \mathrm{~s}$ |
| (c) | Period of a light wave | (iii) | $10^{-15} \mathrm{~s}$ |
| (d) | Period of sound wave | (iv) | $10^{-3} \mathrm{~s}$ |

(1) (a)-(i), (b)-(ii), (c)-(iv), d-(iii)
(2) (a)-(ii), (b)-(i), (c)-(iv), d-(iii)
(3) (a)-(ii), (b)-(i), (c)-(iii), d-(iv)
(4) (a)-(i), (b)-(ii), (c)-(iii), d-(iv)

Ans. (4)
14. Match List-I (Experiment performed) with List-II (Phenomena discovered/ associated) and select the correct option from the options given below the lists :

| List-I |  | List-II |  |
| :--- | :--- | :--- | :--- |
| (a) | Davisson <br> and Germer <br> experiment | (i) | Wave nature <br> of electrons |
| (b) | Millikan's <br> oil drop <br> experiment | (ii) | Charge of an <br> electron |
| (c) | Rutherford <br> experiment | (iii) | Quantisation <br> of energy <br> levels |
| (d) | Franck <br> Hertz <br> experiment | (iv) | Existence of <br> nucleus |

(1) (a)-(i), (b)-(ii), (c)-(iii) (d)-(iv)
(2) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
(3) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(4) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

Ans. (2)
Sol. (a) Davisson - germar give experimental verification for wave nature of electron.
(b) Millikna's formed experiment about change of an electron
(c) Rutherford performed gold foil experiment and found the exisitance of nucleus.
(d) Franck - Hertz gives information about quantisation of energy level.
15. A heavy box is to be dragged along a rough horizontal floor. To do so, person A pushes it at an angle $30^{\circ}$ from the horizontal and requires a minimum force $F_{A}$, while person $B$ pulls the box at an angle $60^{\circ}$ from the horizontal and needs minimum force $F_{B}$. If the coefficient of friction between the box and the floor is $\frac{\sqrt{3}}{5}$, the ratio $\frac{\mathrm{F}_{\mathrm{A}}}{\mathrm{F}_{\mathrm{B}}}$ is :
(1) $\sqrt{\frac{3}{2}}$
(2) $\frac{2}{\sqrt{3}}$
(3) $\frac{5}{\sqrt{3}}$
(4) $\sqrt{3}$

Ans. (2)

Sol.

16. A gas is compressed from a volume of $2 \mathrm{~m}^{3}$ to a volume of $1 \mathrm{~m}^{3}$ at a constant pressure of $100 \mathrm{~N} / \mathrm{m}^{2}$. Then it is heated at constant volume by supplying 150 J of energy. As a result, the internal energy of the gas :
(1) Increases by 250 J
(2) Decreases by 50 J
(3) Decreases by 250 J
(4) Increases by 50 J

Ans. (BONUS)
Sol. Case $1 \quad \mathrm{~F} \cos 30=\left(m g+F_{A} \sin 30\right) \mu$
$\Rightarrow F_{A}=\frac{\mu m g}{\cos 30-\mu \sin 30}$
Case $2 \quad F_{B} \cos 60=(m g-f \sin 60) \mu$

$$
\begin{equation*}
\Rightarrow \quad \mathrm{F}_{\mathrm{B}}=\frac{\mu \mathrm{mg}}{\cos 60+\mu \sin 60} \tag{ii}
\end{equation*}
$$

Dividing (1) by (2) we get

$$
\frac{\mathrm{F}_{\mathrm{A}}}{\mathrm{~F}_{\mathrm{B}}}=\frac{2}{\sqrt{3}}
$$

17. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle $\theta$ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle $\phi$ with the horizontal. Then :

(1) $\sin \phi=\cos \theta$
(2) $\sin \phi=\frac{1}{2} \cos \theta$
(3) $\sin \phi=\frac{3}{4} \cos \theta$
(4) $\sin \phi=\frac{2}{3} \cos \theta$

Ans. (4)

Sol.

$m g r \cos \theta=\frac{1}{2} m v^{2}+r g \sin \phi$
$\frac{\mathrm{v}^{2}}{\mathrm{rg}}=2 \cos \theta-2 \sin \phi$
$\sin \phi=2 \cos \theta-2 \sin \phi$
$3 \sin \phi=2 \cos \theta$
$\sin \phi=\frac{2}{3} \cos \phi$
18. In the diagram shown, the difference in the two tubes of the manometer is 5 cm , the cross section of the tube at $A$ and $B$ is $6 \mathrm{~mm}^{2}$ and $10 \mathrm{~mm}^{2}$ respectively. The rate at which water flows through the tube is $\left(\mathrm{g}=10 \mathrm{~ms}^{-2}\right)$

(1) $10.0 \mathrm{cc} / \mathrm{s}$
(2) $7.5 \mathrm{cc} / \mathrm{s}$
(3) $8.0 \mathrm{cc} / \mathrm{s}$
(4) $12.5 \mathrm{cc} / \mathrm{s}$

Ans. (2)
Sol. $\mathrm{Q}=\mathrm{A}_{1} \mathrm{~A}_{2} \sqrt{\frac{2 \mathrm{~g}\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)}{\left(\mathrm{A}_{1}^{2}-\mathrm{A}_{2}^{2}\right)}}$

$$
=6 \times 10 \times 10^{-4} \sqrt{\frac{2 \times 980(5)}{(-36+100) \times 10^{-2}}}
$$

$=6 \times 10^{-3} \sqrt{\frac{9800 \times 10^{2}}{64}}$

$$
=\frac{6 \times 10^{-3} \times 1000}{8}=7.5
$$

19. Consider a cylinder of mass $M$ resting on a rough horizontal rug that is pulled out from under it with acceleration 'a' perpendicular to the axis of the cylinder. What is $\mathrm{F}_{\text {friction }}$ at point P ? It is assumed that the cylinder does not slip.

(1) Mg
(2) $\frac{\mathrm{Ma}}{3}$
(3) Ma
(4) $\frac{\mathrm{Ma}}{2}$

Ans. (2)

Sol.

$\mathrm{Ma}=\mathrm{f}$
20. A large number of liquid drops each of radius $r$ coalesce to from a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop so formed. The speed of the big drop is (given surface tension of liquid $T$, density $\rho$ )
(1) $\sqrt{\frac{4 T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$
(2) $\sqrt{\frac{T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$
(3) $\sqrt{\frac{2 \mathrm{~T}}{\rho}\left(\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{R}}\right)}$
(4) $\sqrt{\frac{6 T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)}$

Ans. (4)
Sol. T $4 \pi R^{3}\left\{\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{R}}\right\}=\frac{1}{2}\left(\frac{4}{3} \pi \mathrm{R}^{3} \mathrm{~d}\right)$
$V=\sqrt{\frac{6 T}{d}\left\{\frac{1}{r}-\frac{1}{R}\right\}}$
21. The gap between the plates of a parallel plate capacitor of area A and distance between plates d , is filled with a dielectric whose permittivity varies linearly from $\epsilon_{1}$ at one plate to $\epsilon_{2}$ at the other. The capacitance of capacitor is :
(1) $\epsilon_{0}\left(\epsilon_{2}+\epsilon_{1}\right) A / 2 d$
(2) $\in_{0} \mathrm{~A} /\left[\mathrm{d} \ell \mathrm{n}\left(\epsilon_{2} / \epsilon_{1}\right)\right]$
(3) $\epsilon_{0}\left(\epsilon_{1}+\epsilon_{2}\right) A / d$
(4) $\epsilon_{0}\left(\epsilon_{2}-\epsilon_{1}\right) \mathrm{A} /\left[\mathrm{d} \ell \mathrm{n}\left(\epsilon_{2} / \epsilon_{1}\right)\right]$

Ans. (4)
Sol. $\quad \varepsilon=\left(\frac{\varepsilon_{2}-\varepsilon_{1}}{\mathrm{~d}}\right) \mathrm{x}+\varepsilon_{1}$
$\mathrm{dC}=\frac{\varepsilon_{0} \varepsilon \mathrm{~A}}{\mathrm{dx}}$
$C_{e q}=\int d c$
22. A gas molecule of mass $M$ at the surface of the Earth has kinetic energy equivalent to $0^{\circ} \mathrm{C}$. If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than radius of the earth, ( $\mathrm{k}_{\mathrm{B}}$ is Boltzmann constant)
(1) $\frac{819 \mathrm{k}_{\mathrm{B}}}{2 \mathrm{Mg}}$
(2) $\frac{546 \mathrm{k}_{\mathrm{B}}}{3 \mathrm{Mg}}$
(3) 0
(4) $\frac{273 \mathrm{k}_{\mathrm{B}}}{2 \mathrm{Mg}}$

Ans. (1)
Sol. $\frac{3}{2} \mathrm{KT}=\frac{1}{2} \mathrm{mv}^{2}$
$v^{2}=\frac{3 K T}{m}$
$v^{2}=u^{2}+2 a s$
$\mathrm{O}=\frac{3 \mathrm{KT}}{\mathrm{m}}-2 \mathrm{gs}$
$S=\frac{3 K}{2} \frac{(273)}{m g}$
$S=\frac{819 K}{2 m g}$
23. The electric field in a region of space is given by, $\vec{E}=E_{0} \hat{i}+2 E_{0} \hat{\mathrm{j}}$ where $E_{0}=100 \mathrm{~N} / \mathrm{C}$. The flux of this field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly
(1) $0.02 \mathrm{Nm}^{2} / \mathrm{C}$
(2) $0.125 \mathrm{Nm}^{2} / \mathrm{C}$
(3) $3.14 \mathrm{Nm}^{2} / \mathrm{C}$
(4) $0.005 \mathrm{Nm}^{2} / \mathrm{C}$

Ans. (2)
Sol. $\phi=\left(\mathrm{E}_{0} \hat{\mathrm{i}}+2 \mathrm{E}_{0} \hat{\mathrm{j}}\right) \cdot\left(\pi \mathrm{R}^{2}\right) \hat{\mathrm{i}}$
$\phi=100 \times \pi \times(0.02)^{2}$
$=0.125$
24. A ball of mass 160 g is thrown up at an angle of $60^{\circ}$ to the horizontal at a speed of $10 \mathrm{~ms}^{-1}$. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly $\left(\mathrm{g}=10 \mathrm{~ms}^{-2}\right)$
(1) $3.0 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}$
(2) $3.46 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}$
(3) $1.73 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}$
(4) $6.0 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}$

Ans. (1)

Sol. $\overline{\mathrm{L}}=\overline{\mathrm{r}} \times \overline{10}$

$=\frac{160}{1000} \times 10 \times \frac{1}{2} \times \frac{10 \times 10}{2 \times 10}\left(\frac{\sqrt{3}}{2}\right)^{2}$
$=10 \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{4}=3$


In an experiment for determining the gravitational acceleration $g$ of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure.
What is the value of $g$ at the place ?
(1) $9.91 \mathrm{~m} / \mathrm{s}^{2}$
(2) $10.0 \mathrm{~m} / \mathrm{s}^{2}$
(3) $9.87 \mathrm{~m} / \mathrm{s}^{2}$
(4) $9.81 \mathrm{~m} / \mathrm{s}^{2}$

Ans. (3)
Sol. $\mathrm{T}=2 \pi \sqrt{\frac{\ell}{\mathrm{~g}}}$
$\mathrm{T}^{2}=4 \pi^{2} \frac{\ell}{\mathrm{~g}}$
$\mathrm{g}=4 \pi^{2} \frac{\ell}{\mathrm{~T}^{2}}$
$\mathrm{T}^{2}=\frac{4 \pi^{2}}{\mathrm{~g}}$
$\frac{4 \pi^{2}}{\mathrm{~g}}=4$

$$
\mathrm{g}=\pi^{2}=9.87
$$

26. An example of a perfect diamagnet is a superconductor. This implies that when a superconductor is put in a magnetic field of intensity B , the magnetic field Bs inside the superconductor will be such that :
(1) $\mathrm{B}_{\mathrm{s}}<\mathrm{B}$ but $\mathrm{B}_{\mathrm{s}} \neq 0$
(2) $B_{s}=B$
(3) $\mathrm{B}_{\mathrm{s}}=0$
(4) $B_{s}=-B$

Ans. (4)
Sol.
27. The total length of a sonometer wire between fixed ends is 110 cm . Two bridges are placed to divide the length of wire in ratio $6: 3: 2$. The tension in the wire is 400 N and the mass per unit length is $0.01 \mathrm{~kg} / \mathrm{m}$. What is the minimum common frequency with which three parts can vibrate?
(1) 166 Hz
(2) 1000 Hz
(3) 1100 Hz
(4) 100 Hz

Ans. (2)
Sol. $\ell_{1}: \ell_{2}: \ell_{3}=6: 3: 2$
so $\ell_{1}=60 \mathrm{~cm}$
$\ell_{2}=30 \mathrm{~cm}$
$\ell_{3}=20 \mathrm{~cm}$
60, 30, 20
$\frac{\lambda}{2}=10 \mathrm{~cm}$
$\mathrm{f}=\frac{200}{2}=1000 \mathrm{~Hz}$
28. A black coloured solid sphere of radius $R$ and mass $M$ is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature $\mathrm{T}_{0}$. The initial temperature of the sphere is $3 \mathrm{~T}_{0}$. If the specific heat of the material of the sphere varies as $\alpha \mathrm{T}^{3}$ per unit mass with the temperature T of the sphere, where $\alpha$ is a constant, then the time taken for the sphere to cool down to temperature $2 \mathrm{~T}_{0}$ will be
( $\sigma$ is Stefan Boltzmann constant)
(1) $\frac{\mathrm{M} \alpha}{16 \pi \mathrm{R}^{2} \sigma} \ell \mathrm{n}\left(\frac{16}{3}\right)$
(2) $\frac{\mathrm{M} \alpha}{16 \pi \mathrm{R}^{2} \sigma} \ln \left(\frac{3}{2}\right)$
(3) $\frac{\mathrm{M} \alpha}{4 \pi \mathrm{R}^{2} \sigma} \ln \left(\frac{16}{3}\right)$
(4) $\frac{\mathrm{M} \alpha}{4 \pi \mathrm{R}^{2} \sigma} \ln \left(\frac{3}{2}\right)$

Ans. (1)
Sol. $\frac{\mathrm{d} \theta}{\mathrm{dt}}=-\frac{\sigma \mathrm{Ae}}{\mathrm{ms}}\left(\theta^{4}-\theta_{0}{ }^{4}\right)$

$$
\frac{\mathrm{d} \theta}{\mathrm{dt}}=-\frac{\sigma \mathrm{Ae}}{\mathrm{~m}^{-} \alpha \theta^{3}}\left(\theta^{4}-\theta_{0}{ }^{4}\right)
$$


$\int \frac{\theta^{3}}{\theta^{4}-\theta_{0}{ }^{4}} \mathrm{~d} \theta=-\frac{\sigma \mathrm{Ae}}{\mathrm{m}^{-} \alpha} \int \mathrm{dt}$
let $\theta^{4}-\theta_{0}^{4}=x$
$4 \theta^{3} d \theta=d x$
$\int \frac{\mathrm{dx}}{4 \mathrm{x}}=-\frac{\sigma A \mathrm{e}}{\mathrm{m}^{2} \alpha} \mathrm{t}$
$\frac{1}{4}[\log \mathrm{x}]=-\frac{\sigma \mathrm{Ae}}{\mathrm{m}^{-} \alpha}[\mathrm{t}]_{0}^{\mathrm{t}}$
$\frac{1}{4}\left[\log \left(\theta^{4}-\theta_{0}^{4}\right)\right]_{3 \mathrm{~T}_{0}}^{2 \mathrm{~T}_{0}}=-\frac{\sigma A \mathrm{Ae}}{\mathrm{m}^{-} \alpha} \mathrm{t}$
$\frac{1}{4} \log \left(\frac{\left(2 \mathrm{~T}_{0}\right)^{4}-\mathrm{T}_{0}^{4}}{\left(3 \mathrm{~T}_{0}\right)^{4}-\mathrm{T}_{0}^{4}}\right)=-\frac{\sigma \mathrm{Ae}}{\mathrm{m}^{-} \alpha} \mathrm{t}$
$\mathrm{t}=-\frac{\alpha \mathrm{m}}{4 \sigma \mathrm{Ae}} \ln \frac{\mathrm{T}_{0}^{4}}{80 \mathrm{~T}_{0}^{4}}$
$=\frac{\alpha \mathrm{M}}{4 \sigma \mathrm{Ae}} \ln \frac{3}{16}$
$=\frac{\alpha M}{4 \sigma\left(4 \pi R^{2}\right) \mathrm{e}} \ln \frac{3}{16}$
$\mathrm{t}=\mathrm{t}=\frac{\alpha \mathrm{M}}{16 \pi \sigma \mathrm{R}^{2}} \ln \frac{16}{3}$
29. Figure shows a circular area of radius R where a uniform magnetic field $\vec{B}$ is going into the plane of paper and increasing in magnitude at a constant
 rate. In that case, which of the following graphs, drawn schematically, correctly shows the variation of the induced electric field $\mathrm{E}(\mathrm{r})$ ?
(1)

(2)

(3)

(4)


Ans. (4)
Sol. $B=B_{0} t$
$r>R$
$\phi \bar{\varepsilon} \cdot \mathrm{de}=\frac{\mathrm{d} \phi}{\mathrm{dt}}$

$\Rightarrow \varepsilon(2 \pi r)=B \pi r^{2}$
$\mathrm{e} \propto \mathrm{r}$
$r>R$
$\varepsilon(2 \pi \mathrm{r})=\mathrm{B} \pi \mathrm{R}^{2}$
$\varepsilon \propto \frac{1}{\mathrm{r}}$

30. The diameter of the objective lens of microscope makes an angle $\beta$ at the focus of the microscope. Further, the medium between the object and the lens is an oil of refractive index $n$. Then the resolving power of the microscope.
(1) Increases with decreasing value of $n$
(2) Increases with increasing value of $\frac{1}{n \sin 2 \beta}$
(3) Increases with decreasing value of $\beta$
(4) Increases with increasing value of $n \sin 2 \beta$

Ans. (4)

