Mathematics

(Chapter – 2) (Polynomials)

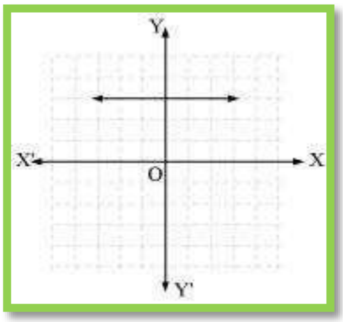
(Class – X)

Exercise 2.1

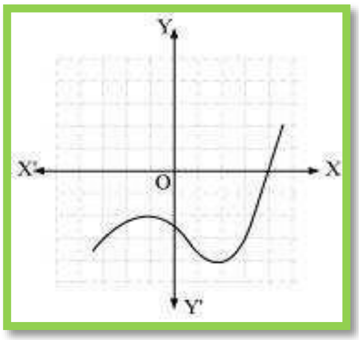
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

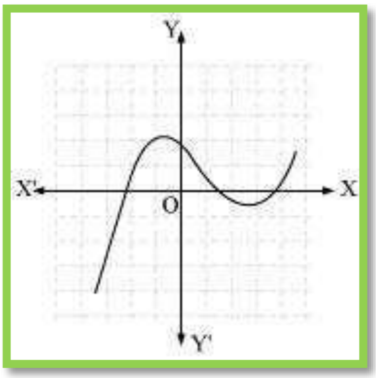
(i)



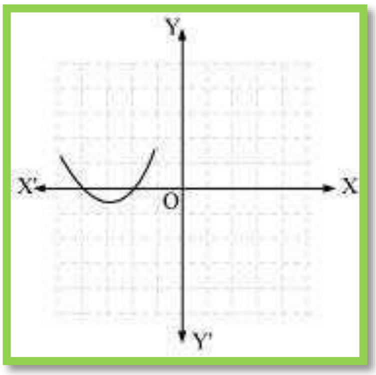
(ii)



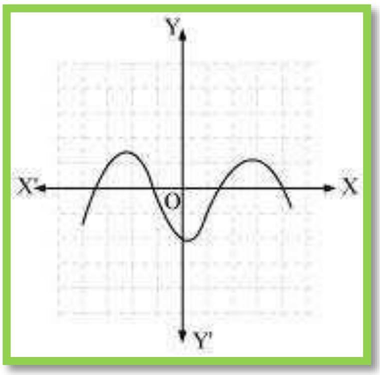
(iii)



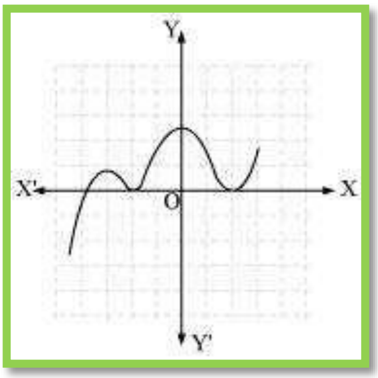
(iv)



(v)



(v)



Answer 1:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Mathematics

(Chapter – 2) (Polynomials)
(Class X)

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8$ $\quad$ $(\text{ii})\, 4s^2-4s+1$ $\quad$ $(\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u$ $\quad$ $(\text{v})\, t^2-15$ $\quad$ $(\text{vi})\, 3x^2-x-4$

Answer 1:

$(\text{i}) \quad x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x-4=0$ or $x+2=0$, i.e., when $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$(\text{ii}) \quad 4s^2-4s+1=(2s-1)^2$

The value of $4s^2-4s+1$ is zero when $2s-1=0$, i.e., $s=\frac{1}{2}$ Therefore,

the zeroes of $4s^2-4s+1$ are ½ and ½ .

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

$(\text{iii}) \quad 6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when $3x+1=0$ or $2x-3=0$, i.e.,

$x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\begin{aligned}(\text{iv})\quad & 4u^2 + 8u = 4u^2 + 8u + 0 \\ & = 4u(u+2)\end{aligned}$$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e.,
$u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

$$\begin{aligned}(\text{v})\quad & t^2 - 15 \\ & = t^2 - 0t - 15 \\ & = (t - \sqrt{15})(t + \sqrt{15})\end{aligned}$$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when
$t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-(\text{Coefficient of } t)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes = $\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

$=(3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e.,

when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are 4/3 and −1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer 2:

(i) $\frac{1}{4},-1$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
$$\text{If } a = 4, \text{ then } b = -1,\ c = -4$$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$
$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$
$$\text{If } a = 3, \text{ then } b = -3\sqrt{2},\ c = 1$$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$
$$\text{If } a = 1, \text{ then } b = 0,\ c = \sqrt{5}$$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
$$\text{If } a=1, \text{ then } b=-1,\ c=1$$

Therefore, the quadratic polynomial is x^2-x+1.

$(\mathrm{v})\quad -\frac{1}{4},\frac{1}{4}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$
$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$
$$\text{If } a=4, \text{ then } b=1,\ c=1$$

Therefore, the quadratic polynomial is $4x^2+x+1$.

$(\mathrm{vi})\quad 4,\ 1$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
$$\text{If } a=1, \text{ then } b=-4,\ c=1$$

Therefore, the quadratic polynomial is x^2-4x+1.

Mathematics

(Chapter – 2) (Polynomials)

(Class – X)

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

Answer 1:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$

$q(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\quad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \qquad + 6 \\
 & + \qquad\quad - \\
\hline
 & 7x - 9 \\
\hline
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$

$q(x) = x^2 + 1 - x = x^2 - x + 1$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\
\hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\
\hline
 & 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2 - 5x + 6 \\
 & \quad 2x^2 \qquad - 4 \\
 & \quad - \qquad\quad + \\
\hline
 & \qquad -5x + 10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer 2:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0.t - 3$

$$
\begin{array}{r}
2t^2 + 3t + 4 \\
t^2 + 0.t - 3 \overline{\big)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
2t^4 + 0.t^3 - 6t^2 \qquad\qquad \\
- \quad - \quad + \qquad\qquad\quad \\
\hline
3t^3 + 4t^2 - 9t - 12 \\
3t^3 + 0.t^2 - 9t \qquad \\
- \quad - \quad + \qquad\quad \\
\hline
4t^2 + 0.t - 12 \\
4t^2 + 0.t - 12 \\
- \quad - \quad + \quad \\
\hline
0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1, \; 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r}
3x^2 - 4x + 2 \qquad\qquad\quad \\
x^2 + 3x + 1 \overline{\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
3x^4 + 9x^3 + 3x^2 \qquad\qquad \\
- \quad - \quad - \qquad\qquad\quad \\
\hline
-4x^3 - 10x^2 + 2x + 2 \\
-4x^3 - 12x^2 - 4x \quad \\
+ \quad + \quad + \qquad \\
\hline
2x^2 + 6x + 2 \\
2x^2 + 6x + 2 \\
\hline
0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$\begin{array}{r|l}
 & x^2-1 \\
\hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & -\quad+\quad- \\
\hline
 & -x^3\qquad +3x+1 \\
 & -x^3\qquad +3x-1 \\
 & +\qquad\quad -\quad + \\
\hline
 & \qquad\qquad\quad 2 \\
\hline
\end{array}$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer 3:

$$p(x)=3x^4+6x^3-2x^2-10x-5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & \quad 6x^3+3x^2-10x-5 \\
 & \quad 6x^3+0x^2-10x \\
 & \quad -\quad -\quad + \\
\hline
 & \qquad 3x^2+0x-5 \\
 & \qquad 3x^2+0x-5 \\
 & \qquad -\quad -\quad + \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}
$$

$$
\begin{aligned}
3x^4+6x^3-2x^2-10x-5 &= \left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right) \\
&= 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)
\end{aligned}
$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$ or $x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$ -1 and -1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer 4:

$p(x)=x^3-3x^2+x+2 \qquad \text{(Dividend)}$

$g(x)$ = ? (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x-2)$

$$\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Answer 5:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x) =$ deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm, $p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = (2)\ (3x^2 + x + 1) + 0$

Thus, the division algorithm is satisfied.

(ii) deg $q(x) =$ deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$ $g(x) = x^2$ $q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1. Checking for division algorithm, $p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$ $x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$ $g(x) = x^2$ $q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$ $x^3 + 1 = (x^2) \times x + 1$ $x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Mathematics

(Chapter – 2) (Polynomials)

(Class – X)

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer 1:

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, ½ , 1, and −2 are the zeroes of the given polynomial.
Comparing the given polynomial with $ax^3 + bx^2 + cx + d$,

we obtain $a = 2$, $b = 1$, $c = -5$, $d = 2$

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$,

we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time

= (2)(1) + (1)(1) + (2)(1) =2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer 2:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If a = 1, then b = −2, c = −7, d = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial , $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$ find a and b.

Answer 3:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are $a - b$, $a + a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

p = 1, q = −3, r = 1, t = 1

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1-b, 1, 1+b$

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, a = 1 and $b = \sqrt{2}$ or $-\sqrt{2}$

Question 4:

]It two zeroes of the polynomial , $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ find other zeroes.

Answer 4:

Given 2 + √3 and 2 – √3 are zeroes of the given polynomial.

So, (2 + √3)(2 – √3) is a factor of polynomial.

Therefore,[x – (2 + √3)][x – (2 – √3)] = $x^2 + 4 - 4x - 3$

= $x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & -\quad +\quad - \\
\cline{2-2}
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-2x \\
 & +\quad -\quad + \\
\cline{2-2}
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & +\quad -\quad + \\
\cline{2-2}
 & 0
\end{array}
$$

Clearly, $x^4-6x^3-26x^2+138x-35 = \left(x^2-4x+1\right)\left(x^2-2x-35\right)$

$\left(x^2-2x-35\right)$ is also a factor of the given

It can be observed that polynomial $\left(x^2-2x-35\right) = (x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ or $x+5=0$

Or x = 7 or −5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial, x^2-2x+k the remainder comes out to be $x + a$, find k and a.

Answer 5:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}
$$

It can be observed that $(-10+2k)x + (10 - a - 8k + k^2)$ Will be 0.

Therefore, $(-10+2k)$ = 0 and $(10 - a - 8k + k^2)$ = 0

For $(-10+2k)$ = 0, 2 k =10 And thus, k = 5

For $(10 - a - 8k + k^2)$ = 0

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$